WALAILAK JOURNAL
http://wjst.wu.ac.th Applied Mathematics

First Integral Method for Systems of (1+1)-Dimensional Dispersive Long Wave

Jafar BIAZAR* **and Mohamad Bagher MEHRLATIFAN**

Department of Applied Mathematics, Faculty of Mathematical Sciences, University of Guilan, Rasht, Iran

(*Corresponding author's e-mail: biazar@guilan.ac.ir)

Received: 12 January 2014, Revised: 10 May 2014, Accepted: 11 June 2014

Abstract

The First Integral method (FIM) is applied to solve a dispersive long wave system. In this method the division theorem, as a statement in commutative Algebra has an important role. To show the ability and the efficiency of this approach an example is provided. Application of FIM to the illustrative example leads to six exact solutions, which it is shown that these six solutions are independent to each other. So there are six different exact solutions.

Keywords: First Integral method, exact solution, dispersive long wave (1+1)-dimensional systems, partial differential equation, non-linear algebraic equations

Introduction

The investigation of exact solutions of non-linear problems has become an interesting subject in the field of non-linear sciences. Nonlinear evolution equations are appeared and studied in different disciplines, such as; Physics, Biology, Chemistry, Plasma, Optical fibers, and computer technology. during the recent years, some other methods have been suggested such as: the Backlund transformation method [1], Darboux transformation [2], Tanh method [3], Extended tanh method [4], Modified extended tanh method [5], Generalized hyperbolic function [6], Variable separation method [7], and First Integral method, first proposed by Feng [8]. Recently, this method is used for some non-linear systems of partial differential equations [9]. This powerful method is widely used by many researchers, for example, [10,11] and references therein.

From our point of view, all these methods have some merits and demerits with respect to the problem considered and there is no unified method that can be used to deal with all types of non-linear partial differential equations. That is why any time that an improvement is made in a particular method to allow it to recover some new solutions to the non-linear partial differential equations, it is always welcomed. The purpose of this paper is to apply a FIM to coupled families of non-linear partial differential equations.

The paper is organized as follows: In Section 2, we briefly present the steps of the FIM. In Section 3, by using the results obtained in Section 2, attempts are made to apply the method to solve the dispersive long wave (1+1)-dimensional systems.

This paper is aimed at finding exact soliton solutions for a system of (1+1)-dimensional dispersive long wave, by the FIM. The dispersive long wave equation has appeared in some studies [12].

First integral method

Consider the following general form of a non-linear partial differential equation;

$$\boldsymbol{F(u, u_t, u_x, u_{tt}, u_{xx}, u_{tx}, \dots) = 0}, \tag{1}$$

where $\boldsymbol{u} = \boldsymbol{u}(\boldsymbol{x}, \boldsymbol{t})$, is the solution of the non-linear PDE of Eq. (1). Furthermore, the following transformations are used to convert the non-linear PDE in (1), into a non-linear ODE;

$$\boldsymbol{u}(\boldsymbol{x}, \boldsymbol{t}) = \boldsymbol{u}(\boldsymbol{\xi}), \qquad \boldsymbol{\xi} = \boldsymbol{x} - \boldsymbol{ct}. \tag{2}$$

where $\boldsymbol{c}$ is a constant.

$$\boldsymbol{G}(\boldsymbol{u}(\boldsymbol{\xi}), \boldsymbol{u}'(\boldsymbol{\xi}), \boldsymbol{u}''(\boldsymbol{\xi}), \dots) = \mathbf{0}. \tag{3}$$

If in this equation, the highest order derivatives here $\boldsymbol{u}''(\boldsymbol{\xi})$, can be presented in terms of the lower derivatives, then by introducing two independent variables;

$$\boldsymbol{X}(\boldsymbol{\xi}) = \boldsymbol{u}(\boldsymbol{\xi}), \qquad \boldsymbol{Y}(\boldsymbol{\xi}) = \frac{\partial \boldsymbol{u}(\boldsymbol{\xi})}{\partial \boldsymbol{\xi}}. \tag{4}$$

The nonlinear Eq. (4) can be presented as the following system.

$$\begin{aligned} &\boldsymbol{X}_{\boldsymbol{\xi}}(\boldsymbol{\xi}) = \boldsymbol{Y}(\boldsymbol{\xi}), \\ &\boldsymbol{Y}_{\boldsymbol{\xi}}(\boldsymbol{\xi}) = \boldsymbol{F}_{\mathbf{1}}\big(\boldsymbol{X}(\boldsymbol{\xi}), \boldsymbol{Y}(\boldsymbol{\xi})\big). \end{aligned} \tag{5}$$

If the integrals of Eq. (5) are under the same conditions of the qualitative theory theorem, for ordinary differential equations [13], then general solutions to (5) can be obtained directly. However, it is so difficult to realize this even for the first integral, because for a given plane autonomous system neither is there a systematic theory to find its integrals, nor a logical way that tells us what the first integrals are. Thus, Division Theorem is used to obtain a first integral of Eq. (5). Let us recall the Division Theorem:

Division theorem:

Suppose that $\boldsymbol{P}(\boldsymbol{w}, \boldsymbol{z})$ and $\boldsymbol{q}(\boldsymbol{w}, \boldsymbol{z})$ are polynomials in $\boldsymbol{C}[\boldsymbol{w}, \boldsymbol{z}]$, and $\boldsymbol{P}(\boldsymbol{w}, \boldsymbol{z})$ which is irreducible in $\boldsymbol{C}[\boldsymbol{w}, \boldsymbol{z}]$. If $\boldsymbol{q}(\boldsymbol{w}, \boldsymbol{z})$ vanishes through all zero points of $\boldsymbol{P}(\boldsymbol{w}, \boldsymbol{z})$, then there exists a polynomial $\boldsymbol{G}(\boldsymbol{w}, \boldsymbol{z})$ in $\boldsymbol{C}(\boldsymbol{w}, \boldsymbol{z})$ such that;

$$\boldsymbol{q}(\boldsymbol{w}, \boldsymbol{z}) = \boldsymbol{P}(\boldsymbol{w}, \boldsymbol{z})\boldsymbol{G}(\boldsymbol{w}, \boldsymbol{z}). \tag{6}$$

Division theorem [14].

Application

In this section, it is aimed to solve the dispersive long wave (1+1)-dimensional system, written in the following form;

$$\begin{aligned} &\boldsymbol{u}_{\boldsymbol{t}} = \left(\boldsymbol{u}^{\mathbf{2}} - \boldsymbol{u}_{\boldsymbol{x}} + \mathbf{2}\boldsymbol{w}\right)_{\boldsymbol{x}}, \\ &\boldsymbol{w}_{\boldsymbol{t}} = (\mathbf{2}\boldsymbol{u}\boldsymbol{w} + \boldsymbol{w}_{\boldsymbol{x}})_{\boldsymbol{x}}. \end{aligned} \tag{7}$$

Minasari and Ganji solved this equation by the Homotopy Perturbation method [15]. By using the transformations $\boldsymbol{u}(\boldsymbol{x}, \boldsymbol{t}) = \boldsymbol{u}(\boldsymbol{\xi})$, $\boldsymbol{w}(\boldsymbol{x}, \boldsymbol{t}) = \boldsymbol{w}(\boldsymbol{\xi})$, and $\boldsymbol{\xi} = \boldsymbol{x} - \boldsymbol{ct}$, Eq. (7) changes into;

$$\begin{aligned} &-\boldsymbol{c}\boldsymbol{u}_{\boldsymbol{\xi}} = (\boldsymbol{u}^{\mathbf{2}}(\boldsymbol{\xi}) - \boldsymbol{u}_{\boldsymbol{\xi}}(\boldsymbol{\xi}) + \mathbf{2}\boldsymbol{w}(\boldsymbol{\xi}))_{\boldsymbol{\xi}}, \\ &-\boldsymbol{c}\boldsymbol{w}_{\boldsymbol{\xi}} = (\mathbf{2}\boldsymbol{u}(\boldsymbol{\xi})\boldsymbol{w}(\boldsymbol{\xi}) + \boldsymbol{w}_{\boldsymbol{\xi}}(\boldsymbol{\xi}))_{\boldsymbol{\xi}}. \end{aligned} \tag{8}$$

Integrating the first equation in (8), once with respect to $\boldsymbol{\xi}$, results in;

$$\boldsymbol{w(\xi) = \tfrac{1}{2}\left(-cu(\xi) + u_\xi(\xi) - u^2(\xi) + r_1\right)}. \tag{9}$$

Substituting Eq. (9) into the second Eq. in (8), results in;

$$\boldsymbol{-\tfrac{1}{2}c\left(-cu - u^2 + u_\xi\right)_\xi = \left(\tfrac{1}{2}\left(-cu - u^2 - u_\xi\right)_\xi + r_1 u + u\left(-cu - u^2 + u_\xi\right)\right)_\xi}, \tag{10}$$

By integrating (10), the following equation will be achieved;

$$\boldsymbol{u_{\xi\xi} = (c^2 - 2r_1)u + 3cu^2 + 2u^3 + 2r_2}. \tag{11}$$

where $\boldsymbol{r_1}$ and $\boldsymbol{r_2}$ are the integration constants, that should be determined. According to the first integral method, by using (4) and (5), one gets;

$$\begin{aligned} &\boldsymbol{\dot{X}(\xi) = Y(\xi)} \\ &\boldsymbol{\dot{Y}(\xi) = (c^2 - 2r_1)X(\xi) + 3cX^2(\xi) + \ 2X^3(\xi) + 2r_2} \end{aligned} \tag{12}$$

Suppose that $\boldsymbol{X(\xi)}$ and $\boldsymbol{Y(\xi)}$ are nontrivial solutions of (12) and $\boldsymbol{q[X, Y]}$ is an irreducible polynomial in the complex domain $\boldsymbol{C[X, Y]}$, such that;

$$\boldsymbol{q[X(\xi), Y(\xi)] = \textstyle\sum_{i=0}^{m} a_i(X)\, Y^i = 0}, \tag{13}$$

where $\boldsymbol{a_i(X)}$ is a polynomial of $\boldsymbol{X}$ and $\boldsymbol{a_m(X) \neq 0}$. Eq. (13) is the first integral to (12). Due to the Division Theorem, there exists a polynomial $\boldsymbol{g(X) + h(X)Y}$, in the complex domain $\boldsymbol{C[X, Y]}$, such that;

$$\boldsymbol{\frac{dq}{d\xi} = \frac{dq}{dX}\frac{dX}{d\xi} + \frac{dq}{dY}\frac{dY}{d\xi} = (g(X)h(X)Y) \textstyle\sum_{i=0}^{m} a_i(X)\, Y^i}. \tag{14}$$

Here, two different values for $\boldsymbol{m}$ are considered, ($\boldsymbol{m = 1, 2}$).

Case 1:

By assuming $\boldsymbol{m = 1}$, in Eq. (14), and by equating the coefficients of $\boldsymbol{Y^i}$ ($\boldsymbol{i = 2, 1, 0}$), on both sides of the Eq. (14), one gets;

$$\boldsymbol{\dot{a}_1(X) = a_1(X)h(X)}, \tag{15}$$

$$\boldsymbol{\dot{a}_0(X) = a_1(X)g(X) + a_0(X)h(X)}, \tag{16}$$

$$\boldsymbol{a_0(X)g(X) = a_1(X)[(c^2 - 2r_1)X(\xi)3cX^2(\xi) + 2X^3(\xi) + 2r_2]}. \tag{17}$$

Since $\boldsymbol{a_1(X)}$ is a polynomial of $\boldsymbol{X}$, is deduced, from (15) that $\boldsymbol{a_1(X)}$ is constant and $\boldsymbol{h(X) = 0.}$ let's take $\boldsymbol{a_1(X) = 1}$. By balancing the degrees of $\boldsymbol{X}$, on sides of Eq. (17), $\boldsymbol{a_1(X)}$, and $\boldsymbol{a_0(X)}$, concluded that$\boldsymbol{deg\ g(X)}$, is just 1. Suppose that$\boldsymbol{g(X) = B_0 + A_1 X}$, ($\boldsymbol{A_1 \neq 0}$) then we will find that;

$$\boldsymbol{a_0(X) = \tfrac{1}{2}A_1 X^2 + B_0 X + A_0}, \tag{18}$$

where $\boldsymbol{A_0}$ is the integration constant. Substitution in Eq. (17) for $\boldsymbol{a_0(X)}$, $\boldsymbol{a_1(X)}$, and $\boldsymbol{g(X)}$ and taking the coefficients of all powers of $\boldsymbol{X}$ to be zero, a system of nonlinear algebraic equations will result. Having solved the resulting system, the following solutions are attained;

$$\boldsymbol{A_1 = 2}, \quad \boldsymbol{B_0 = c}, \quad ,\boldsymbol{r_1 = -A_0},\ \boldsymbol{r_2 = \frac{1}{2}A_0 c}. \tag{19}$$
$$\boldsymbol{A_1 = -2},\ \boldsymbol{B_0 = -c},\ \boldsymbol{r_1 = A_0},\ \boldsymbol{r_2 = -\frac{1}{2}A_0 c}. \tag{20}$$
$$\boldsymbol{A_1 = 2},\ \boldsymbol{B_0 = c},\ \boldsymbol{A_0 = 0},\ \boldsymbol{r_1 = r_2 = 0}. \tag{21}$$

By using the values (19) in Eq. (13), results in;

$$\boldsymbol{Y(\xi) = -(X^2(\xi) + cX(\xi) + A_0)}. \tag{22}$$

Eq. (22) is the first integral of (12). Substituting (22) in the first equation of (12), while $\boldsymbol{A_0 > \frac{1}{4}c^2}$ holds, the exact solutions of Eq. (11) are obtained as follows;

$$\boldsymbol{u(\xi) = -\frac{1}{2}c - \frac{1}{2}\sqrt{4A_0 - c^2}\,tan(\frac{1}{2}\sqrt{4A_0 - c^2}(\xi + \xi_0))}, \tag{23}$$

$$\boldsymbol{w(\xi) = \left(\frac{1}{4}c^2 - A_0\right) sec^2\, (\frac{1}{2}\sqrt{4A_0 - c^2}(\xi + \xi_0))}. \tag{24}$$

where $\boldsymbol{\xi_0}$ is a constant of integration. Then, the exact solution of the dispersive long wave (1+1)-dimensional system (7) can be written as;

$$\boldsymbol{u(x,t) = -\frac{1}{2}c - \sqrt{A_0 - \frac{c^2}{4}}\,tan(\sqrt{A_0 - \frac{c^2}{4}}(x - ct + \xi_0)},$$

$$\boldsymbol{w(x,t) = \left(\frac{1}{4}c^2 - A_0\right) sec^2\, (\sqrt{A_0 - \frac{c^2}{4}}(x - ct + \xi_0)} \tag{25}$$

If $\boldsymbol{A_0 < \frac{1}{4}c^2}$ holds, then the exact solution of (7) can be written as;

$$\boldsymbol{u(x,t) = -\frac{1}{2}c - \sqrt{\frac{c^2}{4} - A_0}\,coth(\sqrt{\frac{c^2}{4} - A_0}(x - ct + \xi_0)},$$
$$\boldsymbol{w(x,t) = 0} \tag{26}$$

Similarly, for the values presented in (20), the following equation will be attained from Eq. (13);

$$\boldsymbol{Y(\xi) = X^2(\xi) + cX(\xi) - A_0}. \tag{27}$$

The exact solution to Eq. (11), is given by;

$$\boldsymbol{u(x,t) = -\frac{1}{2}c - \frac{1}{2}\ \ coth(\sqrt{A_0 + \frac{c^2}{4}}\ (x - ct + \xi_0)(\sqrt{4A_0 + c^2})}, \tag{28}$$

$$\boldsymbol{w(x,t) = 0}. \tag{29}$$

In the same way, for values in (21), the following equation will be resulted from Eq. (13);

$$\boldsymbol{Y(\xi) = -\left(X^2(\xi) + cX(\xi)\right)}. \tag{30}$$

Correspondingly, the exact solutions to Eq. (11) are given by;

$$\boldsymbol{u(x,t)=\frac{c}{-1+e^{c(x-ct+\xi_0)}}},\tag{31}$$

$$\boldsymbol{w(x,t)=-\left(\frac{c\sqrt{exp\,(c(x-ct+\xi_0))}}{-1+e^{c(x-ct+\xi_0)}}\right)^2}.\tag{32}$$

Case 2:

By assuming $\boldsymbol{m=2}$, in Eq. (13), and by equating the coefficients of $\boldsymbol{Y^i}$ $(\boldsymbol{i=3,2,1,0})$ on both sides of (14), the following equations will be revealed;

$$\boldsymbol{\dot{a}_2(X)=a_2(X)h(X)},\tag{33}$$

$$\boldsymbol{\dot{a}_1(X)=a_2(X)g(X)+a_1(X)h(X)},\tag{34}$$

$$\boldsymbol{\dot{a}_0(X)=-2a_2(X)[(c^2-2r_1)X(\xi)+3cX^2(\xi)+2X^3(\xi)+2r_2]+a_1(X)g(X)+a_0(X)h(X)},\tag{35}$$

$$\boldsymbol{a_0(X)g(X)=a_1(X)[(c^2-2r_1)X(\xi)+3cX^2(\xi)+2X^3(\xi)+2r_2]}.\tag{36}$$

Since $\boldsymbol{a_2(X)}$ is a polynomial of $\boldsymbol{X}$, it is deduced from (33) that $\boldsymbol{a_2(X)}$ is constant, and $\boldsymbol{h(X)=0}$, let's take $\boldsymbol{a_2(X)=1}$. Balancing the degrees of $\boldsymbol{X}$,$\boldsymbol{a_1(X)}$, and $\boldsymbol{a_0(X)}$, it can be concluded that $\boldsymbol{deg\ g(X)=0}$ or $\boldsymbol{deg\ g(X)=1}$. Let's take $\boldsymbol{deg\ g(X)=1}$, then $\boldsymbol{g(X)=B_0+A_1X}$, $(\boldsymbol{A_1\neq 0})$ so it is found that;

$$\boldsymbol{a_1(X)=\frac{1}{2}A_1X^2+B_0X+A_0},\tag{37}$$

where $\boldsymbol{A_0}$ is the constant of integration. Substitution in Eq. (36) for $\boldsymbol{a_0(X)}$, $\boldsymbol{a_1(X)}$, and $\boldsymbol{g(X)}$ and taking the coefficients of all powers of $\boldsymbol{X}$ to be zero, a system of non-linear algebraic equations will be achieved. The solution of this system results in the following;

$$\boldsymbol{A_1=4, B_0=2c, r_1=-\frac{1}{2}A_0, r_2=\frac{1}{4}A_0c, d=\frac{1}{4}A_0^2}.\tag{38}$$

$$\boldsymbol{A_1=-4,\ B_0=-2c,\ r_1=\frac{1}{2}A_0, r_2=-\frac{1}{4}A_0c,\ d=-\frac{1}{4}A_0^2}.\tag{39}$$

where $\boldsymbol{d}$ is the constant of integration in (35). Using the values in (38), in Eq. (13), results in;

$$\boldsymbol{Y(\xi)=-\left(X^2(\xi)+cX(\xi)+\frac{1}{2}A_0\right)}.\tag{40}$$

Eq. (40) is a first integral of (12). Substituting (40) into the first equation of (12), while $\boldsymbol{A_0>\frac{1}{2}c^2}$ holds, the exact solution of the system (7) can be written as follows;

$$\boldsymbol{u(x,t)=-\frac{1}{2}c-\frac{1}{2}\sqrt{2A_0-c^2}\,tan\left(\frac{1}{2}\sqrt{2A_0-c^2}(x-ct+\xi_0)\right)},\tag{41}$$

$$\boldsymbol{w(x,t)=-\frac{1}{4}A_0-\frac{1}{4}(2A_0-c^2)sec^2\,(\frac{1}{2}\sqrt{2A_0-c^2}(x-ct+\xi_0))}.\tag{42}$$

where $\boldsymbol{\xi_0}$ is the constant of integration. In the same way, using the values (39) in Eq. (13), results in;

$$\boldsymbol{Y(\xi) = X^2(\xi) + cX(\xi) - \frac{1}{2}(1-\sqrt{2})A_0}. \tag{43}$$

$$\boldsymbol{Y(\xi) = -(X^2(\xi) + cX(\xi) - \frac{1}{2}(1+\sqrt{2})A_0)}. \tag{44}$$

Substituting (43) in the first equation of (12), the exact solution to Eq. (11) may be written as follows;

$$\begin{aligned} &\boldsymbol{u(x,t) = -\frac{1}{2}c - \frac{1}{2}\sqrt{2(1-\sqrt{2})A_0 + c^2}} \times \\ &\boldsymbol{coth\,(\frac{1}{2}\sqrt{2(1-\sqrt{2})A_0 + c^2}(x - ct + \xi_0))}, \\ &\boldsymbol{w(x,t) = \frac{1}{2}A_0(-3+\sqrt{2})}. \end{aligned} \tag{45}$$

Also, a new solution for the dispersive long wave (1+1)-dimensional system (7) will be achieved from (44) by the same method;

$$\begin{aligned} &\boldsymbol{u(x,t) = -\frac{1}{2}c - \frac{1}{2}\sqrt{2(1+\sqrt{2})A_0 + c^2} \times coth\,(\frac{1}{2}\sqrt{2(1+\sqrt{2})A_0 + c^2}(x - ct + \xi_0)}, \\ &\boldsymbol{w(x,t) = -\frac{1}{2}A_0(3+\sqrt{2})} \end{aligned} \tag{46}$$

As a final notion, these solutions are considered as new exact solutions for a dispersive long wave (1+1) dimensional system.

Conclusions

In this paper, the discussion is limited to $\boldsymbol{m = 1, 2}$, because the solutions for $\boldsymbol{m = 3, 4}$ are exactly the same as $\boldsymbol{m = 1, 2}$ or some complicated integral will appear.

The solutions obtained in (25 - 26), (31 - 32), and (41 - 42) are different, about (45) and (46) are almost the same except for the coefficients, for $\boldsymbol{A_0 = 0}$, solutions will be exactly the same and are trivial.

In this study, the First Integral method is described and applied to find exact solutions of dispersive long wave (1+1)-dimensional systems. This approach is led to five nontrivial exact solutions for a dispersive long wave (1+1)-dimensional system. In spite of the fact that these new solutions may be important in physical problems, this study also suggests that one may find different solutions by choosing different methods. This method can be utilized to solve many systems of nonlinear functional equations that arise in the theory of solitons and other related areas of research.

References

[1] MR Miura. *Backlund Transformation*. Springer-Verlag, Berlin, 1978.
[2] VB Matveev and MA Salle. *Darboux Transformations and Solitons*. Berlin, Heidelberg, Springer, 1991.
[3] EJ Parkes and BR Duffy. An automated tanh-function method for finding solitary wave solution to nonlinear evolution equations. *Comput. Phys. Comm.* 1998; **98**, 288-300.
[4] AM Wazwaz. The extended tanh method for new soliton solutions for many forms of the fifth-order KdV equations. *Appl. Math. Comput*. 2007; **184**, 1002-14.
[5] SA Elwakil, SK EL-Labany, SK Zahran and MA Sabry. Modified extended tanh-function method for solving nonlinear partial differential equations. *Phys. Lett. A* 2002; **299**, 179-88.
[6] YT Gao and B Tian. Generalized hyperbolic-function method with computerized symbolic computation to construct the solitonic solutions to nonlinear equations of mathematical physics. *Comput. Phys. Comm*. 2001; **133**, 158-64.

[7] XY Tang and SY Lou. Localized excitations in (2+1)-dimensional systems. *Phys. Rev. E* 2002; **66**, 46601-17.
[8] ZS Feng. The first integral method to study the Burgers-Korteweg-de Vries equation. *Phys. Lett. A* 2002; **35**, 343-9.
[9] K Hosseini, R Ansari and P Gholamin. Exact solutions of some nonlinear systems of partial differential equations by using the first integral method. *J. Math. Anal. Appl*. 2012; **387**, 807-14.
[10] ZS Feng and XH Wang. The first integral method to the two-dimensional Burgers-Korteweg-de Vries equation. *Phys. Lett. A* 2003; **308**, 173-8.
[11] B Lu. The first integral method for some time fractional differential equations. *J. Math. Anal. Appl*. 2012; **395**, 684-93.
[12] MJ Ablowitz and PA Clarkson. *Solitons Nonlinear Evolution Equations and Inverse Scattering*. London Cambridge University Press, 1991.
[13] TR Ding and CZ Li. *Ordinary Differential Equations*. Peking University Press, Peking, 1996.
[14] N Bourbaki. *Commutative Algebra*. Addison-Wesley, Paris, 1971.
[15] ME Minasari, D Ganji and MO Minasari. Jacobi elliptic function solutions of the (1+1) dimensional dispersive long wave equation by Homotopy Perturbation Method. *Numer. Meth. Part. Differ. Equat*. 2008; **24**, 1361-70.