WALAILAK JOURNAL
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# Analytical Solution of MHD Free Convective Flow through Porous Media with Time Dependent Temperature and Concentration

**Pradyumna Kumar PATTNAIK[1,*] and Trilochan BISWAL[2]**

[1]*Department of Mathematics, MIT, Bhubaneswar, Orissa, India*
[2]*Department of Mathematics, VIVTECH, Bhubaneswar, Orissa, India*

**([*]Corresponding author's e-mail: pkpattnaik_07@yahoo.com)**

*Received: 26 April 2014, Revised: 28 August 2014, Accepted: 24 September 2014*

## Abstract

The present study considers the effect of heat and mass transfer on free convective flow of a viscous incompressible electrically conducting fluid past a vertical plate through a porous medium in the presence of a uniform transverse magnetic field and heat source. The novelty of the present study is to analyze the effect of the chemical reaction and permeability of the medium on a viscous fluid flow. The coupled nonlinear partial differential equations are converted to ordinary differential equations by super imposing a solution with a steady and time dependent transient part. Finally, the set of ordinary differential equations are solved by Laplace transformation. It is interesting to note that the magnetic field and permeability of the porous medium are counter-productive in enhancing the velocity whereas the buoyancy force due to thermo and mass free convection are favourable. Moreover, the measure of rates of changes in the boundary surface indicate a change in the streamline flow. The study is well supported by verification of a previous result.

**Keywords:** MHD flow, mass transfer, heat transfer, porous medium, chemical reaction

## Nomenclature

| | | | |
|---|---|---|---|
| $C'$ | Species concentration | $C$ | non-dimensional species concentration |
| $D$ | molecular diffusivity | $G_c$ | Grashof number for mass transfer |
| $G_r$ | Grashof number for heat transfer | $g$ | acceleration due to gravity |
| $K_c'$ | permeability of the medium | $K_p$ | porosity parameter |
| $\alpha$ | thermal diffusivity | $M$ | magnetic parameter |
| $N_u$ | Nusselt number | $P_r$ | Prandtl number |
| $S$ | heat source parameter | $S_c$ | Schmidt number |
| $S_h$ | Sherwood number | $T'$ | temperature of the field |
| $T$ | non-dimensional temperature | $t'$ | time |
| $t$ | non-dimensional time | $u'$ | velocity component along the x-axis |
| $u$ | non- dimensional velocity | $v(t')$ | suction velocity |
| $u_0$ | characteristic velocity | $y'$ | distance along the y-axis |
| $y$ | non-dimensional distance along the y-axis | | |

| | | | |
|---|---|---|---|
| $q_0$ | volumetric heat generation / absorption | | |
| $\beta$ | volumetric coefficient of expansion for heat transfer | | |
| $\beta'$ | volumetric coefficient of expansion with species concentration | | |
| $\upsilon$ | kinematic coefficient of viscosity | $\tau$ | skin friction |
| $\sigma$ | electrical conductivity | $K_c$ | concentration parameter |
| $\phi$ | internal heat generation/ absorption parameter | | |
| $a$ | acceleration | | |

**Introduction**

An important class of 2 dimensional time dependent flow problems dealing with the response of the boundary layer to external unsteady fluctuations of the free stream velocity about a mean value has attracted the attention of many researchers. Magnetohydrodynamic (MHD) flow with heat and mass transfer has been the subject of interest of many researchers because of its varied application in science and technology. Such phenomena are observed in buoyancy induced motions in the atmosphere, in water bodies, quasi–solid bodies such as earth, etc. In natural processes and industrial applications many transportation processes exist where transfer of heat and mass takes place simultaneously as a result of thermal diffusion and diffusion of chemically reactive species.

Several researchers have analyzed the free convective and mass transfer flow of a viscous fluid through a porous medium. The permeability of the porous medium is assumed to be constant while the porosity of the medium may not necessarily be constant. Kim [1] studied the unsteady MHD convective heat past a semi-infinite vertical porous moving plate with variable suction. The problem of 3 dimensional free convective flow and heat transfer through a porous medium with periodic permeability has been discussed by Singh *et al*. [2]. Singh *et al*. [3] have analyzed the heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity. Bathul *et al*. [4] discussed the heat transfer in a 3 dimensional viscous flow over a porous plate moving with harmonic disturbance. Postelnicu [5] studied numerically the influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in a porous media considering Soret and Dufour effects. Ogulu *et al*. [6] studied unsteady MHD flow and heat transfer past an infinite vertical moving plate with variable suction. Das *et al*. [7] analyzed the mass transfer effect on unsteady flow past an accelerated vertical porous plate with suction employing numerical methods.

The basic equations of incompressible MHD flow are non-linear. But there are many interesting cases where the equations become linear in terms of the unknown quantities and may be solved easily. Linear MHD problems are accessible to exact solutions and adopt the approximations that the density and transport properties are constant. No fluid is incompressible but all may be treated as such whenever the pressure changes are small in comparison with the bulk modulus. Ferdows *et al*. [8] analyzed free convection flow with variable suction in presence of thermal radiation. Alam *et al*. [9] studied the Dufour and Soret effect with variable suction on unsteady MHD free convection flow along a porous plate.

Most of the results reported earlier by Muthucumaraswamy *et al*. [10] and Dash *et al*. [11] are related to similar variations of plate temperature and concentration. It is not possible to impose similar conditions on bounding surface so that temperature and concentration obey the same pattern of variation. Therefore, it is envisaged to apply different wall conditions with respect to temperature and concentration variation. Singh and Singh [12] studied the MHD effects on heat and mass transfer in the flow of a viscous fluid with induced magnetic field. Majumder *et al*. [13] have studied the MHD flow past an impulsively started infinite vertical plate in the presence of thermal radiation. Muthucumaraswamy *et al*. [14] have investigated the unsteady flow past an accelerated infinite vertical plate with variable temperature and uniform mass diffusion.

Dash *et al*. [15] have discussed the free convective MHD flow through porous media of a rotating visco-elastic fluid past an infinite vertical porous plate with heat and mass transfer in the presence of a chemical reaction. Das *et al*. [16] studied the mass transfer effects on MHD flow and heat transfer past a vertical porous plate through a porous medium under oscillatory suction and heat source.

Recently, Uwanta *et al*. [17] considered heat and mass transfer with variable temperature and mass diffusion but they confined their work to non-conducting flow. Moving surfaces are very often embedded in an insulating material which serves as a porous medium. In such a case flow is usually encountered by the resistive force due to permeability of the medium. In the present case of slow motion this justifies the linear model (Darcy Model) to account for such a body force. In the present study we have considered the body forces besides the surface forces such as buoyancy due to thermal and mass diffusion, resistive forces such as ponder motive force and Darcy resistive forces. Generally $\phi$, the internal heat generation/absorption parameter is defined as $\frac{\upsilon q_0}{\rho C_p U_0^2}$ where $\upsilon, \rho, C_p$ represent the physical properties (being positive) of the fluid. $U_0^2$ is the square of the characteristic velocity (positive). Only the quantity $q_0$, the rate of volumetric heat generation/absorption, depends on formulation of the physical problem i.e. mathematical model subjected to the present analysis.

Therefore, the objective of the present study is to bring out the outcome of the interacting forces through emerging parameters affecting the flow, heat and mass transfer phenomena. We have applied the Laplace Transform method to solve the equations, avoiding the terms which contribute to non-linearity such as convective acceleration, non-Darcian model term and dissipations such as viscous and Joulian dissipation. The Laplace Transformation can also be applied following an iterative procedure to solve non-linear differential equations. However, without compromising the generality, we have tried to bring out the results reported earlier by Uwanta *et al*. [17] as a special case.

The fluid considered here is viscous, incompressible and electrically conducting. The problem of momentum transport along with thermo and mass diffusion has been considered under the following conditions:

(i) The velocity and temperature of the vertical plate, embedded in a porous medium vary linearly with time.

(ii) The concentration of the plate varies exponentially with time.

(iii) The plate is subject to a uniform transverse magnetic field along the axis normal to the plate generating an electromagnetic force $\vec{J} \times \vec{B} = \left(-\sigma B_0^2 u, 0, 0\right)$ along the main direction of the flow where $\vec{J}$: Current density, $\vec{B}$: Magnetic Induction, $\sigma$: Electrical Conductivity, $B_0$: magnetic field strength.

(iv) The flow through porous media obeys the Darcian model with components $\left(\frac{\mu u}{K_p}, 0, 0\right)$ where $\mu$ and $K_p$ represent the viscosity and permeability of the fluid.

(v) The thermal diffusion and mass diffusion are associated with constant internal heat absorption and exothermic/ endothermic reactions.

(vi) Diffusion of foreign species occurs through the air medium.

The effects of the followings are neglected:

(i) Induced electrical field subsequently Hall current has been neglected due to low magnetic field intensity.

(ii) Soret and Dufour effects are neglected due to the low thermal and concentration gradient.

(iii) Energy dissipation such as viscous dissipation and Joulian dissipation are also neglected as we are considering free convective and slow flow neglecting inertia terms in the transport equations.

**Formulation of the problem**

The unsteady free convective flow of a viscous fluid past an infinite vertical plate embedded in a porous medium with uniform permeability in the presence of a transverse magnetic field is considered. Let the $x'$ – axis be along the plate in the direction of the flow and $y'$ – axis normal to it. Let us consider the magnetic Reynolds number is much less than unity so that the induced magnetic field is neglected in comparison with the applied transverse magnetic field. The basic flow in the medium is, therefore, entirely due to the buoyancy force caused by the temperature difference between the wall and the medium. It is assumed that initially, at $t' \le 0$, the plate as well as fluid are at the same temperature and the concentration of the species is very low so that the Soret and Dufour effects are neglected. When $t' > 0$, the temperature of the plate is instantaneously raised to $T_W{}'$ and the concentration of the species is set to $C_W{}'$.

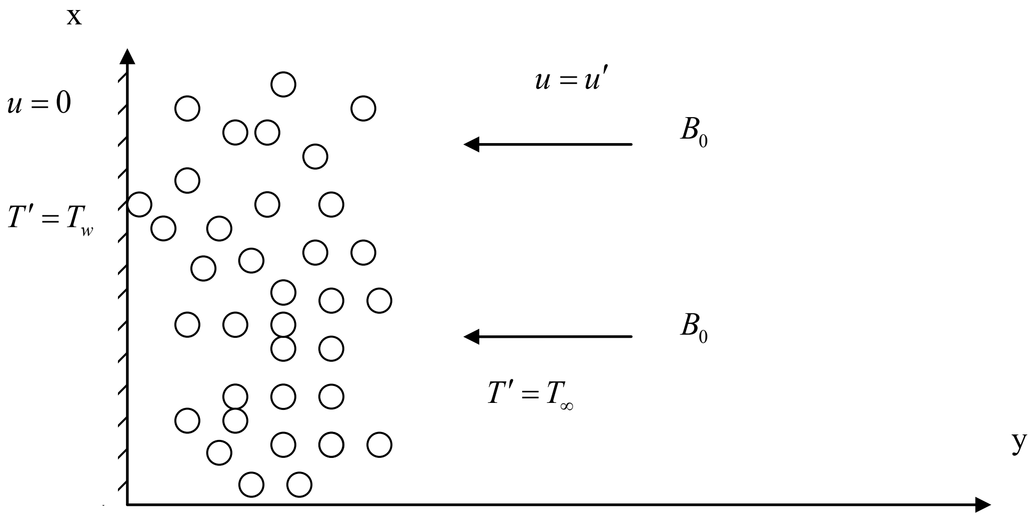


**Figure 1** Flow geometry.

Under the above assumption with the usual Boussinesq's approximation, the governing equations and boundary conditions are given by;

$$\frac{\partial u'}{\partial t'} = \upsilon \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho} u' - \frac{\mu u'}{\rho K'_p} + g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) \tag{1}$$

$$\frac{\partial T'}{\partial t'} = \alpha \frac{\partial^2 T'}{\partial y'^2} + \frac{q_0}{\rho c_p}(T' - T'_\infty) \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} - K'_c(C' - C_\infty{}') \tag{3}$$

Initial and Boundary conditions are;

$$\left.\begin{aligned} &u'=0,\quad T'=T_\infty{}',C'=C_\infty{}' \quad \forall\, y',t'\le 0\\ &t'>0: u'=u_0t'a', T'=T'_\infty+\left(T'_\omega-T'_\infty\right)\frac{u_0^2}{\upsilon}t', C'=C'_\infty+\left(C'_\omega-C'_\infty\right)e^{\frac{u_0^2}{\upsilon}t'} \quad at\ y'=0\\ &u'=0,\quad T'\to T_\infty{}',C'\to C_\infty{}' \quad as\ y'\to\infty \end{aligned}\right\} \tag{4}$$

Introducing the non-dimensional quantities;

$$\left.\begin{aligned} &y=\frac{u_0y'}{\upsilon}, t=\frac{u_0{}^2t'}{\upsilon}, u=\frac{u'}{u_0}, T=\frac{T'-T'_\infty}{T'_w-T'_\infty}, a=\frac{a'\upsilon}{u_0{}^2}, K_c=\frac{K_c{}'\upsilon}{u_0{}^2}\\ &C=\frac{C'-C'_\infty}{C'_w-C'_\infty}, \phi=\frac{\upsilon q_0}{\rho C_P u_0{}^2}, K_P=\frac{u_0{}^2K_P{}'}{\upsilon^2}, M=\frac{\sigma B_0{}^2\upsilon}{\rho u_0{}^2}, P_r=\frac{\upsilon}{\alpha}\\ &G_c=\frac{\upsilon g\beta'(C_w{}'-C'_\infty)}{u_0{}^3}, G_r=\frac{\upsilon g\beta(T_w{}'-T'_\infty)}{u_0{}^3}, S_c=\frac{\upsilon}{D} \end{aligned}\right\} \tag{5}$$

The Eqs. (1), (2) and (3) reduce to following non-dimensional form;

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}+G_rT+G_cC-\left(M+\frac{1}{K_p}\right)u \tag{6}$$

$$\frac{\partial T}{\partial t}=\frac{1}{\mathrm{P}_r}\frac{\partial^2 T}{\partial y^2}-\phi T \tag{7}$$

$$\frac{\partial C}{\partial t}=\frac{1}{S_c}\frac{\partial^2 C}{\partial y^2}-K_cC \tag{8}$$

$$\left.\begin{aligned} &u=0, T=0, C=0 \ \ for\,all\ y,t\le 0\\ &t>0:\ \ u=at, T=t, C=e^t \ \ at\, y=0\\ &u\to 0,\ T\to 0,\ \ C\to 0 \ \ as\ y\to\infty \end{aligned}\right\} \tag{9}$$

**Method of solution**

The solutions of Eqs. (6) - (8) with boundary conditions (9) using the Laplace transformation are;

$$\bar{T}=\frac{1}{s^2}e^{-y\sqrt{(s+\phi)P_r}},\ \bar{C}=\frac{1}{s-1}e^{-y\sqrt{Sc(S+Kc)}} \tag{10}$$

$$\bar{u} = \frac{a}{s^2} e^{-y\sqrt{s+Q}} + \frac{G_r}{s^2[P_r(s+\phi)-(s+Q)]} + \left[ e^{-y\sqrt{s+Q}} - e^{-y\sqrt{(s+\phi)P_r}} \right]$$
$$+ \frac{G_c}{(s-1)\left[S_c(s+K_c)-(s+Q)\right]}\left[ e^{-y\sqrt{S+Q}} - e^{-y\sqrt{(S+K_c)S_c}} \right] \tag{11}$$

The Laplace inversion gives;

$$T = \frac{1}{2}\left[\left(t+\frac{y}{2}\sqrt{\frac{P_r}{\phi}}\right)e^{y\sqrt{P_r\phi}} erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}}+\sqrt{\phi t}\right)+\left(t-\frac{y}{2}\sqrt{\frac{P_r}{\phi}}\right)e^{-y\sqrt{P_r\phi}} erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}}-\sqrt{\phi t}\right)\right] \tag{12}$$

$$C = \frac{e^t}{2}\left[e^{y\sqrt{S_c(K_c+1)}} erfc\left(\frac{y}{2}\sqrt{\frac{S_c}{t}}+\sqrt{(K_c+1)t}\right)+e^{-y\sqrt{S_c(K_c+1)}} erfc\left(\frac{y}{2}\sqrt{\frac{S_c}{t}}-\sqrt{(K_c+1)t}\right)\right] \tag{13}$$

$$U = \frac{a}{2}\left[\left(t+\frac{y}{2\sqrt{Q}}\right)e^{y\sqrt{Q}} erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{Qt}\right)+\left(t-\frac{y}{2\sqrt{Q}}\right)e^{-y\sqrt{Q}} erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{Qt}\right)\right]$$
$$-\frac{\beta_1}{2\alpha_1^2}\left[e^{y\sqrt{Q}} erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{Qt}\right)+e^{-y\sqrt{Q}} erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{Qt}\right)\right]$$
$$+\frac{\beta_1}{2\alpha_1}\left[\left(t+\frac{y}{2\sqrt{Q}}\right)e^{y\sqrt{Q}} erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{Qt}\right)+\left(t-\frac{y}{2\sqrt{Q}}\right)e^{-y\sqrt{Q}} erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{Qt}\right)\right]$$
$$+\frac{\beta_1}{2\alpha_1^2}e^{-\alpha_1 t}\left[e^{y\sqrt{Q-\alpha_1}} erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{(Q-\alpha_1)t}+e^{-y\sqrt{Q-\alpha_1}} erfc\,\frac{y}{2\sqrt{t}}-\sqrt{(Q-\alpha_1)t}\right)\right]$$
$$+\frac{\beta_1}{2\alpha_1^2}\left[e^{y\sqrt{P_r\phi}} erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}}+\sqrt{\phi t}\right)+e^{-y\sqrt{P_r\phi}} erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}}-\sqrt{\phi t}\right)\right]$$
$$-\frac{\beta_1}{2\alpha_1}\left[\left(t+\frac{y}{2}\sqrt{\frac{P_r}{\phi}}\right)e^{y\sqrt{P_r\phi}} erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}}+\sqrt{\phi t}\right)+\left(t-\frac{y}{2}\sqrt{\frac{P_r}{\phi}}\right)e^{-y\sqrt{P_r\phi}} erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}}-\sqrt{\phi t}\right)\right]$$
$$-\frac{\beta_1}{2\alpha_1^2}e^{-\alpha_1 t}\left[e^{y\sqrt{P_r(\phi-\alpha_1)}} erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}}+\sqrt{(\phi-\alpha_1)t}\right)+e^{-y\sqrt{P_r(\phi-\alpha_1)}} erfc\left(\frac{y}{2}\sqrt{\frac{P_r}{t}}-\sqrt{(\phi-\alpha_1)t}\right)\right]$$
$$+\frac{\beta_2 e^t}{2(1+\alpha_2)}\left[e^{y\sqrt{Q+1}} erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{(Q+1)t}\right)+e^{-y\sqrt{Q+1}} erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{(Q+1)t}\right)\right]$$
$$-\frac{\beta_2 e^{-\alpha_2 t}}{2(1+\alpha_2)}\left[e^{y\sqrt{Q-\alpha_2}} erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{(Q-\alpha_2)t}\right)+e^{-y\sqrt{Q-\alpha_2}} erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{(Q-\alpha_2)t}\right)\right]$$

$$-\frac{\beta_2 e^{t}}{2(1+\alpha_2)}\left[e^{y\sqrt{S_c(K_c+1)}}erfc\left(\frac{y}{2}\sqrt{\frac{S_c}{t}}+\sqrt{(K_c+1)t}\right)+e^{-y\sqrt{S_c(K_c+1)}}erfc\left(\frac{y}{2}\sqrt{\frac{S_c}{t}}-\sqrt{(K_c+1)t}\right)\right]$$
$$+\frac{\beta_2 e^{-\alpha_2 t}}{2(1+\alpha_2)}\left[e^{y\sqrt{S_c(K_c-\alpha_2)}}erfc\left(\frac{y}{2}\sqrt{\frac{S_c}{t}}+\sqrt{(K_c-\alpha_2)t}\right)+e^{-y\sqrt{S_c(K_c-\alpha_2)}}erfc\left(\frac{y}{2}\sqrt{\frac{S_c}{t}}-\sqrt{(K_c-\alpha_2)t}\right)\right] \quad (14)$$

where $Q = M+\frac{1}{K_p}, \alpha_1 = \frac{p_r\phi - Q}{p_r - 1}, \alpha_2 = \frac{S_c K_c - Q}{S_c - 1}, \beta_1 = \frac{G_r}{P_r - 1}, \beta_2 = \frac{G_c}{S_c - 1}$

The skin friction at the plate is given by;

$$\tau = -\left(\frac{\partial u}{\partial y}\right)_{y=0}$$
$$= \frac{a(1+2tQ)}{2\sqrt{Q}}erf\left(\sqrt{Qt}\right)+a\sqrt{\frac{t}{\pi}}e^{-Qt}-\frac{\beta_1}{\alpha_1^2}\left[\sqrt{Q}erf\left(\sqrt{Qt}\right)+\frac{1}{\sqrt{t\pi}}e^{-Qt}\right]+\frac{\beta_1}{2\alpha_1}\left[\frac{1+2tQ}{\sqrt{Q}}erf\left(\sqrt{Qt}\right)+2\sqrt{\frac{t}{\pi}}e^{-Qt}\right]$$
$$+\frac{\beta_1}{\alpha_1^2}e^{-\alpha_1 t}\left[\sqrt{Q-\alpha_1}erf\left(\sqrt{(Q-\alpha_1)t}\right)+\frac{1}{\sqrt{\pi t}}e^{-(Q-\alpha_1)t}\right]+\frac{\beta_1}{\alpha_1^2}\left[\sqrt{P_r\phi}\,erf\sqrt{\phi t}+\sqrt{\frac{P_r}{\pi t}}e^{-\phi t}\right]$$
$$-\frac{\beta_1}{2\alpha_1}\left[\sqrt{\frac{P_r}{\phi}}\left(1+2t\phi\right)erf\sqrt{\phi t}+2t\sqrt{\frac{P_r}{\pi\phi}}e^{-\phi t}\right]-\frac{\beta_1 e^{-\alpha_1 t}}{\alpha_1^2}\left[\sqrt{P_r(\phi-\alpha_1)}\,erf\sqrt{(\phi-\alpha_1)t}+\sqrt{\frac{P_r}{\pi t}}e^{-(\phi-\alpha_1 t)}\right]$$
$$+\frac{\beta_2 e^{t}}{\alpha_2+1}\left[\sqrt{Q+1}\,erf\left(\sqrt{(Q+1)t}\right)+\frac{1}{\sqrt{\pi t}}e^{-(Q+1)t}\right]$$
$$-\frac{\beta_2 e^{-\alpha_2 t}}{\left(\alpha_2+1\right)}\left[\sqrt{Q-\alpha_2}\,erf\left(\sqrt{(Q-\alpha_2)t}\right)+\frac{1}{\sqrt{\pi t}}e^{-(Q-\alpha_2)t}\right]$$
$$-\frac{\beta_2 e^{t}}{\alpha_2+1}\left[\sqrt{S_c(K_c+1)}\,erf\left(\sqrt{(K_c+1)t}\right)+\sqrt{\frac{S_c}{t\pi}}e^{-(K_c+1)t}\right]$$
$$+\frac{\beta_2 e^{-\alpha_2 t}}{\alpha_2+1}\left[\sqrt{S_c(K_c-\alpha_2)}\,erf\left(\sqrt{(K_c-\alpha_2)t}\right)+\sqrt{\frac{S_c}{t\pi}}e^{-(K_c-\alpha_2)t}\right] \quad (15)$$

The rate of heat transfer, i.e. heat flux at the $N_u$ is given by;

$$N_u = -\left[\frac{\partial T}{\partial y}\right]_{y=0} = \frac{1}{2}\sqrt{\frac{P_r}{\phi}}erf\left(\sqrt{\phi t}\right)+\sqrt{\frac{P_r t}{\pi}}e^{-\phi t}+t\sqrt{P_r\phi}\,erf\left(\sqrt{\phi t}\right) \quad (16)$$

The mass transfer coefficient, i.e. the Sherwood number ( $S_h$ ) is given by;

$$S_h = -\left[\frac{\partial C}{\partial y}\right]_{y=0} = \sqrt{\frac{S_c}{t\pi}}e^{-K_c t} + \sqrt{S_c(K_c+1)}e^{t} erf\left(\sqrt{(K_c+1)}t\right) \tag{17}$$

**Results and discussion**

An analytical study has been carried out on the MHD flow of a viscous fluid. The effects of the magnetic field, permeability of the medium, heat absorption and chemical reactions are presented with the help of graphs and tables. During computation, those parameters which are kept constant are mentioned in the graph itself. We have compared the results reported by [11] with the help of **Figure 2** (curve where $M = 0$ and $K_p = 100$). It is found that in the absence of a magnetic field and porous medium, the velocity distribution agrees with [11].

**Figures 2 - 5** show that the velocity increases significantly within a few layers near the plate. This is due to the shearing effect of the motion of the plate. The resistive force i.e Lorentz force reduces the velocity both in the presence ( $K_p = 0.5$ ) or absence ( $K_p = 100$ ) of the porous medium. Moreover, a further decrease is marked due to the resistance offered by the porous matrix. This fact is concomitant to without a magnetic field and porous medium ( $M = 0$ and $K_p = 100$ ) when the velocity attains a maximum. It is also observed that an increase in Grashof number and modified Grashof number accelerate the velocity due to thermo and mass buoyancy forces throughout the flow domain with or without a porous medium. Thus, it is concluded that the effect of buoyancy forces overrides the resistance offered due to magnetic and permeability parameters (**Figures 3** and **5**).

**Figure 4** displays the role of the Prandtl number ($P_r$), an important parameter which measures the relative effect of kinematic viscosity and thermal diffusivity. In the present case, $P_r$ being less than one, the velocity decreases as the value of $P_r$ increases both in the presence or absence of the porous medium.

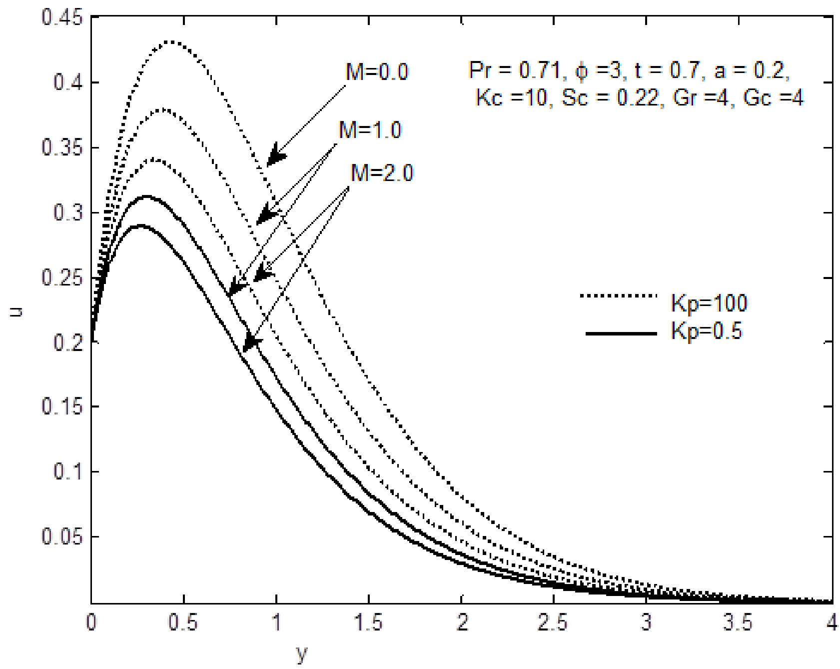


**Figure 2** Variation of M and Kp on velocity profile.

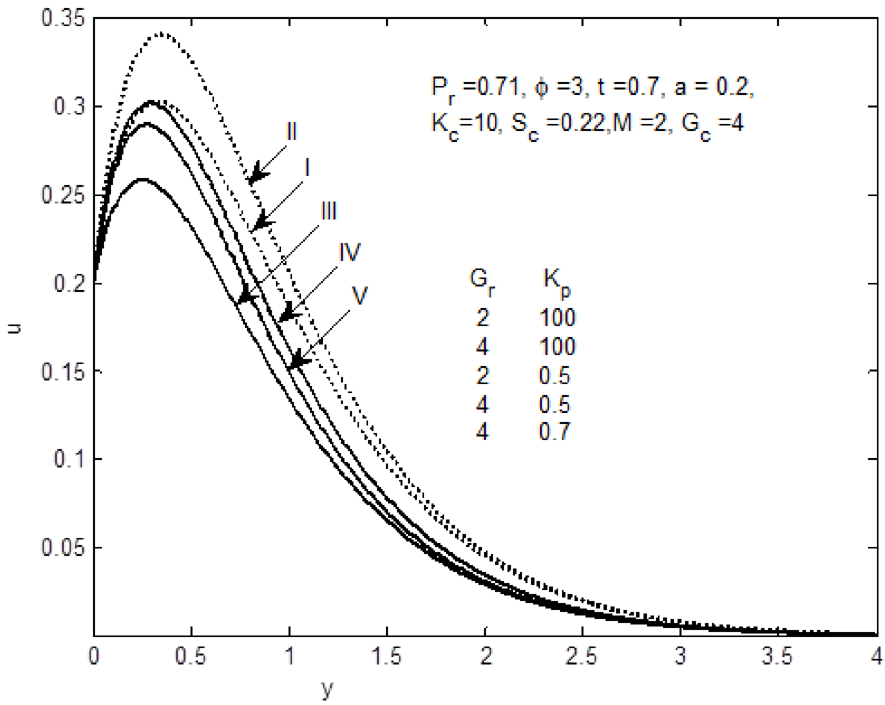


**Figure 3** Variation of Gr on velocity profile.

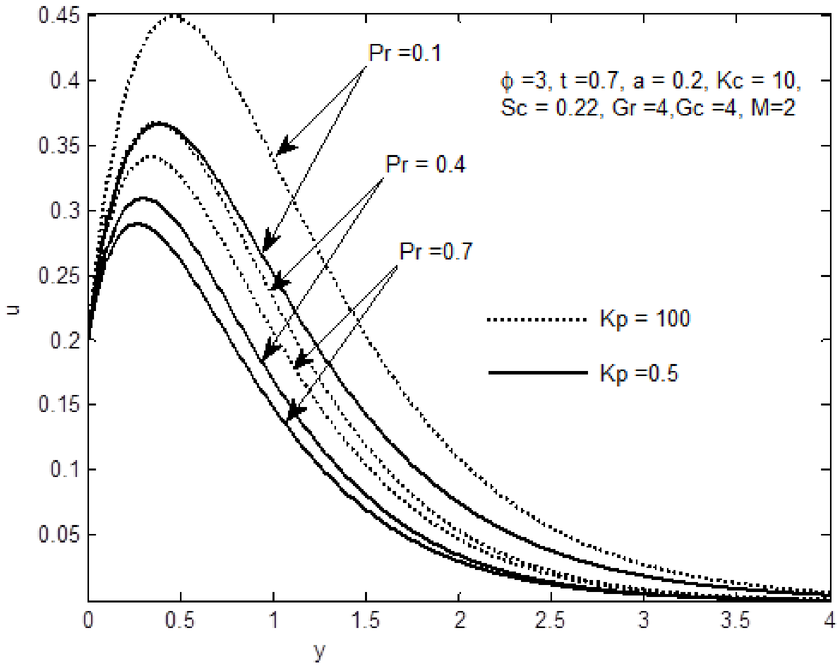


**Figure 4** Variation of Pr on velocity profile.

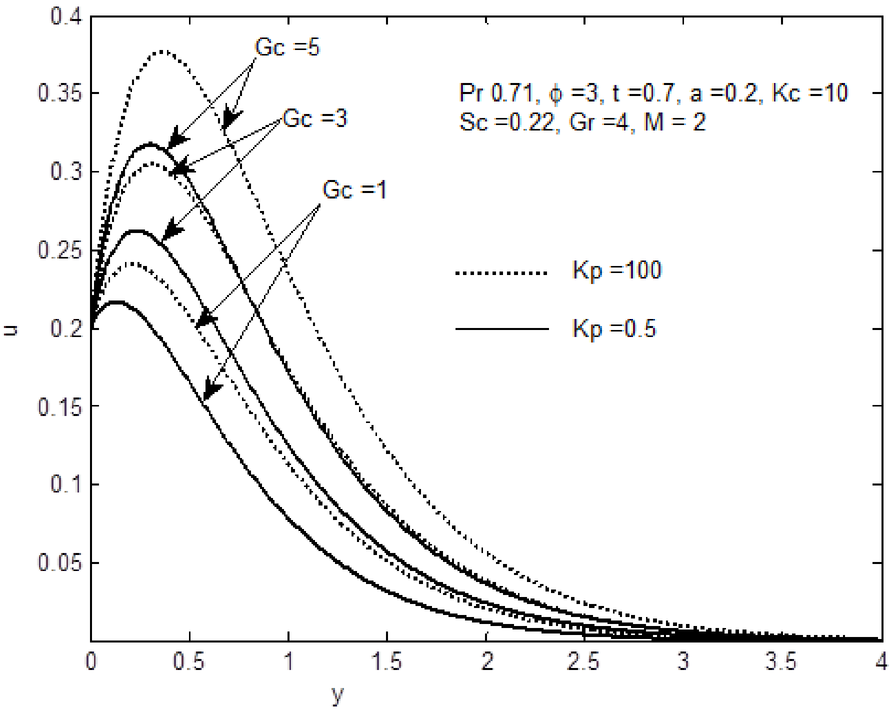


**Figure 5** Variation of Gc on velocity profile.

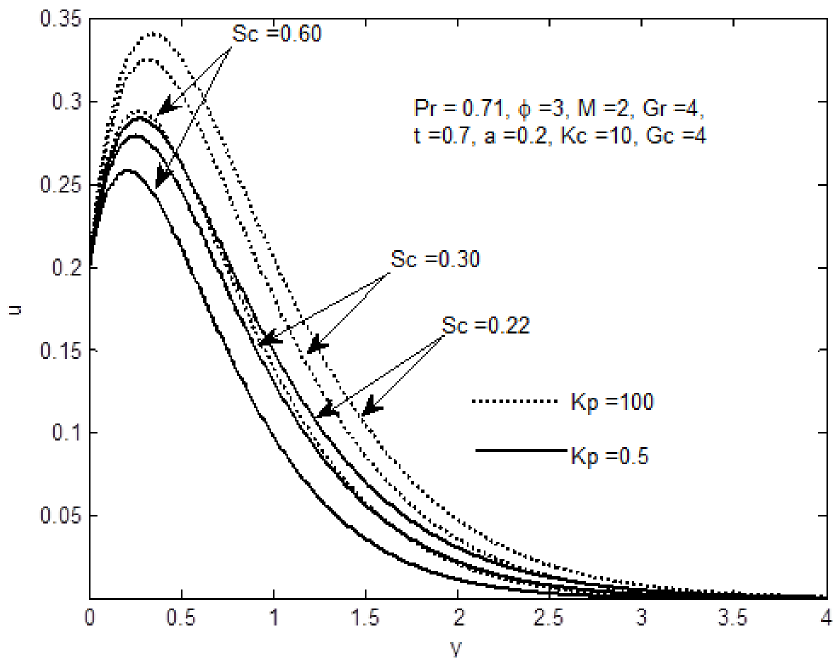


**Figure 6** Variation of Sc on velocity profile.

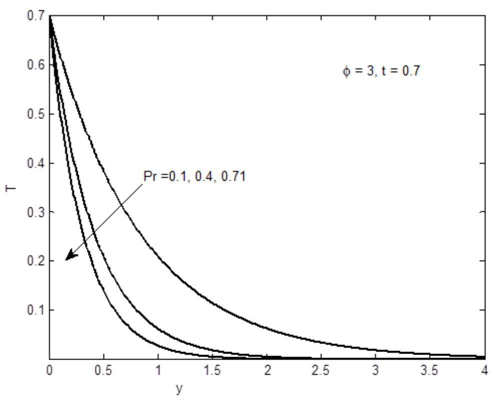


**Figure 7** Variation of Pr on temperature profile.

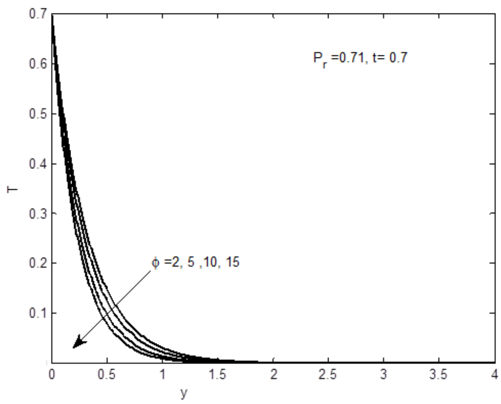


**Figure 8** Variation of ϕ on temperature profile.

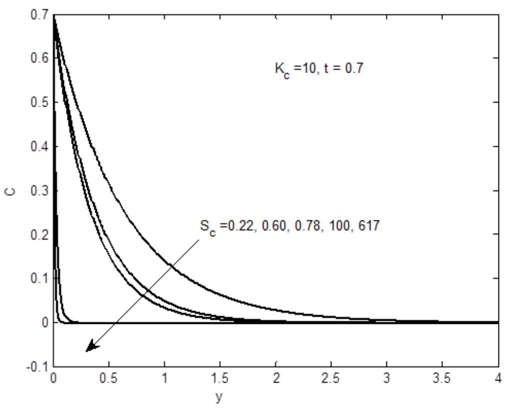


**Figure 9** Variation of Sc on concentration profile.

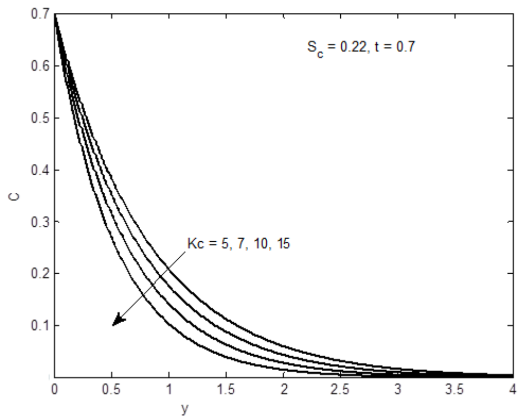


**Figure 10** Variation of Kc on concentration profile.

**Figures 4** and **6** show that the $P_r$ and Schmidt number $(S_C)$ affect the flow in a similar manner. Thus, the response of mass diffusivity property of the fluid is same as that of thermal diffusivity. Further, it is seen that heavier species decrease the velocity also. The data considered here corresponds to Helium and Hydrogen $S_C = 0.30$ and 0.22 in the air medium respectively. From the above observations it is evident that the magnetic field, permeability of the medium, thermal and mass diffusion decelerate the fluid particles resulting in a thinner boundary layer whereas buoyancy forces accelerate them.

**Figures 7** and **8** exhibit the effects of the $P_r$ and internal heat absorption parameter $(\phi)$ on temperature distribution. An increase in $P_r$ and $\phi$ leads to a reduced temperature at all points. An increase in $P_r$ which implies a reduction in thermal diffusivity of the fluid and an increase in $\phi$ that means an increase in the internal heat absorption, obviously reduces the fluid temperature along with an increase in the wall temperature.

**Figure 9** exhibits the concentration distribution for various species in an air medium ($S_C$ = 0.22, 0.60 and 0.78) and aqueous medium ($S_C$ = 100 and 617). The reduction in concentration at all the layers is marked for higher values of $S_c$ i.e. for heavier species, the variation is qualitatively similar to the variation of temperature in respect of $P_r$. One striking result is noticed in the case of aqueous medium with ($S_C$ = 100 and 617). Practically, no variation occurs in concentration in the flow domain only it assumes the plate concentration.

**Figure 10** exhibits the effect of the destructive chemical reaction on the concentration profiles. It is seen that an increase in the first order rate of the destructive chemical reaction causes a decrease in the concentration level at all points.

**Table 1** shows that the presence of a porous medium reduces the skin friction whereas a higher $P_r$ fluid increases it. One striking feature is that the presence of both magnetic field and porous medium renders the skin friction negative whereas in the absence of the porous medium the skin friction remains positive. Therefore, it is suggested that for a streamline flow the strength of the magnetic field and permeability of the porous medium need to be taken care of.

**Table 2** shows that rate of heat transfer increases with a higher $P_r$ fluid whereas the heat absorption parameter decreases it.

**Table 3** presents the value of the Sherwood number ($S_h$), which is a measure of the rate of change of concentration at the surface of the plate. It is seen that an increase in $K_C$ and $S_C$ leads to an increase in the rate of change of concentration at the plate thereby reducing the flow.

**Table 1** Skin friction coefficient (τ).

| **M** | $K_p$ | $P_r$ | $G_r$ | $G_c$ | **ϕ** | **τ** |
|---|---|---|---|---|---|---|
| 2 | 100 | 1.6 | 0.5 | 0.1 | 0.8 | 2.063313 |
| 2 | 0.5 | 1.6 | 0.5 | 0.1 | 0.8 | 0.23049 |
| 3 | 100 | 1.6 | 0.5 | 0.1 | 0.8 | 1.364556 |
| 2 | 100 | 2 | 0.5 | 0.1 | 0.8 | 2.591956 |
| 2 | 100 | 1.6 | 1 | 0.1 | 0.8 | 1.473819 |
| 2 | 100 | 1.6 | 0.5 | 0.5 | 0.8 | 1.99461 |
| 2 | 100 | 1.6 | 0.5 | 0.1 | 0.5 | 2.360807 |
| 2 | 0.5 | 1.6 | 0.5 | 0.1 | 0.8 | 0.23049 |
| 3 | 0.5 | 1.6 | 0.5 | 0.1 | 0.8 | −1.30266 |

**Table 2** Nusselt number ($N_u$).

| $P_r$ | **ϕ** | $N_u$ |
|---|---|---|
| 1.6 | 0.8 | 1.61349617 |
| 1.6 | 0.5 | 1.75021931 |
| 1.8 | 0.8 | 1.75705764 |
| 1.8 | 0.5 | 1.91786758 |
| 2 | 0.8 | 1.89765018 |
| 2 | 0.5 | 2.08290428 |

**Table 3** Sherwood number ($S_h$).

| $S_c$ | $K_c$ | $S_h$ |
|---|---|---|
| 1.3 | 1 | 3.32280003 |
| 1.3 | 2 | 4.00568187 |
| 1.6 | 1 | 3.68631565 |
| 1.6 | 2 | 4.44390504 |
| 1.78 | 1 | 3.8881457 |
| 1.78 | 2 | 4.68721398 |

**Conclusions**

The fluid motion is resisted by the Lorentz force, a force of electromagnetic origin and it is further assisted by the permeability of the porous medium whereas it is accelerated by the buoyancy force. Further, it is concluded that a higher $P_r$ fluid as well as a higher $S_c$ fluid (heavier species) are counterproductive in accelerating fluid motion and it is further assisted by the presence of a porous medium. Fluid with low thermal diffusivity reduces the temperature which is assisted by the presence of heat absorption. The effect of the $P_r$ and $S_c$ are the same on velocity and temperature distribution in the present study. It is interesting to note that the destructive reaction does not enhance the concentration distribution. The rates of change at the boundary surface play a significant role in controlling the entire flow, heat and mass distribution phenomena.

**Acknowledgement**

The authors express sincere thanks to the reviewers for their valuable comments and suggestions. The authors are also thankful to Professor GC Dash, Dr. SR Mishra and Dr. SN Sahoo, Department of Mathematics, Siksha 'O' Anusandhan University for their helpful suggestions.

**References**

[1] YJ Kim. Unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. *Int. J. Eng. Sci.* 2000; **38**, 833-45.
[2] KD Singh and R Sharma. Three dimensional free convective flow and heat transfer through a porous medium with periodic permeability. *Indian J. Pure Appl. Math*. 2002; **33**, 941-9.
[3] AK Singh and NP Singh. Heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity. *Indian J. Pure Appl. Math.* 2003; **34**, 429-42.
[4] S Bathul. Heat transfer in three dimensional viscous flow over a porous plate moving with harmonic disturbances. *Bull. Pure Appl. Sci. E: Math. Stat.* 2005; **24**, 69-97.
[5] A Postelnicu. Influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects. *Int. J. Heat Mass Tran.* 2004; **47**, 1467-72.
[6] A Ogulu and J Prakash. Heat transfer to unsteady magneto hydrodynamics flow past an infinite moving vertical plate with variable suction. *Phys. Scr.* 2006; 74, 232-9.
[7] SS Das, SK Sahoo and GC Dash. Numerical solution of mass transfer effects on unsteady flow past an accelerated vertical porous plate with suction. *Bull. Malays. Math. Sci. Soc.* 2006; **29**, 33-42.
[8] M Ferdows, MA Sattar and MNA Siddiqui. Numerical approach on parameters of the thermal radiation interaction with convection in boundary layer flow at a vertical plate with variable suction. *Thammasat Int. J. Sci. Tech.* 2004; **9**, 19-28.
[9] MS Alam, MM Rahman and MA Samad. Dufour and Soret effects on unsteady MHD free convection and mass transfer flow past a vertical plate in a porous medium. *Nonlinear Anal. Model. Contr.* 2006; **11**, 211-26.
[10] R Muthucumaraswamy, MS Raj and VSA Subramanian. Exact solution of flow past an accelerated infinite vertical plate with heat and mass flux. *Int. J. Appl. Mech. Eng.* 2009; **14**, 585-92.
[11] N Das, GC Dash and JP Panda. Heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity with heat source. *Model. Meas. Contr. B* 2010; **79**, 52-65.
[12] NP Singh and AK Singh. MHD effects on Heat and mass transfer in flow of a viscous fluid with induced magnetic field. *Indian J. Pure Appl. Phys.* 2000; **38**, 182-9.
[13] MK Majumder and RK Deka. MHD flow past an impulsively started infinite vertical plate in the presence of thermal radiation. *Rom. J. Phys.* 2007; **52**, 565-73.
[14] R Muthucumaraswamy, M Sundarraj and V Subhramanian. Unsteady flow past an accelerated infinite vertical plate with variable temperature and uniform mass diffusion. *Int. J. Appl. Math. Mech.* 2009; **5**, 51-6.
[15] GC Dash, PK Rath, N Mohapatra and PK Dash. Free convective MHD flow through porous media of a rotating visco-elastic fluid past an infinite vertical porous plate with heat and mass transfer in the presence of a chemical reaction. *Model. Meas. Contr. B* 2009; **78**, 21-37.
[16] SS Das, A Satapathy, JK Das and JP Panda. Mass transfer effects on MHD flow and heat transfer past a vertical porous plate through a porous medium under oscillatory suction and heat source. *Int. J. Heat Mass Tran.* 2009; **52**, 5962-9.
[17] IJ Uwanta and MN Sarki. Heat and mass transfer with variable temperature and exponential mass diffusion. *Int. J. Comput. Eng. Res.* 2012; **2**, 1487-94.