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Heat Transfer in MHD Squeezing Flow using Brinkman Model

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Abstract

This study investigates squeezing flow of viscous incompressible fluid in a highly permeable medium between 2 parallel, permeable, unsteadily rotating plates in the presence of a magnetic field and radiation. The plates at time t^* are separated by a distance $H(1-\alpha t^*)^{1/2}$. Using a similarity transformation, the governing equations have been transformed into a system of non-linear ordinary differential equations. The resulting equations have been solved numerically by a shooting method. Graphs are presented to depict the temperature and heat transfer profiles. The results show a decline in the temperature profiles under the effect of enhanced radiation.

Keywords: Squeezing flow, permeable plates, porous medium, heat transfer, MHD, radiation

Introduction

The unsteady squeezing flow of a viscous incompressible fluid between 2 parallel plates moving normal to their own planes occurs in many hydro-dynamical machines, particularly in turbo-machinery. The rotor-stator system of turbo-machinery can be idealized by a system of 2 plates, in which one plate is in rotation and the other is stationary. The movement of underground water through hydraulic pumps can be simulated by the motion of 2 impermeable plates moving towards or apart from each other. The high performance of a turbo-machine is attributable to the occurrence of self-sustained, vortex-induced oscillating flow in the radial direction coupled with the effect of the centrifugal and Coriolis forces. These forces induce the heat transfer processes. The 2 plate problem with magnetic field has promising application in the field of hydromagnetic lubrication. These squeezing flows are also useful in polymer processing, compression and injection moulding. Earlier studies of squeezing flows involved the solution of a Reynolds equation. A study involving full Navier-Stokes equations is more useful in the analysis of porous thrust bearing and squeeze films involving high velocities.

Kuhn and Yates [1] considered the pressure distribution of a thin liquid film between axially oscillating parallel circular plates by taking inertia terms into account. Hunt [2] examined pressure distribution in a plane fluid film subject to normal sinusoidal excitation. He showed that the maximum and minimum pressures in the fluid depend on the amplitude of oscillation. He found a good correlation between theoretical and experimental work. A similarity solution for the Navier-Stokes equations was given by Gupta and Gupta [3] for the unsteady flow between 2 plates approaching or receding from each other symmetrically. Bhatt and Hamza [4] presented similarity solutions for the squeeze film between 2 rotating naturally permeable plates.

Rajvanshi [5] considered the flow between 2 parallel plates, when both perform normal sinusoidal oscillations. In this paper, Navier-Stokes equations have been solved for;

(i) where the Reynolds number R , based on maximum velocity of the oscillating plates, is so small that inertia terms can be disregarded,

- (ii) where R is not small, but the terms of the order of εR can be omitted, where ε is the non-dimensional amplitude of the oscillation of the plates,
 - (iii) where neither R nor εR can be neglected.

In a subsequent paper, Rajvanshi [6] studied the effect of slip velocity and axial current induced pinch on load capacity and film thickness-time of squeeze-film between annular plates. Singh and Rajvanshi [7] investigated the flow between 2 parallel pulsating plates. The flow profiles of the fluid squeezed between rotating plates have been a subject matter of extensive investigation with Gauthier *et al.* [8], Schouveiler *et al.* [9] and Serre *et al.* [10].

Many heat transfer phenomena occur in the form of electromagnetic waves which are a result of changes in the electronic configuration of the atoms. Thus, heat transfer by radiation becomes an important tool for various engineering applications, such as nuclear power plants, gas turbines, and space satellites, to name a few.

Hossain and Takhar [11] investigated the effect of radiation on mixed convection along a vertical plate with uniform free stream velocity and surface temperature, using the Roseland diffusion approximation. Hossain et al. [12] studied the radiation effects on flow past a vertical plate with free convection. Free convection flow past a moving plate under the effect of radiation was investigated by Raptis and Perdikis [13]. The magnetohydrodynamic (MHD) flow in the presence of solar radiation was studied by Chamkha [14]. The radiation effects on free convection flow past a semi-infinite vertical plate with mass transfer was presented by Chamkha et al. [15]. The unsteady MHD free convection flow through a porous vertical flat plate immersed in a porous medium with radiation was analyzed by Samad and Rahman [16]. Postelnicu [17] investigated the onset of a Darcy-Brinkman convection using a thermal non-equilibrium model. The analytic approximate solutions for unsteady two-dimensional and axisymmetric squeezing flows between parallel plates were found by Rashidi et al. [18] .The unsteady MHD free convective flow and heat transfer between heated inclined plates with magnetic field in the presence of radiation effects were studied by Sharma et al. [19]. Ali and Shahzad [20] extended the MHD flow to a non-Newtonian fluid past a vertical stretching sheet. They included convective boundary conditions for analyzing the flow. The radiation effects on unsteady flow through a porous medium channel with velocity and temperature slip boundary condition were considered by Chauhan and Kumar [21]. They used the Crank-Nicolson implicit difference scheme to solve the initial value boundary value problem numerically. Recently, Rajput and Sahu [22] analyzed the natural convection in an unsteady hydromagnetic Couette flow through a vertical channel in the presence of radiation using the Laplace transform technique. Rajvanshi et al. [23] investigated the MHD squeezing flow of a viscous incompressible fluid in a highly permeable porous medium contained between two permeable rotating plates using the Brinkman model. The effect of radiation and mass transfer on MHD free convection flow past an impulsively started isothermal vertical plate with dissipation was studied by Sangapatnam et al. [24]. The influence of MHD, radiation, and mass transfer on unsteady free convection flow past a heated vertical plate in a porous medium with viscous dissipation was investigated by Prasad and Reddy [25].

In quite a few industrial applications, the lubrication fluid is injected into the main flow to avoid contact between the 2 plates. For air-lubricated bearings, the injection is made through a porous medium. Therefore, in the present study, the squeezing flow of a viscous incompressible fluid in a highly permeable porous medium contained between 2 permeable rotating plates is studied using the Brinkman [26] model. The problem has applications to special porous media, such as mushy zones and ferromagnetic fluids, where magnetic drag plays a significant role. The solutions are obtained for an optically thin medium with relatively low density, and are valid for fluids with a thickness much larger than the wall roughness. The numerical results showing the effect of radiation on temperature and heat transfer are analyzed graphically.

Materials and methods

We consider a thin film of a highly permeable medium saturated with Newtonian fluid squeezed between two parallel plates with different permeability. The plates placed at a distance $h(t^*)$, at any time t^* , are allowed to rotate in their own planes about the z^* -axis with different angular velocities. The upper plate is set in motion along the z^* -axis with velocity $\frac{dz^*}{dt^*}$ towards the lower plate, which remains at a fixed position $z^*=0$.

The governing equations for flow through porous medium as suggested by Brinkman [26] are;

$$\frac{\partial u^*}{\partial r^*} + \frac{u^*}{r^*} + \frac{\partial w^*}{\partial z^*} = 0 \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} + u^* \frac{\partial u^*}{\partial r^*} + w^* \frac{\partial u^*}{\partial z^*} - \frac{v^{*2}}{r^*} = -\frac{1}{\rho} \frac{\partial p}{\partial r^*} + \bar{v} \left(\frac{\partial^2 u^*}{\partial r^{*2}} + \frac{1}{r^*} \frac{\partial u^*}{\partial r^*} - \frac{u^*}{r^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}} \right) - \frac{vu^*}{k} - \frac{\sigma B^2 u^*}{\rho}$$
(2)

$$\frac{\partial v^*}{\partial t^*} + \frac{u^*v^*}{r^*} + u^* \frac{\partial v^*}{\partial r^*} + w^* \frac{\partial v^*}{\partial z^*} = \bar{v} \left(\frac{1}{r^*} \frac{\partial v^*}{\partial r^*} - \frac{v^*}{r^{*2}} + \frac{\partial^2 v^*}{\partial r^{*2}} + \frac{\partial^2 v^*}{\partial z^{*2}} \right) - \frac{v v^*}{k} - \frac{\sigma B^2 v^*}{\rho}$$
(3)

$$\frac{\partial w^*}{\partial t^*} + u^* \frac{\partial w^*}{\partial r^*} + w^* \frac{\partial w^*}{\partial z^*} = -\frac{1}{\rho} \frac{\partial p}{\partial z^*} + \bar{v} \left(\frac{\partial^2 w^*}{\partial r^{*2}} + \frac{1}{r^*} \frac{\partial w^*}{\partial r^*} + \frac{\partial^2 w^*}{\partial z^{*2}} \right) - \frac{v}{k} w^* \tag{4}$$

$$\rho C_p \left(\frac{\partial T^*}{\partial t^*} + u^* \frac{\partial T^*}{\partial r^*} + w^* \frac{\partial T^*}{\partial z^*} \right) = \bar{\kappa} \left[\frac{1}{r^*} \frac{\partial}{\partial r^*} \left(r^* \frac{\partial T^*}{\partial r^*} \right) + \frac{\partial^2 T^*}{\partial z^{*2}} \right] - \frac{\partial q^*}{\partial v^*}$$
 (5)

where u^* , v^* , w^* are the velocity components in the direction of r^* , θ^* and z^* respectively, and p the pressure, v the kinematic viscosity, v the effective kinematic viscosity in porous medium, v the electrical conductivity, v the specific heat at constant pressure, v the permeability of the porous medium, v the effective thermal conductivity in porous medium, v the radiation heat flux, v the fluid density, and v the magnetic field. Owing to symmetrical considerations, v to v the fluid density, and v the magnetic field.

Assuming the medium to be optically thin and with relatively low density, the radiative heat flux as represented by Cogley *et al.* [27] is taken as;

$$\frac{\partial q^*}{\partial y^*} = 4I(T^* - T_0)$$
 with $I = \int_0^\infty K_{\lambda W} \frac{\partial P}{\partial T^*} d\lambda$

where $K_{\rm w}$ is the absorption coefficient at the wall and P is Planck's constant.

With a view to make the physical quantities dimensionless, we introduce the characteristic length, time, and angular velocity as H, α^{-1} and Ω respectively. The lower and the upper plate are assumed to rotate with angular velocities $\Omega_1(1-\alpha t^*)^{-1}$ and $\Omega_2(1-\alpha t^*)^{-1}$ respectively. The permeabilities of the lower plate, the upper plate and the porous medium are taken in the forms $k_1(1-\alpha t^*)$, $k_2(1-\alpha t^*)$ and $k_0(1-\alpha t^*)$, respectively. The solution of governing equations is valid for $t^* < \frac{1}{\alpha}$.

The boundary conditions on the plates are assumed in the form;

$$\frac{\partial u^*}{\partial z^*} = \frac{\sigma_1 u^*}{\sqrt{k_1^*}}, \quad \frac{\partial v^*}{\partial z^*} = \frac{\sigma_1}{\sqrt{k_1^*}} \left(v^* - \frac{\Omega_1 r^*}{1 - \alpha t^*} \right), w^* = 0 \quad \text{on } z^* = 0$$

$$\frac{\partial u^*}{\partial z^*} = -\frac{\sigma_2 u^*}{\sqrt{k_2^*}}, \quad \frac{\partial v^*}{\partial z^*} = -\frac{\sigma_2}{\sqrt{k_2^*}} \left(v^* - \frac{\Omega_2 r^*}{1 - \alpha t^*} \right), w^* = \frac{dh}{dt^*} \quad \text{on } z^* = h(t^*)$$
(6)

where σ_1 and σ_2 represent slip parameters for lower and upper disks respectively. Following Wang [28], we introduce the following non-dimensional quantities;

$$u^*(y) = \frac{\alpha \, r^* \, f'(y)}{2(1 - \alpha t^*)} \,, \quad v^*(y) = \frac{r^* \Omega \, g(y)}{(1 - \alpha t^*)} \,, \quad w^*(y) = \frac{-\alpha \, H \, f(y)}{\sqrt{(1 - \alpha t^*)}}$$
(7)

and

$$B = \frac{B_0}{\sqrt{1-\alpha t^*}}, L = \frac{4I(1-\alpha t^*)}{\rho C_n \alpha}$$
 (8)

The temperature T^* is given by;

$$T^* = \frac{T_0 a(y)}{1 - \alpha t^*} + T_0 \tag{9}$$

where

$$y = \frac{z^*}{H\sqrt{1-\alpha t^*}}\tag{10}$$

Non-dimensional quantities are also introduced as $\phi_1 = \frac{\overline{\nu}}{\nu}$, $\phi_2 = \frac{\overline{x}}{x}$, where κ is the thermal conductivity of the fluid.

With a view to maintain uniformity, we replace $r^* \to r$, $z^* \to z$ and $t^* \to t$.

Using Eqs. (7) - (10), in the governing Eqs. (2) - (5), we obtain;

$$\frac{1}{\rho} \frac{\partial p}{\partial r} = \frac{r\alpha^2}{4(1-\alpha t)^2} \left[\frac{f'''}{Re^s} - y f'' - 2f' + 2f f' - (f')^2 + N^2 g^2 - \frac{\beta f'}{Re^s} \right]$$
(11)

$$\frac{g''}{Re^{s}} - \frac{\beta g}{Re^{s}} - y g' - 2g - 2f'g + 2f g' = 0$$
 (12)

$$\frac{1}{\rho} \frac{\partial p}{\partial y} = \frac{-\alpha^2 H}{2(1-\alpha t)} \left[\frac{f''}{Re^s} - y f' - f' + 2f f' - 2\lambda_4 f \right]$$
 (13)

$$\frac{\phi_2 a''}{\phi_1 Pr Re^s} + 2 f a' - a' y - 2a(L+1) = 0$$
 (14)

where $M^2 = (H B_0)^2 \sigma \mu^{-1}$, Re^S (Squeeze Reynolds Number) $= \frac{\rho \alpha H^2}{2 \overline{\mu}}$,

$$N = \frac{2\Omega}{\alpha}, \lambda_3 = \frac{H}{\sqrt{k_0}}, \lambda_4 = \frac{v}{\alpha k_0}, \beta = \frac{1}{\phi_1} (\lambda_3^2 + M^2)$$
 (15)

Eq. (1) is satisfied identically.

N may also be interpreted as follows;

$$N = \frac{2\Omega}{\alpha} = \frac{Re^{R}}{Re^{s}} \tag{16}$$

Where $Re^R = \frac{\rho H^2}{\mu}$ is the Rotational Reynolds number. Therefore, N defines the ratio of the Rotational Reynolds number to Squeeze Reynolds number.

Elimination of p between Eqs. (11) and (13) gives;

$$f^{iv} = Re^{s}(3f'' + yf''' - 2ff''' - 2N^{2}gg') + \beta f''$$
(17)

It is assumed that the plates are maintained at different temperatures given by;

$$T^* = T_0 \text{ on } z^* = 0$$

$$T^* = T_0 + \frac{T_0}{(1 - \alpha t^*)}$$
 on $z^* = h(t^*)$ (18)

The modified boundary conditions are;

$$f''(0) = \lambda_1 f'(0), g'(0) = \lambda_1 \{g(0) - 1\}, f(0) = 0, a(0) = 0$$

$$f''(1) = -\lambda_2 f'(1), g'(1) = -\lambda_2 \{g(1) - s\}, f(1) = 1/2, a(1) = 1$$
 (19)

The slip parameters are given by;

$$\lambda_i = \frac{\sigma_i H}{\sqrt{k_i}} \tag{20}$$

where σ_i , i = 1, 2 represents the slip parameter for the lower and upper plates respectively. Ratio of the rotational velocities of the plates is defined as;

$$S = \frac{\Omega_2}{\Omega_1} \tag{21}$$

such that;

s = 1, depicts that the plates are rotating with the same angular velocities in the same direction,

s > 0, shows that the plates are rotating in the same direction,

s = 0, depicts that the upper plate is stationary, and

s < 0, relates to the rotation of the plates in opposite directions.

Results and discussion

The MATLAB differential equation solver bvp4c is used to find the numerical solution of the problem. For large mechanical devices, such as turbine generator rotor lines, the fluid is oil. Air is used for lighter mechanisms, such as dentist drills or computer disk drives. We have drawn the temperature profiles for Pr = 11.4 and Pr = 0.71. The variation in radiation parameter has been calculated and depicted graphically.

Temperature profiles

Figure 1 shows the variation of temperature profiles of the porous medium under the effect of radiation with M = 0.01, N = 80, Pr = 11.4, $Re^S = 0.02$, $\lambda_1 = \lambda_2 = 0.05$, $\phi_1 = \phi_2 = 1$, s = 0.5. At large Prandtl numbers the momentum transfer plays a dominant role. By increasing the radiation parameter, the transfer of heat from the boundary increases. This results in decrease in the temperature profiles for Pr = 11.4. The effect of radiation for Pr = 0.71 on the temperature profiles has been depicted in **Figure 2**, with M = 0.01, N = 80, Pr = 0.71, $Re^S = 0.02$, $\lambda_1 = \lambda_2 = 0.05$, $\phi_1 = \phi_2 = 1$, s = 0.5. At low Prandtl numbers, the ratio of momentum transfer to the heat transfer is small. There is decrease in the temperature of the porous medium squeezed between the plates with increase in radiation. Here, radiation plays a dominant role. The temperature profiles do not have a significant qualitative difference.

In **Figure 3**, the variation of the parameter ϕ_1 on the temperature profile has been depicted with M = 0.01, N = 80, L = 0.4, P = 11.4, $Re^S = 0.02$, $\lambda_1 = \lambda_2 = 0.05$, $\phi_2 = 1$, s = 0.5. As the influence of effective kinematic viscosity is increased, the temperature profiles start decreasing. The effect of variation of ϕ_2 on the temperature profiles is also shown in **Figure 4**, with M = 0.01, N = 80, L = 0.4, P = 11.4, P = 11.4

Heat transfer profiles

Following Hamza [29], the Nusselt number is defined as;

$$Nu^* = -\frac{H}{T_0} \frac{\partial T}{\partial z^*} = -\frac{\alpha'(y)}{(1-\alpha t)^{3/2}}$$

We further define;

$$Nu = Nu^* (1 - \alpha t)^{3/2} = -a'(y)$$
(22)

The heat transfer profiles for Pr = 11.4 and Pr = 0.71, versus permeability of the lower plate along the x-axis, have been depicted in **Figures 5** and **6**, with M = 0.01, N = 80, $Re^S = 0.02$, $\lambda_2 = 0.05$, $\phi_1 = \phi_2 = 1$, s = 0.5. As the radiation parameter is increased, the heat transfer first decreases, and after the value $\lambda_1 = 0.58$ with $\alpha(y) = 0.9983$, it starts increasing for both Pr = 11.4 and Pr = 0.71.

The heat transfer profiles for Pr = 11.4 and Pr = 0.71, versus permeability of the upper plate along the x-axis, have been depicted in **Figures 7** and **8**, with M = 0.01, N = 80, $Re^S = 0.02$, $\lambda_1 = 0.05$, $\phi_1 = \phi_2 = 1$, s = 0.5. The heat transfer increases as permeability of the upper plate is increased. Under the effect of enhanced radiation, the heat transfer profiles decrease, but with increase in the permeability of upper plate, these decrease. The trend is reversed after λ_2 takes the value 0.58, with $\alpha(y) = 1$ for both graphs.

Conclusions

Under the enhanced effect of radiation, the temperature profiles of the porous medium squeezed between the plates are lowered both for air and water at freezing point. The increase in the ratio of effective thermal conductivity in porous medium to thermal conductivity of the fluid results in an increase in the temperature profiles. The heat transfer profiles show a change in behavior and, with an increase in radiation, the magnitude is enhanced. The effect has been analyzed for air and water at freezing point.

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