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Effect of Radiation and Magnetohydrodynamic Free Convection Boundary Layer Flow on a Solid Sphere with Convective Boundary Conditions

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Abstract

In this paper, the effect of radiation on magnetohydrodynamic free convection boundary layer flow on a solid sphere with convective boundary conditions, in which the heat is supplied through a bounding surface of finite thickness and finite heat capacity, is considered. The basic equations of the boundary layer are transformed into a non-dimensional form and reduced to nonlinear systems of partial differential equations and solved numerically using an implicit finite difference scheme known as the Keller-box method. Numerical solutions are obtained for the wall temperature, the heat transfer coefficient, local Nusselt number and the local skin friction coefficient, as well as the velocity and temperature profiles. The features of the flow and heat transfer characteristics for various values of the Prandtl number Pr, magnetic parameter M, radiation parameter N_R, the conjugate parameter γ, and the coordinate running along the surface of the sphere, x are analyzed and discussed.

Keywords: Convective boundary conditions, free convection, magnetohydrodynamic, radiation effects, solid sphere

Introduction

The effect of radiation on boundary layer flow and heat transfer problems can be quite significant at high operating temperatures such as gas turbines, nuclear power plant, and thermal energy store (Bataller [1]). Since processes in engineering areas occur at high temperature, the study of the effect of radiation becomes very important for the design of equipment. Molla *et al*. [2] studied the natural convection laminar flow from an isothermal sphere immersed in a viscous incompressible optically dense fluid in the presence of radiation effects. The laminar boundary layer flow over a moving plate in a moving fluid with convective surface boundary conditions and in the presence of thermal radiation has been considered by Ishak *et al*. [3]. Salleh *et al.* [4] presented the effect of radiation free convection boundary layer flow over a permeable horizontal flat plate embedded in a porous medium with mixed thermal boundary conditions.

The application of magnetohydrodynamics plays an important role in agriculture, engineering and petroleum industries. Ganesan and Palani [5], Alam *et al.* [6] and Molla *et al.* [7] studied the viscous dissipation and magnetohydrodynamic effect on a natural convection flow past an inclined plate and over a sphere in the presence of heat generation, respectively.

The analysis of flow and heat transfer characteristics for laminar free, mixed and forced convection about a sphere has been studied by Chen and Mucoglu [8]. Nazar *et al.* [9,10] considered the free convection boundary layer flows on a sphere in a viscous and micropolar fluid with 2 boundary conditions namely, constant heat flux (CHF) and constant wall temperature (CWT), respectively. The natural convection heat and mass transfer from a sphere in a micropolar fluid with constant wall temperature and concentration was presented by Cheng [11]. Lastly, Salleh *et al.* [12,13] considered the free convection boundary layer flow on a sphere with Newtonian heating in viscous and micropolar fluids, respectively.

Another class of boundary conditions which has been given attention recently is convective boundary conditions (CBC), in which the heat is supplied through a bounding surface of finite thickness and finite capacity and the interface temperature is not known a priori but depends on the intrinsic properties of the systems, (see Merkin [14]). Aziz [15] studied the similarity solution for the forced convection boundary layer flow over a flat plate by applying convective boundary conditions. It is shown in his paper that similarity solutions are possible if the convective heat transfer of the plate is proportional to $x^{-1/2}$, where x is the coordinate measured along the plate. Makinde and Aziz [16] discussed the problem of magnetohydrodynamic mixed convection from a vertical flat plate embedded in a porous medium with a convective boundary condition. Further, the similarity solutions for flow and heat transfer over a static permeable plate and the radiation effects on the thermal boundary layer flow over a moving plate with convective boundary conditions have been studied by Ishak [17]. Merkin and Pop [18] studied the forced convection flow of a uniform stream over a flat surface and Yao *et al*. [19] presented the heat transfer of a viscous fluid flow over a stretching/shrinking sheet with a convective boundary condition. Recently, the numerical solution for stagnation point flow over a stretching surface with convective boundary conditions and solved numerically by using the shooting method has been studied by Mohamed *et al.* [20].

Motivated by the above mentioned studies, therefore, the aim of the present paper is to study the effect of radiation on magnetohydrodynamic free convection boundary layer flow on a solid sphere with convective boundary conditions. The governing dimensional boundary layer equations are first transformed into a system of non-dimensional equations via the non-dimensional variables, and then into non-similar equations before they are solved numerically by the Keller-box method, as described in the book Cebeci and Bradshaw [21].

Mathematical analysis

Consider a heated sphere of radius a, which is immersed in a viscous and incompressible fluid of ambient temperature, T_∞. We assume that the equations and surface of the sphere is subjected to a convective boundary condition (CBC), as shown in **Figure 1**.

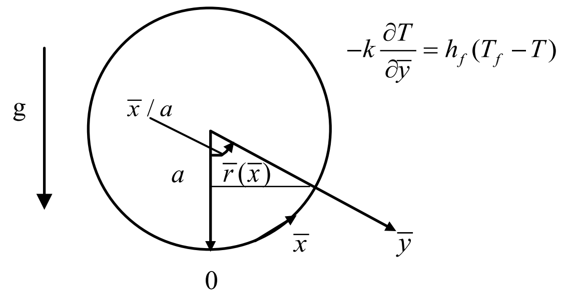

Figure 1 Physical model and coordinate system.

Under the Boussinesq and boundary layer approximations, the basic equations are;

$$\frac{\partial}{\partial \bar{x}}(\bar{r}\,\bar{u})+\frac{\partial}{\partial \bar{y}}(\bar{r}\,\bar{v})=0 \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}=\nu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}+g\beta(T-T_\infty)\sin\left(\frac{\bar{x}}{a}\right)-\frac{\sigma\beta^2}{\rho}\bar{u} \tag{2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}}+\bar{v}\frac{\partial T}{\partial \bar{y}}=\alpha\frac{\partial^2 T}{\partial \bar{y}^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial \bar{y}} \tag{3}$$

subject to the boundary conditions see (Salleh *et al.* [12] and Aziz [15]);

$$\bar{u}=\bar{v}=0,\ -k\frac{\partial T}{\partial \bar{y}}=h_f(T_f-T)\ \text{ at } \bar{y}=0$$
$$\bar{u}\to 0,\ T\to T_\infty \text{ as } \bar{y}\to\infty\,, \tag{4}$$

where $\bar{r}(\bar{x})=a\sin(\bar{x}/a)$. $\bar{u}$ and $\bar{v}$ are the velocity components along the $\bar{x}$ and $\bar{y}$ directions, respectively. T is the local temperature. q_r is the radiative heat flux. g is the gravity acceleration. β is the thermal expansion coefficient. ν is the kinematic viscosity. ρ is the fluid density. σ is the electric conductivity. c_p the specific heat. α is the thermal diffusivity. T_f is the temperature of the hot fluid. $k=\alpha\rho c_p$ is the thermal conductivity and h_f is the heat transfer coefficient for the convective boundary conditions.

We introduce now the following non-dimensional variables (Salleh *et al.* [12] and Aziz [15]);

$$x=\frac{\bar{x}}{a},\ y=Gr^{1/4}\left(\frac{\bar{y}}{a}\right),\ r=\frac{\bar{r}}{a},$$
$$u=\left(\frac{a}{\nu}\right)Gr^{-1/2}\bar{u},\ v=\left(\frac{a}{\nu}\right)Gr^{-1/4}\bar{v},\ \theta=\frac{T-T_\infty}{T_f-T_\infty} \tag{5}$$

where $Gr=g\beta(T_f-T_\infty)\dfrac{a^3}{\nu^2}$ is the Grashof number for convective boundary conditions.

Using the Rosseland approximation for radiation (Bataller, [22]) the radiative heat flux is simplified as;

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial \bar{y}} \tag{6}$$

where σ^* and k^* are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively.

We assume that the temperature differences within the flow through a porous medium such that the term T^4 may be expressed as a linear function of temperature. Hence, expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, we get;

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{7}$$

Substituting variables (5) - (7) into (1) - (3) then become;

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) = 0, \tag{8}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \theta \sin x - Mu, \tag{9}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\left(1 + \frac{4}{3}N_R\right)\frac{\partial^2 \theta}{\partial y^2}, \tag{10}$$

where $\Pr = \dfrac{\nu}{\alpha}$ is the Prandtl number. $M = \dfrac{\sigma\beta^2 a^2}{\nu\rho Gr^{1\backslash 2}}$ is the magnetic parameter and $N_R = \dfrac{4\sigma^* T_\infty^3}{\alpha k^* \rho c_\rho}$ is the radiation parameter. The boundary conditions (4) become;

$$u = v = 0,\ \frac{\partial \theta}{\partial y} = -\gamma(1-\theta) \text{ at } y = 0$$
$$u \to 0,\ \theta \to 0 \text{ as } y \to \infty \tag{11}$$

where $\gamma = ah_f Gr^{-1/4}/k$ is the conjugate parameter for the convective boundary conditions. To solve Eqs. (8) to (10), subject to the boundary conditions (11), we assume the following variables;

$$\psi = xr(x)f(x,y),\ \theta = \theta(x,y), \tag{12}$$

where ψ is the stream function defined as;

$$u = \frac{1}{r}\frac{\partial \psi}{\partial y} \text{ and } v = -\frac{1}{r}\frac{\partial \psi}{\partial x}, \tag{13}$$

which satisfies the continuity Eq. (8). Thus, (9) and (10) become;

$$\frac{\partial^3 f}{\partial y^3} + (1 + x\cot x) f\frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial f}{\partial y}\right)^2 + \frac{\sin x}{x}\theta - M\frac{\partial f}{\partial y} = x\left(\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x \partial y} - \frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}\right), \tag{14}$$

$$\frac{1}{\Pr}\left(1 + \frac{4}{3}N_R\right)\frac{\partial^2 \theta}{\partial y^2} + (1 + x\cot x) f\frac{\partial \theta}{\partial y} = x\left(\frac{\partial f}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial \theta}{\partial y}\right), \tag{15}$$

subject to the boundary conditions;

$$f = \frac{\partial f}{\partial y} = 0,\ \frac{\partial \theta}{\partial y} = -\gamma(1-\theta) \text{ at } y = 0$$

$$\frac{\partial f}{\partial y} \to 0,\ \theta \to 0 \text{ as } y \to \infty \tag{16}$$

It can be seen that at the lower stagnation point of the sphere, $x \approx 0$, Eqs. (14) and (15) reduce to the following ordinary differential equations;

$$f''' + 2ff'' - f'^2 + \theta - Mf' = 0 \tag{17}$$

$$\frac{1}{\Pr}\left(1 + \frac{4}{3}N_R\right)\theta'' + 2f\theta' = 0 \tag{18}$$

and the boundary conditions (16) become;

$$\begin{aligned} &f(0) = f'(0) = 0,\ \theta'(0) = -\gamma(1-\theta(0)) \\ &f' \to 0,\ \theta \to 0 \text{ as } y \to \infty \end{aligned} \tag{19}$$

where primes denote the differentiation with respect to y.

The physical quantities of interest in this problem are the local skin friction coefficient C_f and the local Nusselt number Nu and they can be written as;

$$C_f = \frac{Gr^{-3/4}a^2}{\mu\nu}\tau_w,\quad Nu = \frac{a\,Gr^{-1/4}}{k\left(T_f - T_\infty\right)}q_w, \tag{20}$$

where

$$\tau_w = \mu\left(\frac{\partial \overline{u}}{\partial \overline{y}}\right)_{\overline{y}=0},\ q_w = -k\left(\frac{\partial T}{\partial \overline{y}}\right)_{\overline{y}=0} + q_r \tag{21}$$

Using the non-dimensional variables (5) and Rosseland approximation for radiation (6) with the boundary condition (11) into Eqs. (20) and (21), we get;

$$C_f = x\frac{\partial^2 f}{\partial y^2}(x,0)\quad N_u = \gamma\left(1 + \frac{4}{3}N_R\right)(1-\theta(x,0)) \tag{22}$$

Results and discussion

The nonlinear system of partial differential Eqs. (14) and (15) subject to the boundary conditions (16) were solved numerically using an efficient, implicit finite-difference method known as the Keller-box method (KBM) for convective boundary conditions as described in the book by Cebeci and Bradshaw [21]. The solution is obtained by the following 4 steps:

1) Reduce Eqs. (14) and (15) to a first-order system.
2) Write the difference equations using central differences.
3) Linearize the resulting algebraic equations by Newton's method, and write them in a matrix-vector form.
4) Solve the linear system by the block tridiagonal elimination technique.

The step size $\Delta y = 0.01$ to 0.04 is sufficient to provide accurate numerical results and the edge of the boundary layer y_∞ had to be adjusted for different values of parameters to maintain accuracy. The parameters which are considered in the numerical results, namely the magnetic parameter *M*, the radiation parameter N_R, the Prandtl number Pr, the conjugate parameter γ and the coordinate running along the surface of the sphere, *x*. To validate the nonlinear system of ordinary differential Eqs. (17) and (18) with the boundary conditions (19) are solved numerically using the Runge-Kutta-Fehlberg method (RKF) for certain values of parameters.

Table 1 presents a comparison between the 2 methods (RKF and KBM) for the values of the wall temperature $\theta(0)$ and the heat transfer coefficient $-\theta'(0)$, with various values of the magnetic parameter *M* when the Prandtl number Pr = 0.7, the radiation parameter $N_R = 0$ and the conjugate parameter $\gamma = 0.1$. It is clear that the value of the maximum error between the 2 methods is very small, where *Max error* = $= 2\times10^{-5}$ and *Max error* $= 2\times10^{-6}$, for the values of $\theta(0)$ and $-\theta'(0)$, respectively. This indicates that the agreement between the RKF and KBM is very good. We can conclude that the comparison between this methods works efficiently for the present problem and we are also confident that the results presented here are accurate.

Table 1 Comparison between RKF and KBM of solving Eqs. (17) and (18) for various values of *M* when Pr =0.7, $N_R = 0$ and $\gamma = 0.1$.

	$\theta(0)$			$-\theta'(0)$		
M	**RKF**	**KBM**	**Error**	**RKF**	**KBM**	**Error**
0	0.238060	0.238051	0.000009	0.076194	0.076195	0.000001
5	0.333988	0.333977	0.000011	0.066601	0.066602	0.000001
10	0.360093	0.360078	0.000015	0.063991	0.063992	0.000001
15	0.372829	0.372816	0.000013	0.062717	0.062718	0.000001
20	0.380449	0.380469	0.000020	0.061955	0.061953	0.000002

Table 2 Values of the heat transfer coefficient $-(\partial\theta/\partial y)(x,0)$ at the lower stagnation point of the sphere, $x \approx 0$, when Pr = 0.7, 7, without the effect of radiation and magnetohydrodynamic (i.e. *M* = 0, $N_R = 0$) and $\gamma \to \infty$.

Pr = 0.7			**Pr = 7**		
Huang and Chen [23]	**Nazar *et al*. [9]**	**Present**	**Huang and Chen [23]**	**Nazar *et al*. [9]**	**Present**
0.4574	0.4576	0.457582	0.9581	0.9595	0.959498

The numerical solutions for the KBM start at the lower stagnation point of the sphere, $x \approx 0$, with initial profiles as given by Eqs. (17) and (18), and proceed around the sphere up to $x = 90^o$ because the data unstable after this point.

The values of the heat transfer coefficient $-(\partial\theta/\partial y)(x,0)$ at the lower stagnation point of the sphere, $x \approx 0$, when Pr = 0.7, 7, without the effect of radiation and magnetohydrodynamic (i.e. *M* = 0, $N_R = 0$)

and $\gamma \to \infty$ are shown in **Table 2**. In order to verify the accuracy of the present method, the present results are compared with those reported by Huang and Chen [23] and Nazar *et al*. [9]. It is found that the agreement between the previously published results with the present ones is excellent.

Table 3 show the values of the wall temperature $\theta(x,0)$, the heat transfer coefficient $-(\partial\theta/\partial y)(x,0)$ and the skin friction coefficient $(\partial^2 f/\partial y^2)(x,0)$ at the lower stagnation point of the sphere, $x \approx 0$, for various values of N_R when Pr = 0.7, $\gamma = 0.1$ and M = 0, 5. It is observed that, when the magnetic parameter M is fixed, an increase in the radiation parameter N_R causes the values of $\theta(x,0)$, $-(\partial\theta/\partial y)(x,0)$ and $(\partial^2 f/\partial y^2)(x,0)$ to increase. On other hand, when N_R is fixed and M increases, the value of $\theta(0,y)$ increases but both values of $(\partial^2 f/\partial y^2)(x,0)$ and $-(\partial\theta/\partial y)(x,0)$ decrease.

Table 3 Values of the wall temperature $\theta(x,0)$, the heat transfer coefficient $-(\partial\theta/\partial y)(x,0)$ and the skin friction coefficient $(\partial^2 f/\partial y^2)(x,0)$ at the lower stagnation point of the sphere, $x \approx 0$, for various values of N_R when Pr= 0.7, M = 0, 5 and $\gamma = 0.1$.

N_R	**M = 0**			**M = 5**		
	$\theta(0,y)$	$-(\partial\theta/\partial y)$	$(\partial^2 f/\partial y^2)$	$\theta(0,y)$	$-(\partial\theta/\partial y)$	$(\partial^2 f/\partial y^2)$
0	0.238051	0.076195	0.260067	0.333977	0.066602	0.135855
1	0.285971	0.166607	0.333039	0.368856	0.147266	0.152026
2	0.311935	0.252290	0.371648	0.381992	0.226602	0.158119
3	0.328807	0.335597	0.396153	0.388949	0.305525	0.161346
4	0.340704	0.417554	0.413230	0.393268	0.384264	0.163349
5	0.349603	0.498638	0.425880	0.396232	0.462889	0.164722

Figures 2 and **3** illustrate the variation of the wall temperature $\theta(x,0)$ at the lower stagnation point of the sphere, $x \approx 0$, with the radiation parameter N_R and magnetic parameter M when Pr =0.7 and γ = 0.05, 0.1, 0.2, respectively. It is found that increasing the value of M, N_R and the conjugate parameter γ caused an increase in the wall temperature $\theta(x,0)$.

Figures 4 and **5** show the temperature $\theta(0,y)$ and velocity profiles $(\partial f/\partial y)(0,y)$, when Pr = 7, M = 5, $N_R = 0,1,3,5$ and $\gamma = 0.1$, respectively. It is found that as N_R increases, the temperature and velocity profiles increase. It means that higher radiation occurs for higher values of temperature, which cause an increase in the velocity as well.

The temperature $\theta(0,y)$ and velocity profiles $(\partial f/\partial y)(0,y)$ when Pr = 0.7, $N_R = 1$, M = 5, 10, 15 and $\gamma = 0.1$, are presented in **Figures 6** and **7**, respectively. From these figures we can see that when the value of M increases, the temperature profiles increase, but the velocity profiles decrease along the y direction.

Variation of the local Nusselt number Nu and the local friction coefficient C_f with various values of x when Pr = 0.7, $N_R = 1$, M =5, 10, 15 and $\gamma = 0.1$ are plotted in **Figures 8** and **9**, respectively. It is found that as M increases, both values of the local Nusselt number and the local skin friction coefficient decrease from zero at the lower stagnation point along the x direction.

Figures 10 and **11** display the local Nusselt number Nu and the local friction coefficient C_f with various values of x when Pr = 0.7, M = 5, $N_R = 0,1,3,5$ and $\gamma = 0.1$, respectively. It is found that as N_R increases, both values of the local Nusselt number and the local skin friction coefficient increase. So the effect of the radiation parameter N_R on the local Nusselt number is more than of the effect of N_R on the local skin friction coefficient.

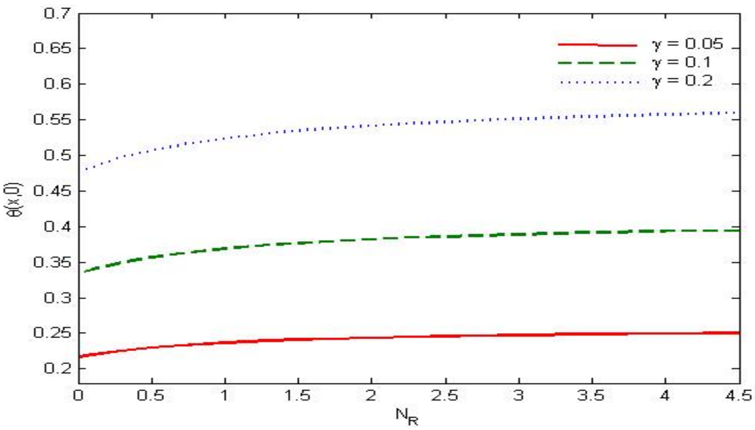

Figure 2 Variation of the wall temperature $\theta(x,0)$, at the lower stagnation point of the sphere, $x \approx 0$, with N_R when Pr =0.7, M = 5 and $\gamma =$ 0.05, 0.1, 0.2.

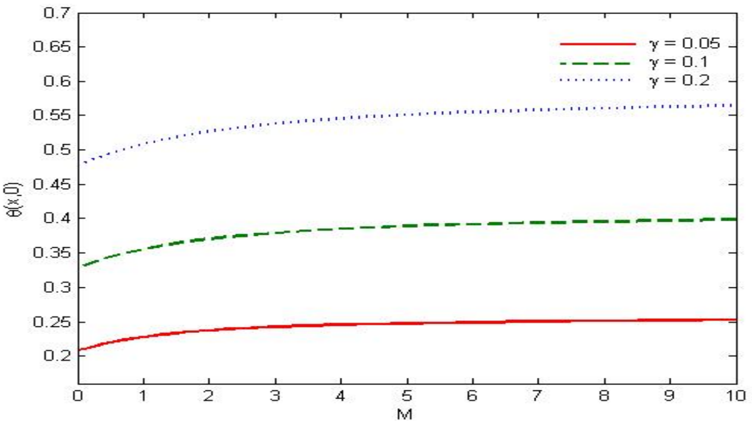

Figure 3 Variation of the wall temperature $\theta(x,0)$, at the lower stagnation point of the sphere, $x \approx 0$, with M when Pr =0.7, $N_R = 3$ and $\gamma =$ 0.05, 0.1, 0.2.

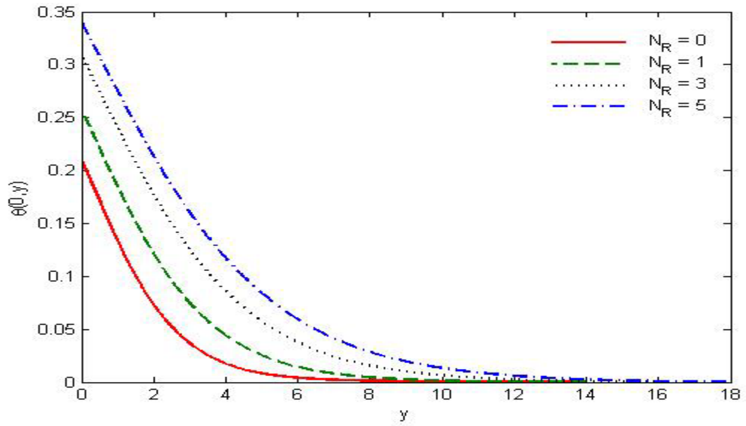

Figure 4 Temperature profiles $\theta(0,y)$ when Pr = 7, *M*= 5, $N_R = 0,1,3,5$ and $\gamma = 0.1$.

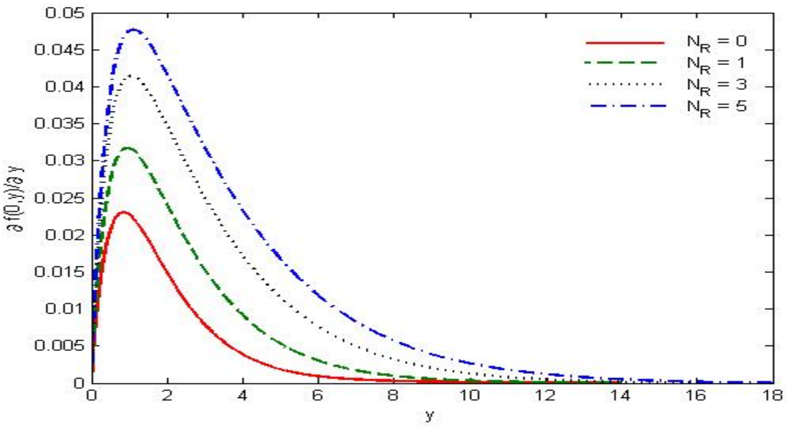

Figure 5 Velocity profiles $(\partial f/\partial y)(0,y)$, when Pr = 7, *M*= 5, $N_R = 0,1,3,5$ and $\gamma = 0.1$.

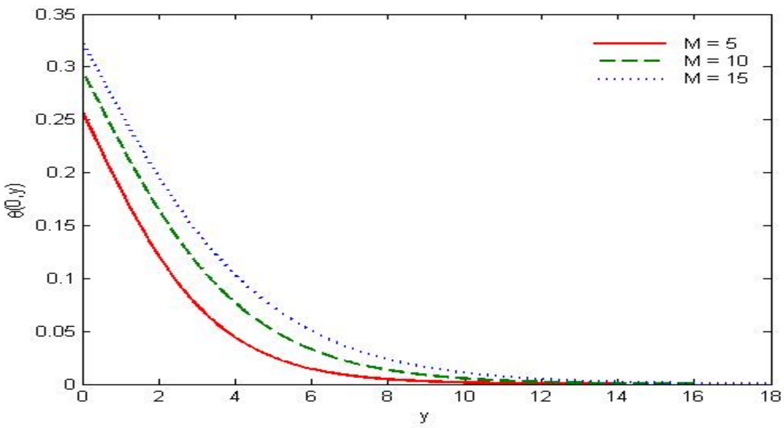

Figure 6 Temperature profiles $\theta(0,y)$ when Pr = 7, $N_R = 1$, M = 5, 10, 15 and $\gamma = 0.1$.

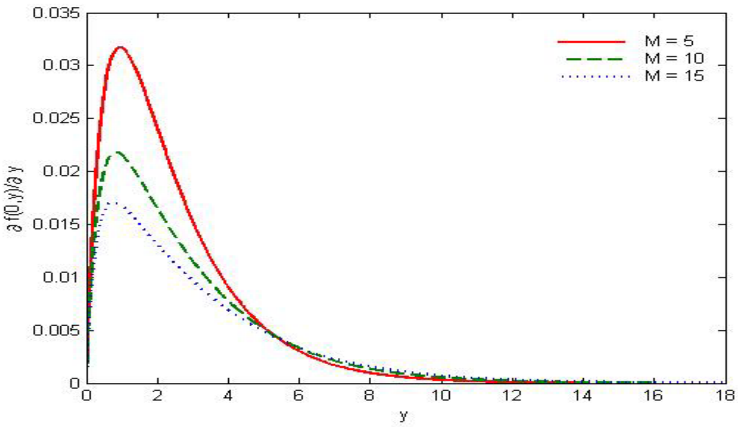

Figure 7 Velocity profiles $(\partial f/\partial y)(0,y)$, when Pr = 7, $N_R = 1$, M = 5, 10, 15 and $\gamma = 0.1$.

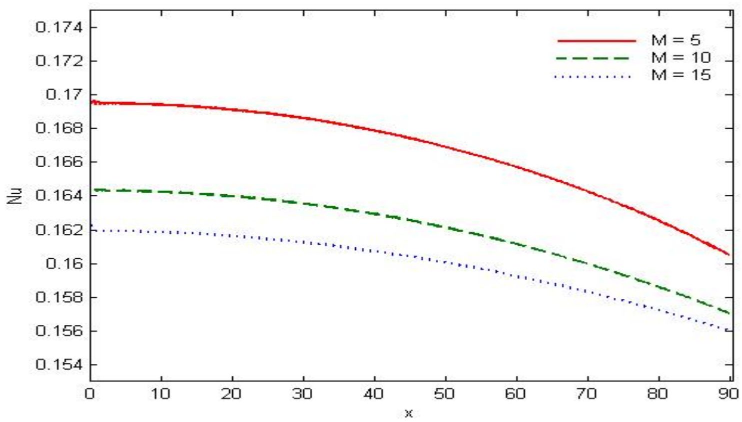

Figure 8 Variation of the local Nusselt number Nu with x when Pr = 0.7, $N_R = 1$, M = 5, 10, 15 and $\gamma = 0.1$.

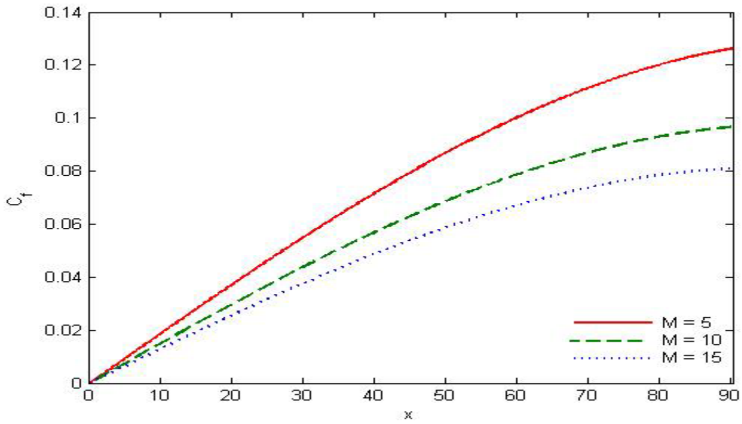

Figure 9 Variation of the local skin friction coefficient, C_f with x when Pr = 0.7, $N_R = 1$, M = 5, 10, 15 and $\gamma = 0.1$.

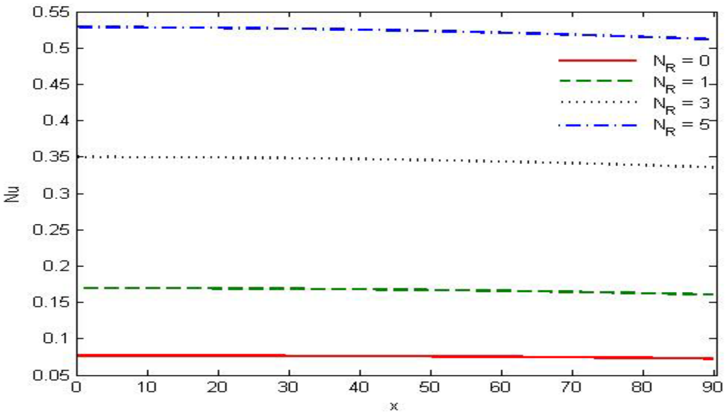

Figure 10 Variation of the local Nusselt number Nu with x when Pr = 0.7, M= 5, $N_R = 0,1,3,5$ and $\gamma = 0.1$.

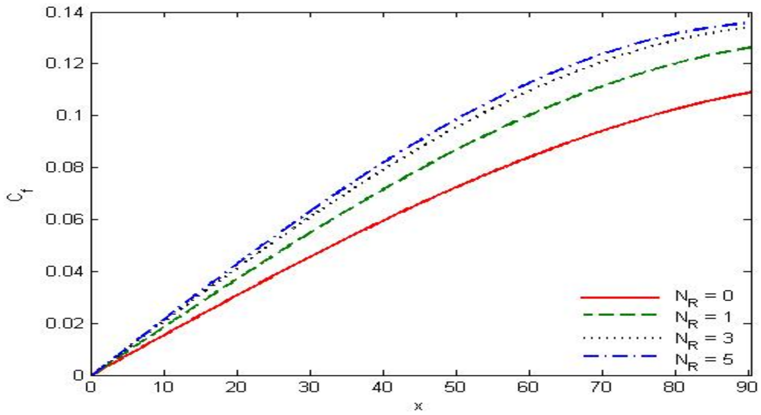

Figure 11 Variation of the skin friction coefficient, C_f with x when Pr = 0.7, M = 5, $N_R = 0,1,3,5$ and $\gamma = 0.1$.

Conclusions

In this paper, we have numerically studied the effect of radiation on magnetohydrodynamic free convection boundary layer flow on a solid sphere with convective boundary conditions. It shows how the Prandtl number Pr, magnetic parameter M, thermal radiation parameter N_R, conjugate parameter γ and the coordinate running along the surface of the sphere, x affect the values of the temperature profiles $\theta(0,y)$, heat transfer coefficient $-(\partial\theta/\partial y)(0,y)$, the skin friction coefficient $(\partial^2 f/\partial y^2)(0,y)$, the local Nusselt number Nu and the local friction coefficient C_f. The conclusions arise as follows;

1) The agreement between RKF and KBM are very good for solving the nonlinear system of ordinary differential equations and partial differential equations, respectively.

2) When γ and M are fixed, as N_R increases, the values of temperature, velocity profiles, skin friction coefficient and the heat transfer coefficient increases, while when γ and N_R are fixed, as M increases, the value of the temperature profiles increases, and velocity profiles, skin friction coefficient

and heat transfer coefficient decrease. On other hand, an increasing value of M, N_R and the conjugate parameter γ caused an increase in the wall temperature.

3) When γ and N_R are fixed, as M increases, the both values of the local Nusselt number and the local skin friction coefficient decrease, and if γ and M are fixed, as N_R increases, the local Nusselt number and the local skin friction coefficient increase.

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