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Unsteady MHD Pulsatile Blood Flow through Porous Medium in Stenotic Channel with Slip at Permeable Walls Subjected to Time Dependent Velocity (Injection/Suction)

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Abstract

The flow through porous boundaries is of great importance, both in technological and biophysical flows. The present paper is concerned with the study of unsteady pulsatile flow of blood through porous medium in a time dependent constricted porous channel subjected to time dependent suction/injection at the walls of the channel. The blood flow is subjected to a constant transverse magnetic field, considering blood as an incompressible electrically conducting fluid. Due to the permeability of the arterial wall, the no-slip condition at the wall is no longer valid, and one has to consider the slip condition at the channel wall because of more realistic approach. Perturbation analysis is used to solve the system of equations governing the flow. With a view to illustrating the applicability of the mathematical model developed here, the analytic explicit expressions of axial velocity, volumetric flow rate and wall shear stress are obtained. The computed numerical results are presented graphically for different values of the physical parameters of interest, to depict the variations in axial velocity, volumetric flow rate and wall shear stress.

Keywords: Pulsatile blood flow, time dependent stenosis, porous channel, slip velocity, suction/injection, MHD, porous media

Introduction

The study of the flow through a channel with permeable walls not only possesses a theoretical appeal, but also models biological and engineering systems. The major activity of the entire cardiovascular system is to supply blood to tissues under a sufficient pressure gradient in order to exchange materials through the arterial wall. This is a 2-way exchange: the nutrients are carried to tissues and cells, and the fluid returns, along with the waste from cellular metabolism. Small arteries are thin-walled and consist of endothelial cells. They contain ultra-microscopic pores through which substances of various molecular sizes can penetrate inside and pass into the lumen of the arteries from the surrounding tissues. One of the most important features of small arteries is the permeability of their walls. Examples of this are found in living organisms are fluid transport mechanisms, for example, blood flow in the circulatory system and transpiration cooling.

Many investigators have theoretically studied the flow of blood through permeable walls. Elshehawey and Husseny [1] studied the peristaltic transport of a magneto-fluid with porous boundaries. Fluid entering the flow region through one plate at the same rate as it left through the other plate was considered. Sinha and Misra [2] investigated the blood flow through an artery with permeable wall. Makinde and Osalusi [3] studied steady magnetohydrodynamics (MHD) flow in a 2 dimension channel with permeable boundaries. Steady MHD flow through a circular vertical pipe with permeable boundaries has been investigated by Elangovan and Ratchagar [4]. Makinde and Chinyoka [5] studied the unsteady MHD flow in a porous 2 dimension channel with one wall impermeable and the other porous. Recently, Sattar and Waheedullah [6] investigated the unsteady flow of a viscoelastic fluid through porous medium bounded by two porous plates. It was assumed that one plate is injected with certain constant velocity and that the other sucked it off with the same velocity. Xin-Hui Si *et al*. [7] studied the asymmetric laminar flow in a porous channel with expanding or contracting walls. Homotopy analysis method (HAM) was employed to obtain expressions for the velocity fields.

Due to the permeability of the arterial wall, the no-slip condition at the wall is no longer valid, and one has to consider the slip condition at the artery wall. The slip condition plays an important role in shear skin, spurt and hysteresis effects. The boundary conditions relevant to flowing fluids are very important in predicting fluid flows in many applications. The fluids that exhibit boundary slip have important technological applications, such as in polishing valves of artificial hearts and internal cavities [8]. For many fluids, such as particulate fluids, the motion is still governed by the Navier-Stokes equations, but the usual no-slip condition at the boundary should be replaced by the slip condition given by Beavers and Joseph [9]. In this case, the use of slip boundary condition in preference to the no-slip condition was due to the fact that the walls allowed the fluid particles to slip. Much recent research has been made in the subject of slip boundary conditions [2-4,8,10-12].

In human circulatory system, blood flow, under normal conditions, depends on the pumping action of the heart. The pumping action of the heart produces a pressure gradient throughout the arterial and venous network. Pulsatile flow occurs in many areas of engineering fluid dynamics, like pressure surges in pipelines, cavitations in hydraulic systems, pumping of slurries, refrigeration systems, combustion mechanisms, de-watering devices, and cardiovascular biomechanics. Considerable attention has been given to the study of the problems of pulsatile flow of fluids in channels of various cross-sections, due to their increasing application in the analysis of blood flow and in the flows of other biological fluids [13].

The study of pulsatile flow in a porous channel or porous pipe has recently becomes the object of scientific research because of its importance in some practical phenomena, such as transpiration cooling and gaseous diffusion. Particularly, the study of pulsatile flow in a porous channel is useful in understanding the process of dialysis of blood in artificial kidneys and in industrial applications in relation to heat exchange efficiency. Also, the pulsatile flow between permeable walls is important in understanding blood flow in the circulatory system, where the nutrients are supplied to tissues of various organs and waste products are removed. In 1971, Wang [14] studied the interesting problem of pulsatile flow in a porous channel bounded by rigid walls. Many researchers studied the effect of slip velocity at permeable boundaries [2-4]. Recently, Eldesoky [8] investigated the unsteady pulsatile flow of blood through porous medium in an artery under the influence of periodic body acceleration and slip condition in the presence of magnetic field, considering blood as an incompressible electrically conducting fluid.

Most of the researchers dealing with steady incompressible laminar flow with uniform injection or suction have attempted to determine the axial pressure variation, wall shear stress on the porous walls, and shapes of the velocity profiles within the tube. The unsteady suction problem was considered by Tsangaris *et al.* [15]. The case of periodic suction for flow through parallel plates was considered by Ramanamurthy *et al.* [16]. The steady flow of micropolar fluid through a circular pipe under a transverse magnetic field with constant suction/injection at the walls of the tube has been investigated by Murthy and Bahali [17]. Recently, many researchers studied the injection/suction at the permeable walls [18-20].

The study of flow of an electrically conducting fluid through a channel with permeable walls not only possesses a theoretical appeal but also provides a model for many biological and engineering problems, such as MHD generators, plasma studies, nuclear reactors, geothermal energy extraction, the boundary layer control in the field of aerodynamics, blood flow problems, etc. The application of MHD in physiological flow is of growing interest. The flow of blood can be controlled by applying the appropriate magnetic field. Many researchers have shown that blood is an electrically conducting fluid. The Lorentz force will act on the constituent particles of blood, and this force will oppose the motion of blood and thus reduce its velocity. This decelerated blood flow may help in the treatment of certain cardiovascular diseases and in diseases with accelerated blood circulation, such as hypertension, hemorrhage etc. Therefore, it is essential to study the blood flow in presence of a magnetic field. Much work has been done in this field by various investigators [8,13,21,22].

Atherosclerosis is a disease of the cardiovascular system which involves a hardening of the arteries due to the deposition of plaque. Localized atherosclerotic constrictions in arteries, known as arterial stenoses, are predominantly found in the internal carotid artery which supplies blood to the brain, the coronary artery which supplies blood to the cardiac muscles, and the femoral artery which supplies blood to the lower limbs. Blockage of more than about 70 % (by area) of the artery is considered clinically significant, since it presents significant health risks for the patient [23]. Complete closure of the artery can occur if a blood-clot becomes lodged in the stenosis, and this can lead to a stroke or a heart attack. In addition to this, moderate as well as severe stenoses can have long-term health consequences. First, the presence of a constriction results losses which can reduce blood supply through the artery and also impose an additional load on the heart. Secondly, the fluctuations in the blood flow downstream of the stenosis can damage and weaken the internal wall (intima) of the artery. It is accepted that both wall pressure and shear stress play a role in this.

For many decades, cardiovascular disease has been one of the most severe diseases, causing a large number of deaths worldwide each year, especially in developed countries. Most of these cases are associated with some form of abnormal flow of blood in stenotic arteries. In the presence of a stenosis, normal blood flow through the artery is disturbed, resulting in blood recirculation and wall shear stress oscillation near the stenosis. The heart has to increase blood pressure to impel the blood to pass through the narrowing region so as to enforce blood circulation. If the heart works too hard and the blood cannot flow well, heart attack may occur. In order to understand blood flow behavior in arteries so as to provide sufficient information for clinical purposes, intensive research has been carried out worldwide for both normal and stenotic arteries. Mekheimer *et al*. [24-26] studied the unsteady pulsatile flow through a vertical constricted annulus with heat transfer and investigated magnetic field and Hall current influences on blood flow through a stenotic artery. They also studied induced magnetic field influences on blood flow through an anisotropically tapered elastic artery with overlapping stenosis in an annulus. The effects of MHD and hematocrits on blood flow in an artery with multiple mild stenosis have been investigated by Verma and Parihar [27]. Nagarani and Sarojamma [28] investigated the effect of body acceleration on pulsatile flow of Casson fluid through a mild stenosed artery. Mishra and Verma [29] studied the effect of porous parameter and stenosis on wall shear stress for the flow of blood in human body. Sinha *et al*. [30] studied the mathematical modeling of blood flow in a porous vessel with double stenosis in the presence of an external magnetic field.

The main objective of the present paper is to study the combined effect of magnetic field and permeable wall slip velocity on the unsteady pulsatile flow of blood through a porous medium in a time dependent constricted porous channel subjected to time dependent suction/injection at the walls of the channel, considering blood as an incompressible electrically conducting fluid. The governing continuity and Navier-Stokes equations are solved by perturbation technique. In the following sections, the problem is formulated, analyzed and discussed.

Mathematical model

The simulation model of the stenosed porous channel is depicted in **Figure 1**. The channel walls are located at a distance 2H apart with reference to a Cartesian coordinate system (x, y). Let the x-axis be taken along the axis of the channel, while the y-axis is the transverse coordinate normal to the x axis. Let us consider the pulsatile flow of blood as an electrically conducting, unsteady, viscous, incompressible and Newtonian fluid in the presence of a constant magnetic field of strength B_o acting perpendicular to the channel. We assume that the magnetic Reynolds number of the flow is taken to be small enough that the induced magnetic and electric field can be neglected. We consider that the wall of the channel is permeable, so that finite fluid exchange can take place across the wall. The channel is filled with a homogeneous porous media.

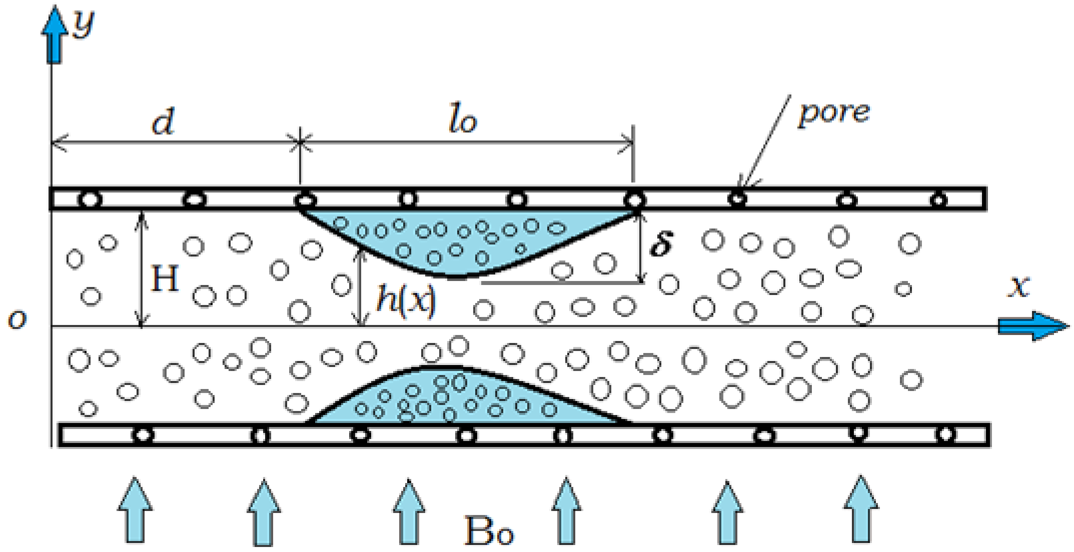

Figure 1 Schematic representation of the model geometry.

Under the above assumptions, in two dimensions, the governing equations of continuity and momentum (in x and y-direction) are given in dimensional form as;

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\left(\frac{\upsilon}{k}\right)u-\frac{\sigma B_O^2 u}{\rho}, \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\upsilon\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\left(\frac{\upsilon}{k}\right)v, \tag{3}$$

where u denotes the velocity component along the x-axis, v the velocity component along the y-axis, t time, υ the kinetic viscosity, p the blood pressure, ρ the density of the blood, σ the electrical conductivity, and k the permeability of porous medium.

The absorption of fluid at the walls is accounted by prescribing the flow flux as Makinde and Chinyoka [5];

$$\int_{-H}^{H} u\,dy = U f\left(\frac{x}{2H}\right)\left(1+\varepsilon e^{i\omega t}\right), \tag{4}$$

where U is the initial characteristic flow velocity (i.e. at $x = 0$) and $f\left(x/2H\right)$ is the flux function that describes the rate of fluid absorption through the permeable wall. We assume that the fluid is injected or sucked off through the channel walls with a time dependent velocity V given by;

$$V = V_O\left(1+\varepsilon e^{i\omega t}\right), \tag{5}$$

where V_O is the uniform transpiration velocity (for injection $V_O > 0$ and for suction $V_O < 0$), ε is the small amplitude of oscillation and the value of ε is necessarily less than unity $\varepsilon < 1$. Since the fluid medium is filled with homogeneous porous material and the normal component of velocity $v = V$ is independent of x and y, Eq. (1) reduces to $\frac{\partial u}{\partial x} = 0$. Therefore, the axial velocity u is function of y and t only $u = u\left(y,t\right)$ and Eq. (3) simply reduces to $-\frac{\partial p}{\partial y} = \rho V_o\left(i\,\omega\varepsilon e^{i\omega t}\right)+\frac{\mu V_o}{k}\left(1+\varepsilon e^{i\omega t}\right)$.

Under the above assumptions, the axial momentum equation reduces to;

$$\frac{\partial u}{\partial t}+V_O\left(1+\varepsilon e^{i\omega t}\right)\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\frac{\partial^2 u}{\partial y^2}-\left(\frac{\upsilon}{k}\right)u-\frac{\sigma B_O^2 u}{\rho}. \tag{6}$$

Since the flow of blood is pulsatile heart pumping, the unsteady pressure gradient can be approximated as [8,13];

$$-\frac{\partial p}{\partial x} = P_S+\varepsilon P_O\cos\left(\omega t\right), \tag{7}$$

where P_S is the steady part of the pressure gradient and P_O is the pulsatile amplitude.

The geometry of the stenosis, which is assumed to be symmetric, is given by [30-32];

$$\begin{aligned} h'(x) &= H\left[1-\frac{\delta}{2}\left\{1+\cos\frac{2\pi}{l_o}\left(x-d-\frac{l_o}{2}\right)\right\}\right]\left(1-\left[\cos(\omega t)-1\right]\varepsilon e^{-\omega\varepsilon t}\right), \quad d \le x \le d+l_o \\ &= H\left(1-\left[\cos(\omega t)-1\right]\varepsilon e^{-\omega\varepsilon t}\right), \qquad otherwise \end{aligned} \tag{8}$$

where δ is the maximum projection (height of the throat) of the stenosis located at $x = d+\frac{l_o}{2}$, l_o is the stenosis length, and *d* indicates its location. Also, $h'(x)$ is the variable height of the channel at the stenosed portion, and ε is the amplitude of oscillation.

Assuming that the flow of the blood is symmetric about the centerline of the channel (y=0), we focus our attention to the flow in the region $0 \le y \le h(x)$ only.

For small permeability, the boundary condition proposed by Beavers and Joseph [9] was simplified by Saffman [33] as $\frac{du}{dy} = \frac{\eta}{\sqrt{k_1}}u$ where η is a constant depending only upon the properties of the porous material and not on its structure, and k_1 is the permeability of porous material of the wall. We can replace the value of $\frac{\eta}{\sqrt{k_1}}$ by β. This condition holds well in the case of unsteady flows, and even if we take MHD effects into account [34]. Although the slip condition looks simple, analytically it is much more difficult than the no-slip condition, and the boundary conditions on the wall of the porous channel are prescribed as follows;

$$\frac{\partial u\left(y,t\right)}{\partial y} = 0 \quad \text{at} \quad y = 0 \tag{9}$$

$$u(y,t) = \beta\frac{\partial u(y,t)}{\partial y} \quad \text{at} \quad y = h(x), \tag{10}$$

Let us introduce the following dimensionless quantities;

$$u^* = \frac{u}{V_O}, \quad x^* = \frac{x}{H}, \quad y^* = \frac{y}{H}, \quad t^* = \frac{t\,\omega}{2\pi}, \quad P^* = \frac{P}{\rho\,V_O^2}, \quad \tau^* = \frac{\mu\,V_O}{H}\tau \tag{11}$$

Using these dimensionless quantities in Eqs. (6), (9) and (10) then drop the stars we obtain;

$$\frac{\alpha^2}{2\pi\,\mathrm{Re}}\frac{\partial u}{\partial t} + \left(1+\varepsilon\,e^{2\pi i t}\right)\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{\mathrm{Re}}\frac{\partial^2 u}{\partial y^2} - \left(M + \frac{1}{\lambda}\right)u, \tag{12}$$

$$\frac{\partial u\left(y,t\right)}{\partial y} = 0 \quad \text{at} \quad y = 0, \tag{13}$$

$$u(y,t) = Kn\,\frac{\partial u(y,t)}{\partial y} \quad \text{at} \quad y = h(x). \tag{14}$$

The geometry of the stenosis in dimensionless form is given by;

$$\begin{aligned} h(x) &= \left(1-\left[\cos(2\pi t)-1\right]\varepsilon e^{-2\pi\varepsilon t}\right)\left[1-\frac{\delta}{2}\left\{1+\cos\frac{2\pi}{l_o}\left(x-d-\frac{l_o}{2}\right)\right\}\right], \quad d \le x \le d+l_o \\ &= 1-\left[\cos(2\pi t)-1\right]\varepsilon e^{-2\pi\varepsilon t}, \quad \textit{otherwise} \end{aligned} \tag{15}$$

where the magnetic parameter M , the Womersley parameter α, the Reynolds number Re, the permeability parameter of porous medium (Darcian linear drag parameter) λ and the Knudsen number Kn are defined respectively by;

$$M = \frac{\sigma B_o^2 H}{\rho V_o}, \qquad \alpha = H\sqrt{\frac{\rho\omega}{\mu}}, \qquad \mathrm{Re} = \frac{H V_o}{\upsilon}, \qquad \lambda = \frac{k V_o}{\upsilon H} \qquad and \qquad Kn = \frac{\beta}{H} \tag{16}$$

Solution of the problem

We seek the solution of the governing equations on the form;

$$u(y,t) = u_o(y) + \varepsilon u_1(y)\, e^{2\pi i\, t}, \tag{17}$$

where ε is the small amplitude of oscillation, and hence we can assume square and higher order terms of ε to be of negligibly small magnitude. Also, u_o and u_1 are the velocity of steady state and transient state respectively. Substituting (17) into (12), (13) and (14), and comparing the coefficients of zero and first order terms of ε on both sides, we obtain;

(i) Steady state (zero order)

$$\frac{1}{\mathrm{Re}}\frac{d^2u_o}{dy^2} - \frac{du_o}{dy} - \left(M + \frac{1}{\lambda}\right)u_o = -P_S, \tag{18}$$

Subject to boundary conditions;

$$\frac{du_o}{dy} = 0 \qquad \text{at} \qquad y = 0, \tag{19}$$

$$u_o = Kn\frac{d\,u_o}{d\,y} \qquad \text{at} \qquad y = h(x). \tag{20}$$

By solving (13) we can get the general solution on the form;

$$u_o = C_1 e^{m_1 y} + C_2 e^{m_2 y} + \frac{P_S}{\mathrm{Re}\left(M + \frac{1}{\lambda}\right)}, \tag{21}$$

where

$$m_1 = 0.5\left(\mathrm{Re} + \sqrt{\mathrm{Re}^2 + 4\,\mathrm{Re}\left(M + \frac{1}{\lambda}\right)}\right),$$

$$m_2 = 0.5\left(\mathrm{Re} - \sqrt{\mathrm{Re}^2 + 4\,\mathrm{Re}\left(M + \frac{1}{\lambda}\right)}\right).$$

Substituting by the boundary conditions (19) and (20), we obtain the constants;

$$C_1 = \frac{m_2\left(\dfrac{P_S}{\mathrm{Re}\left(M + \dfrac{1}{\lambda}\right)}\right)}{m_1 m_2 Kn\left(e^{m_1 h(x)} - e^{m_2 h(x)}\right) + m_1 e^{m_2 h(x)} - m_2 e^{m_1 h(x)}}, \tag{22}$$

$$C_2 = \frac{-m_1\left(\dfrac{P_S}{\mathrm{Re}\left(M + \dfrac{1}{\lambda}\right)}\right)}{m_1 m_2 Kn\left(e^{m_1 h(x)} - e^{m_2 h(x)}\right) + m_1 e^{m_2 h(x)} - m_2 e^{m_1 h(x)}},. \tag{23}$$

(ii) Transient state (first order)

$$\frac{1}{\mathrm{Re}}\frac{d^2 u_1}{dy^2} - \left(1 + \varepsilon\, e^{2\pi i t}\right)\frac{du_1}{dy} - \left(M + \frac{1}{\lambda} + \frac{i\alpha^2}{\mathrm{Re}}\right)u_1 = \frac{du_o}{dy} - P_O \cos\left(2\pi t\right)e^{-2\pi i t}, \tag{24}$$

Subject to boundary conditions;

$$\frac{du_1}{dy} = 0 \qquad \text{at} \qquad y = 0, \tag{25}$$

$$u_1 = Kn\frac{du_1}{dy} \qquad \text{at} \qquad y = h(x). \tag{26}$$

By solving (24) we can get the general solution of the form;

$$u_1 = C_3 e^{m_3 y} + C_4 e^{m_4 y} + C_5 e^{m_1 y} + C_6 e^{m_2 y} + C_7, \tag{27}$$

where

$$m_3 = 0.5\left(\left(\mathrm{Re}^*\right) + \sqrt{\left(\mathrm{Re}^*\right)^2 + 4\,\mathrm{Re}\left(M + \frac{1}{\lambda} + \frac{i\alpha^2}{\mathrm{Re}}\right)}\right),$$

$$m_4 = 0.5\left(\left(\mathrm{Re}^*\right) - \sqrt{\left(\mathrm{Re}^*\right)^2 + 4\,\mathrm{Re}\left(M + \frac{1}{\lambda} + \frac{i\,\alpha^2}{\mathrm{Re}}\right)}\right),$$

$$\mathrm{Re}^* = \mathrm{Re}\left(1 + \varepsilon e^{2\pi i t}\right) \approx \mathrm{Re}$$

Substituting by the boundary conditions (19) and (20), we obtain the constants $C_3, \ldots.., C_7$ as follows;

$$C_3 = \frac{C_{31} + C_{32} + C_7 m_4}{m_3 e^{m_4 h(x)} - m_4 e^{m_3 h(x)} + m_3 m_4 Kn\left(e^{m_3 h(x)} - e^{m_4 h(x)}\right)}, \tag{28}$$

$$C_{31} = C_5\left[m_4 e^{m_1 h(x)} - m_1 e^{m_4 h(x)} + m_1 m_4 Kn\left(e^{m_4 h(x)} - e^{m_1 h(x)}\right)\right], \tag{29}$$

$$C_{32} = C_6\left[m_4 e^{m_2 h(x)} - m_2 e^{m_4 h(x)} + m_2 m_4 Kn\left(e^{m_4 h(x)} - e^{m_2 h(x)}\right)\right], \tag{30}$$

$$C_4 = -\frac{m_1 C_5 + m_2 C_6 + m_3 C_3}{m_4},$$

and

$$C_5 = \frac{\mathrm{Re}\, m_1 C_1}{m_1^2 - m_1 \mathrm{Re} - \mathrm{Re}\left(M + \frac{1}{\lambda} + \frac{i\,\alpha^2}{\mathrm{Re}}\right)}, \tag{31}$$

$$C_6 = \frac{\mathrm{Re}\, m_2 C_2}{m_2^2 - m_2 \mathrm{Re} - \mathrm{Re}\left(M + \frac{1}{\lambda} + \frac{i\,\alpha^2}{\mathrm{Re}}\right)}, \tag{32}$$

$$C_7 = \frac{P_O \cos(2\pi t) e^{-2\pi i t}}{\left(M + \frac{1}{\lambda} + \frac{i\,\alpha^2}{\mathrm{Re}}\right)}. \tag{33}$$

Substituting the expressions of u_o and u_1 from the Eqs. (21) and (27) in Eq. (17), we obtain the expression for the velocity $u(y,t)$ as;

$$u(y,t)=C_1 e^{m_1 y}+C_2 e^{m_2 y}+\frac{P_S}{\mathrm{Re}\left(M+\frac{1}{\lambda}\right)}+\varepsilon\, e^{2\pi i t}\left(C_3 e^{m_3 y}+C_4 e^{m_4 y}+C_5 e^{m_1 y}+C_6 e^{m_2 y}+C_7\right), \tag{34}$$

Rate of flow

The volumetric flow rate Q is defined by;

$$Q=\int_0^{h(x)} u(y,t)\, dy. \tag{35}$$

Substituting from Eq. (34) in Eq. (35) and calculating the integration yields;

$$Q=C_1\frac{e^{m_1 h(x)}-1}{m_1}+C_2\frac{e^{m_2 h(x)}-1}{m_2}+\frac{P_S\, h(x)}{\mathrm{Re}\left(M+\frac{1}{\lambda}\right)}+ \varepsilon\, e^{2\pi i t}\left[C_3\frac{e^{m_3 h(x)}-1}{m_3}+C_4\frac{e^{m_4 h(x)}-1}{m_4}+C_5\frac{e^{m_1 h(x)}-1}{m_1}+C_6\frac{e^{m_2 h(x)}-1}{m_2}+C_7 h(x)\right]. \tag{36}$$

Shear stress at the wall

The non-dimensional wall shear stress is given by the relation;

$$\tau_w=\left[\frac{\partial u}{\partial y}\right]_{y=h(x)}. \tag{37}$$

Substituting from Eq. (34) in Eq. (37), the wall shear stress can be written as;

$$\tau_w=C_1 m_1 e^{m_1 h(x)}+C_2 m_2 e^{m_2 h(x)}+\varepsilon\, e^{2\pi i t}\left[C_3 m_3 e^{m_3 h(x)}+C_4 m_4 e^{m_4 h(x)}+C_5 m_1 e^{m_1 h(x)}+C_6 m_2 e^{m_2 h(x)}\right]. \tag{38}$$

Numerical results and discussion

In the present paper, we are concerned with studying the combined effect of magnetic field and permeable wall slip velocity on the unsteady pulsatile flow of blood through porous medium in a time dependent constricted porous channel subjected to time dependent suction/injection at the walls of the channel, considering blood as an incompressible electrically conducting fluid. The analytical expressions of axial velocity, volumetric flow rate, and wall shear stress derived in the preceding section have been computed numerically and plotted for different values of Knudsen number Kn, the maximum projection (height of the throat) of the stenosis δ, magnetic parameter M, Darcian linear drag parameter λ and

Reynolds number Re. We have run our code for the parameters ($\alpha = 0.5,\ P_O = 7.0,\ l_o = 10.0,\ t = 1.0,$ $P_s = 10.0$, and $\varepsilon = 0.01$) related to a realistic physical problem similar to the ones used by other authors [3,5,31]. The profiles of axial velocity versus dimensionless transverse y-coordinate at the throat of the stenosis for various physical parameters are shown in **Figures 2 - 5**.

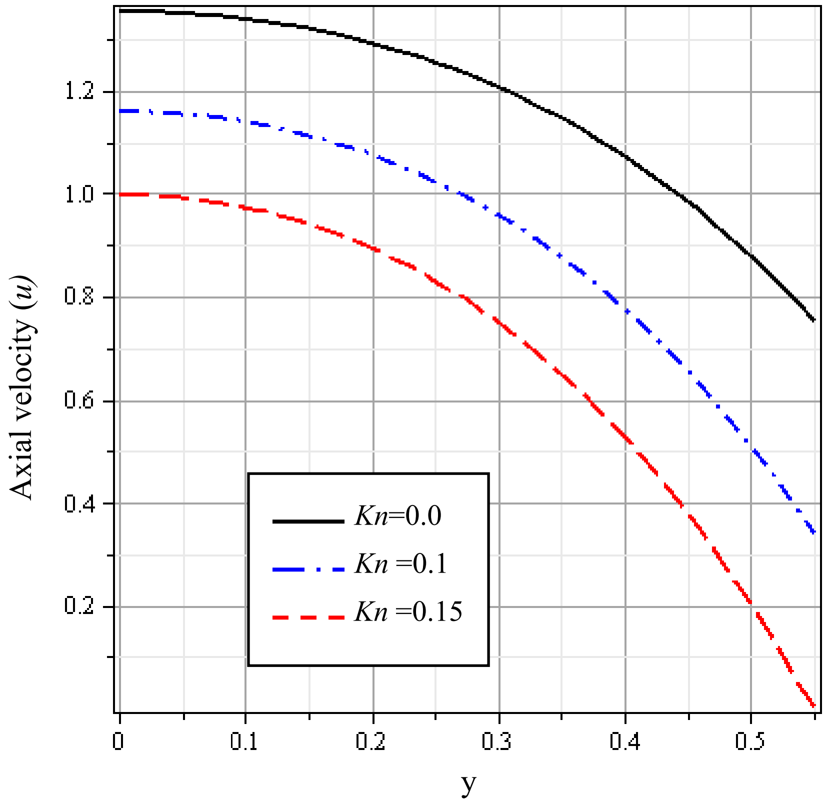

Figure 2 Axial velocity profiles versus y at the throat of the stenosis for different values of Knudsen number *Kn* when $\mathrm{Re} = 1.0,\ M = 2.0,\ \delta = 0.25,$ and $\lambda = 0.30$.

Figure 2 depicts that the parabolic axial velocity of blood has a maximum value at the centerline of the channel and a minimum value at the walls. Also, it is clear from this figure that the axial velocity of blood decreases with increasing Knudsen number *Kn*, meaning that the flowing fluid is slowed down in the axial direction.

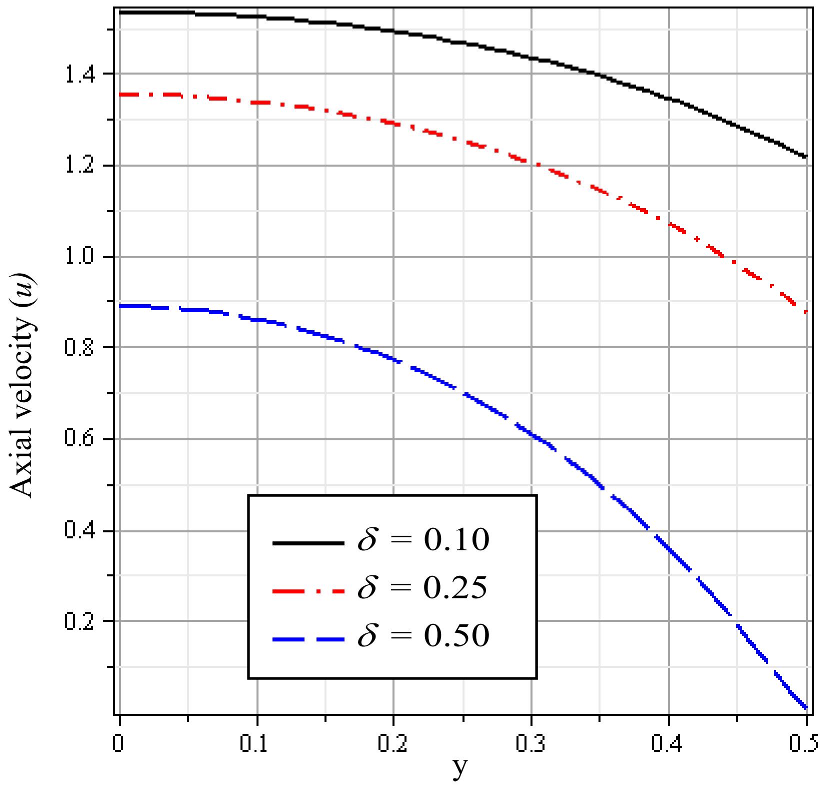

Figure 3 Axial velocity profiles versus y at different depths of the stenosis δ when $\text{Re} = 1.0$, $M = 2.0$, $Kn = 0.1$, and $\lambda = 0.30$.

Figure 3 represents the pulsating axial velocity profiles at different locations of the channel constriction, as well as different depths. It is clear that from this figure that the axial velocity strongly decreases near the walls of the channel, as well as in the centerline of the channel with the increasing effects of constriction height. **Figure 4** demonstrates the effect of magnetic field on the pulsating axial velocity in the stenosed region of the vessel. The axial velocity of the blood decreases with the increase of magnetic field, which is in good agreement with studies carried out by Shit and Roy [32] in **Figure 2**. It is observed that the axial velocity decreases with increasing magnetic parameter M. It indicates that the blood velocity can be reduced by applying suitable magnetic field strength. Thus, the reduction in blood velocity can be used with surgical patients during surgery. **Figure 5** shows that the pulsating axial velocity of the blood increases with increasing values of the Darcian linear drag parameter λ, where increasing the values of λ corresponds to a rise in permeability, and so has less resistance to flow.

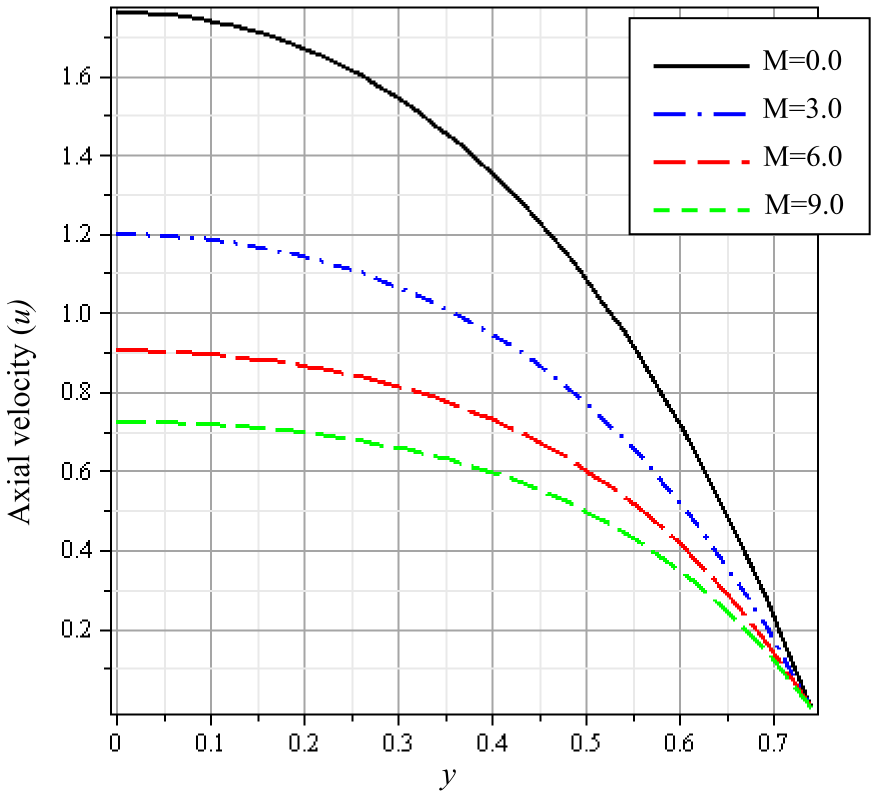

Figure 4 Axial velocity profiles versus y at the throat of the stenosis for different values of magnetic parameter M when $\mathrm{Re} = 1.0$, $Kn = 0.1$, $\delta = 0.25$, and $\lambda = 0.30$.

It is very important to note that an increase in the positive value of flow Reynolds number (Re) represents an increase in the fluid injection, while an increase in the negative value of flow Reynolds number represents an increase in the fluid suction. From **Figure 6**, it is observed that the pulsating axial velocity decreases with increasing Reynolds number. It means that, at strong injection, it is observed that the axial velocity profiles are concave in an upwards direction with decreasing curvature as Re increases. Moreover, for very high values of Re, the graph is almost rectilinear.

The variations of volumetric flow rate of the blood along the dimensionless axial *x*-coordinate of the channel for various physical parameters are shown in **Figures 7 - 11**. It is observed from **Figure 7** that the flow rate has a maximum value at the ends of the stenosis, while it has a minimum value at the throat of the stenosis. Moreover, the flow rate decreases with increasing slip parameter (Knudsen number Kn). From **Figures 8, 9, 11**, it is seen that the volumetric flow rate decreases with increasing height of the stenosis δ, the magnetic parameter M, and the Reynolds number Re, while the volumetric flow rate increases with increasing Darcian linear drag parameter λ, as shown in **Figure 10**.

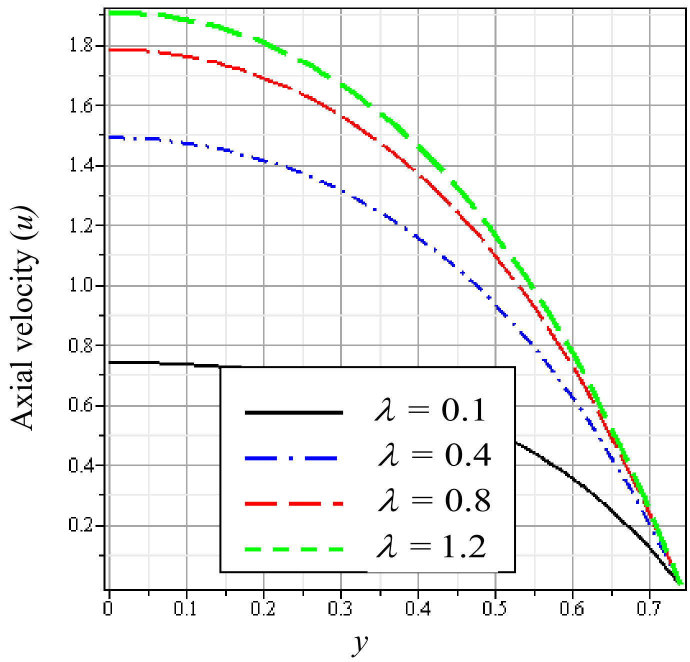

Figure 5 Axial velocity profiles versus y at the throat of the stenosis for different values of Darcian linear drag parameter λ when $\mathrm{Re} = 1.0$, $Kn = 0.1$, $\delta = 0.25$, and $\lambda = 0.30$.

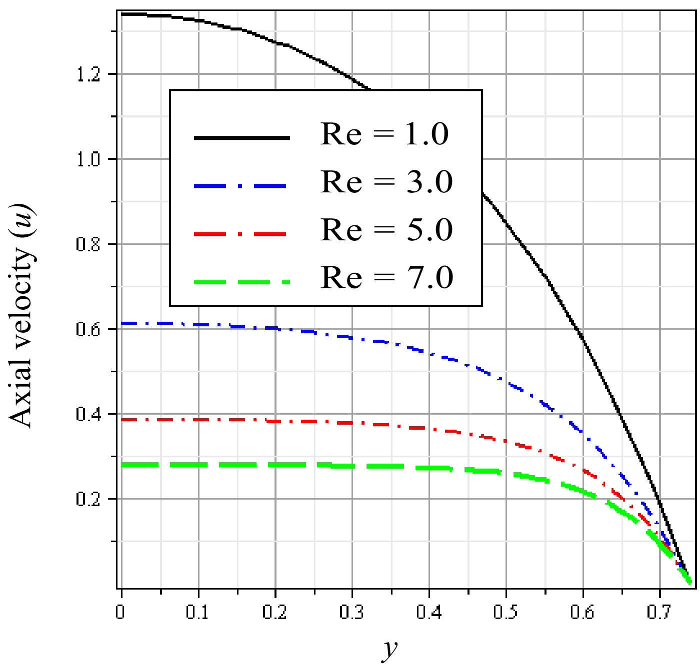

Figure 6 Axial velocity profiles versus y at the throat of the stenosis for different values of Reynolds number Re when $M = 2.0$, $Kn = 0.1$, $\delta = 0.25$, and $\lambda = 0.30$.

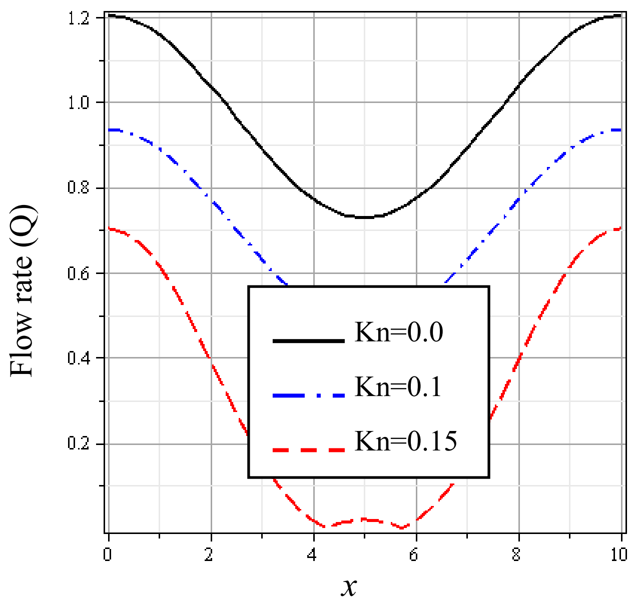

Figure 7 Variation of volumetric flow rate for different values of Knudsen number Kn when $\mathrm{Re} = 1.0$, $M = 2.0$, $\delta = 0.25$, and $\lambda = 0.30$.

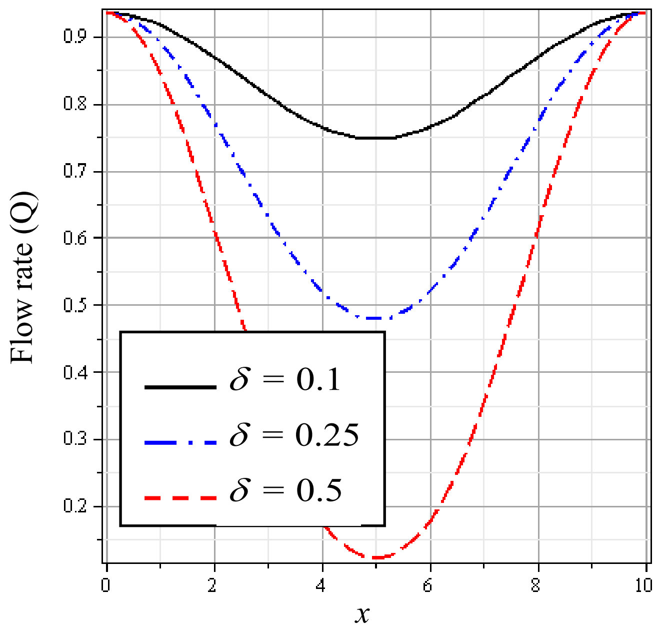

Figure 8 Variation of volumetric flow rate for different depths of the stenosis when $\mathrm{Re} = 1.0$, $M = 2.0$, $\delta = 0.25$, and $\lambda = 0.30$.

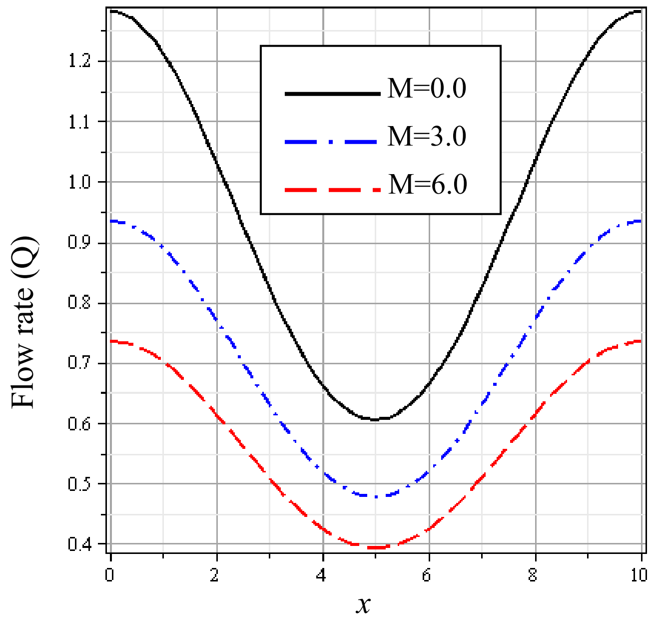

Figure 9 Variation of volumetric flow rate for different values of magnetic parameter M when $\mathrm{Re} = 1.0$, $Kn = 0.01$ $\delta = 0.25$, and $\lambda = 0.30$.

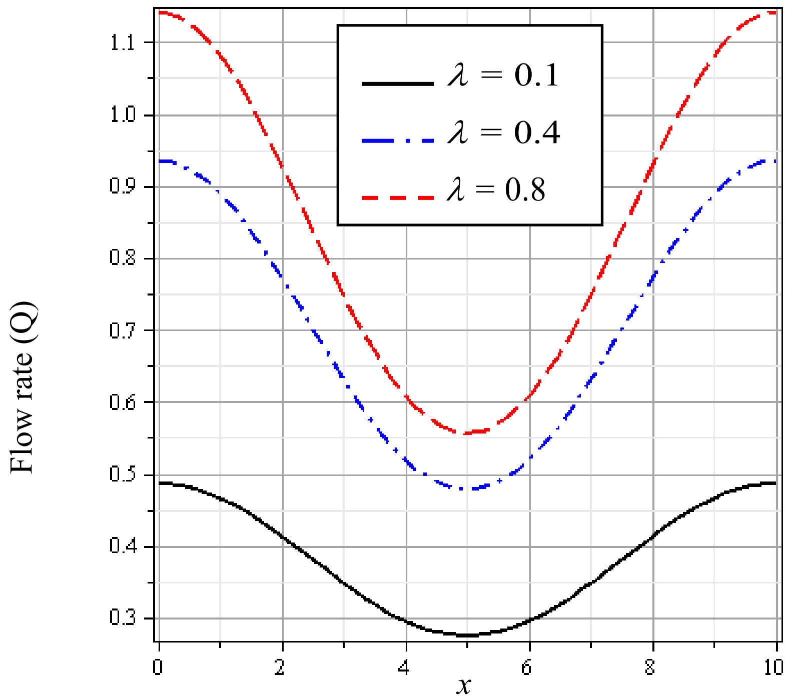

Figure 10 Variation of volumetric flow rate for different values of Darcian linear drag parameter λ when $\mathrm{Re} = 1.0$, $Kn = 0.01$ $\delta = 0.25$, and $M = 2.0$.

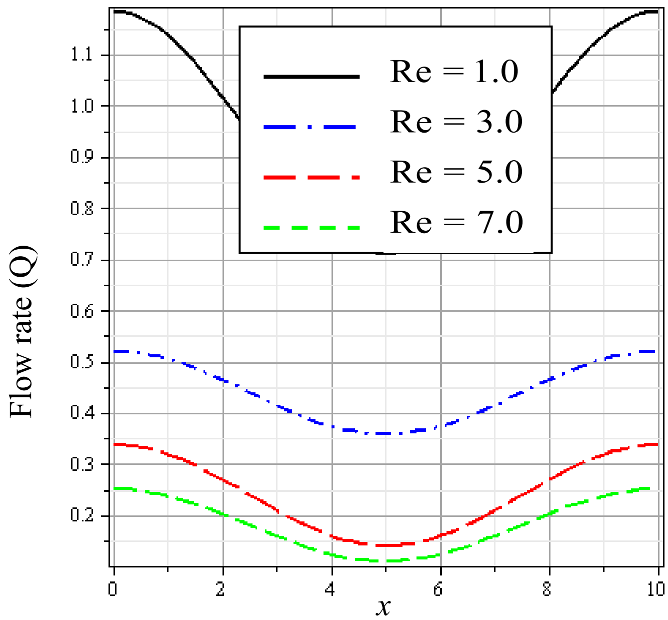

Figure 11 Variation of volumetric flow rate for different values of Reynolds number Re when $M = 2.0$ $Kn = 0.01$ $\delta = 0.25$, and $\lambda = 0.30$.

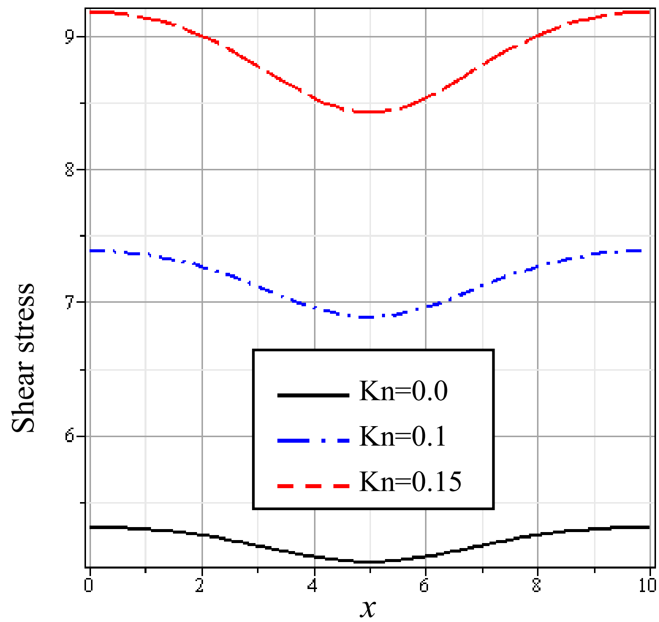

Figure 12 Variation of wall shear stress for different values of Knudsen number Kn when $M = 2.0$ $\mathrm{Re} = 1.0$ $\delta = 0.25$, and $\lambda = 0.30$.

It is a widely accepted fact that wall shear stress plays an important role in the development of arterial diseases. Hence, it is important to study the effects of the physical parameters on wall shear stress. **Figure 12** gives the variation of wall shear stress for different values of the slip parameter (Knudsen number Kn). It may be observed that the wall shear stress increases with increasing slip parameter. Alternatively, the wall shear stress decreases with increasing height of the stenosis δ, as shown in **Figure 13**.

Figure 14 gives the variation of the wall shear stress for different values of the magnetic parameter M. It may be observed that the wall shear stress decreases with increasing magnetic parameter. Conversely, as shown in **Figure 15**, the wall shear stress increases with increasing values of Darcian linear drag parameter λ. **Figure 16** depicts the variation of the wall shear stress for different values of Reynolds number Re. It is worthwhile to note that the wall shear stress decreases as the Reynolds number Re increases. It also noted that, at strong injection, the wall shear stress seems to be rectilinear.

Figure 17 indicates the variation of the pressure gradient ($-\partial p / \partial y$) versus time at different values of Darcian linear drag parameter λ. It is observed that the pressure gradient and its effect decreases with increased Darcian linear drag parameter λ.

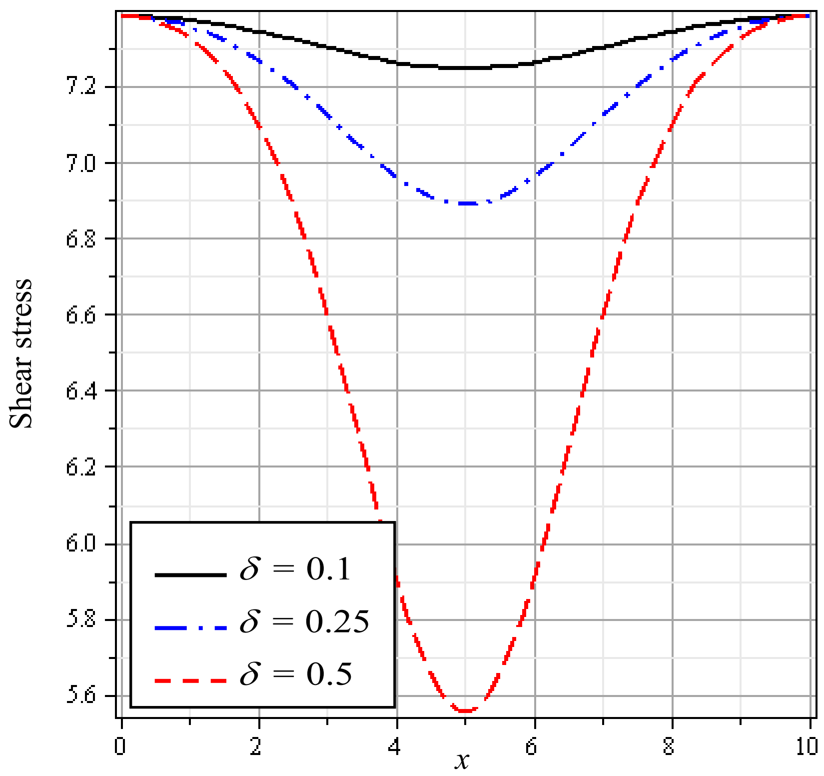

Figure 13 Variation of wall shear stress for different values of depths of the stenosis δ when $M = 2.0$ $\mathrm{Re} = 1.0$ $Kn = 0.01$ and $\lambda = 0.30$.

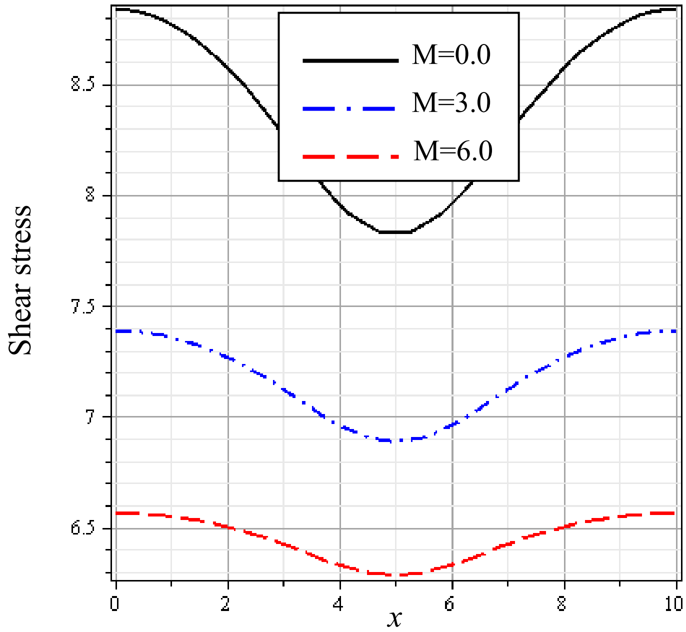

Figure 14 Variation of wall shear stress for different values of magnetic parameter M when $\delta = 0.25$ $\mathrm{Re} = 1.0$ $Kn = 0.01$ and $\lambda = 0.30$.

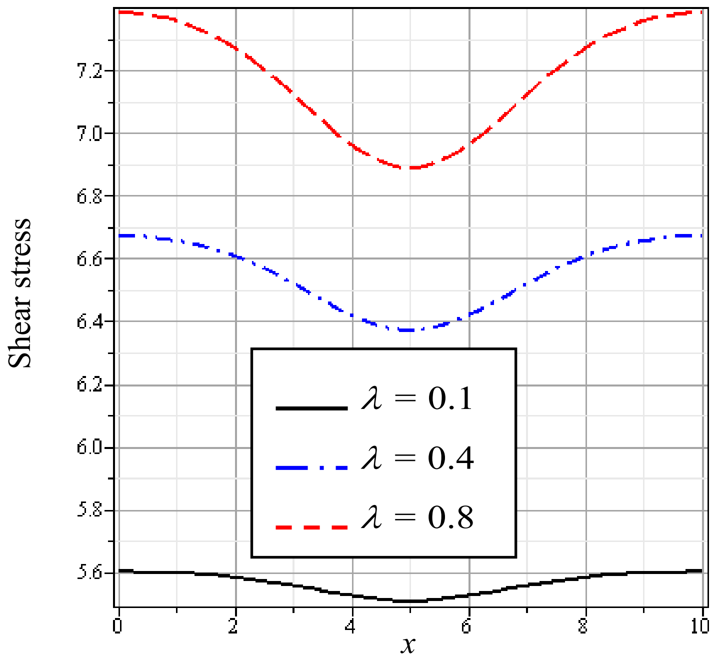

Figure 15 Variation of wall shear stress for different values of Darcian linear drag parameter λ when $M = 2.0$ $\mathrm{Re} = 1.0$ $Kn = 0.01$ and $\delta = 0.25$.

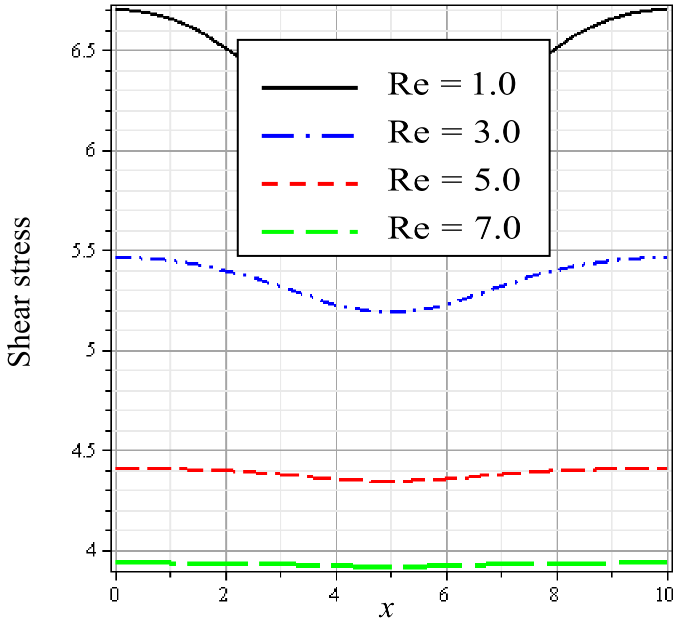

Figure 16 Variation of wall shear stress for different values of Reynolds number Re when $M = 2.0$ $\delta = 0.25$ $Kn = 0.01$ and $\lambda = 0.30$.

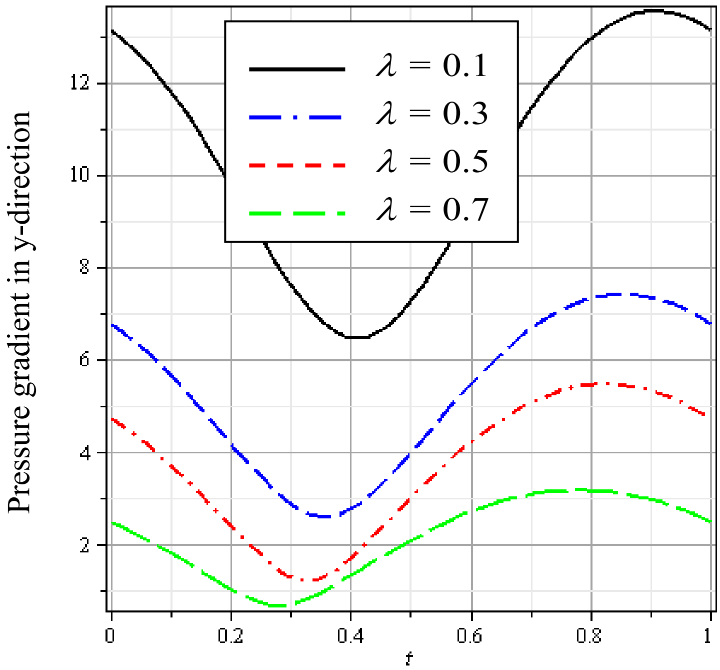

Figure 17 Variation of pressure gradient $-\partial p / \partial y$ for different values of Darcian linear drag parameter λ when $\varepsilon = 0.3$.

Conclusions

The present study deals with a theoretical investigation of unsteady pulsatile blood flow through porous medium in a constricted porous channel subjected to time dependent suction/injection at the walls of the channel, considering blood as an incompressible electrically conducting fluid. Also, the study investigates the combined effect of magnetic field and permeable wall slip velocity. The prime concern in our present study has been to assess the role of velocity slip in blood flow through arteries, and to determine those regions where the velocity is low and also the regions where the wall shear stress is low. Thus, the study bears the potential for further exploration of the causes and development of arterial diseases like atherosclerosis and atheroma.

Consideration of slip velocity at the permeable wall has been of prime concern in the study. From the computational results, it may be concluded that;

1) The pulsatile axial velocity u of the blood at the throat of the stenosis decreases with increasing slip parameter Kn, depth of the stenosis δ, magnetic parameter M, and Reynolds number Re (effect of injection or suction) while it increases with increasing Darcian linear drag parameter λ.

2) The volumetric flow rate Q of the blood along the longitudinal x-axis decreases with increasing slip parameter Kn, depth of the stenosis δ, magnetic parameter M, and Reynolds number Re (effect of injection or suction) while it increases with increasing Darcian linear drag parameter λ.

3) It is a widely accepted fact that wall shear stress plays an important role in the development of arterial diseases. The shear stress τ_w at the permeable wall increases with increasing slip parameter Kn and Darcian linear drag parameter λ, while it decreases with increasing depth of the stenosis δ, magnetic parameter M, and Reynolds number Re (effect of injection or suction).

4) The pressure gradient decreases with increasing Darcian linear drag parameter λ. Also, the pressure gradient in the y-direction is affected only by time and Darcian linear drag parameter λ.

5) The present model gives a most general form of velocity expression, volumetric flow rate and wall shear stress, from which the other mathematical models can easily be obtained by proper substitutions, such as the results of Shit and Roy [32] which have been recovered by taking the Knudsen number Kn = 0.

Acknowledgments

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