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Accuracy of VIM, HPM and ADM in Solving Nonlinear Equations for the Steady Three-Dimensional Flow of a Walter's B Fluid in Vertical Channel

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Abstract

In this paper, the steady three-dimensional flow of a Walter's B fluid in a vertical channel is investigated. It is assumed that the fluid is injected into the passage through one side of the channel. The combined effects of viscoelasticity and inertia are considered. By using the appropriate similarity transformations for the velocity components and temperature, the basic equations governing flow and heat transfer are reduced to a set of ordinary differential equations. These equations are solved approximately, subject to the relevant boundary conditions, with a numerical technique. In the present study, three powerful analytical methods of Variational iteration method (VIM), Homotopy perturbation method (HPM) and Adomian decomposition method (ADM) are introduced to overcome this shortcoming. Then, VIM, HPM and ADM are used to solve nonlinear equations in fluids. These methods are useful and practical for solving the nonlinear equation in fluids. Comparison of the results obtained by all three methods and exact solutions reveals that all three methods are tremendously effective.

Keywords: Flow of a Walter's B fluid, Adomian decomposition method (ADM), Variational iteration method (VIM), Homotopy perturbation method (HPM).

Introduction

Most scientific problems and phenomena, such as the flow of fluids, occur nonlinearly. Except in a limited number of these problems, there are difficulties in finding the exact analytical solutions. Therefore, approximate analytical solutions have been introduced. Among the most effective and convenient approximate analytical solutions for both weakly and strongly nonlinear equations are the Variational iteration method (VIM) [7-9,11,19], the Homotopy perturbation method (HPM) [10,12-16] and the Adomian decomposition method (ADM) [7-9]. Perturbation methods [3] provide the most versatile tools available in nonlinear analysis of engineering problems, but their limitations hamper their application:

1) Perturbation method is based on assuming a small parameter. An overwhelming majority of nonlinear problems, especially those having strong nonlinearity, have no small parameters at all.

2) The approximate solutions obtained by perturbation methods, in most cases, are valid only for small values of the small parameter. The perturbation solutions are generally uniformly valid as long as a specific system parameter is small. However, the approximations cannot be relied on fully, because there is no criterion on which the small parameter should exist. Thus, it is essential to check the validity of the approximations numerically and/or experimentally.

The aim of this paper is to develop VIM, HPM and ADM for systems of nonlinear differential equations. To show the accuracy of VIM, HPM and ADM, the flow of fluids equation is solved. The results of VIM, HPM and ADM are then compared with the results obtained by numerical solution. For this purpose, after a brief introduction of VIM, HPM and ADM, and the flow of fluids problem, the approximate solution is found. Finally, the results of VIM, HPM and ADM are compared with the numerical results.

Materials and methods

Homotopy perturbation method

Consider the following function;

$$A(u) - f(r) = 0 \tag{1}$$

with the boundary condition of;

$$B(u, \frac{\partial u}{\partial n}) = 0 \tag{2}$$

where A (u) is defined as follows;

$$A(u) = L(u) + N(u) \tag{3}$$

The Homotopy perturbation structure is shown as;

$$H(v,p) = L(v) - L(u_0) + p.L(u_0) + p\left[N(v) - f(r)\right] = 0 \tag{4}$$

or

$$H(v,p) = (1-p)\left[L(v) - L(u_0)\right] + p\left[A(v) - f(r)\right] = 0 \tag{5}$$

where

$$v(r,p) : \Omega \times [0,1] \to R \tag{6}$$

Obviously, considering Eqs. (4) and (5),

$$H(v,0) = L(v) - L(u_0) = 0\,, H(v,1) = A(v) - f(r) = 0 \tag{7}$$

is obtained, where p ∈ [0, 1] is an embedding parameter and u0 is the first approximation that satisfies the boundary condition. The process of the changes in p from zero to unity is that of v(r, p) changing from u_0 to u_r. v is considered as;

$$v = v_0 + p.v_1 + p^2.v_2 \tag{8}$$

and the best approximation is;

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + \cdots \tag{9}$$

The above convergence is discussed in [26,27].

Variational iteration method

To clarify the basic ideas of VIM, the following differential equation is considered;

$$Lu + Fu = g(t) \tag{10}$$

where L is a linear operator, F is a nonlinear operator and g (t) is a heterogeneous term.

According to VIM a correction functional can be written down as follows;

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda \left(Lu_n(\tau) + F\tilde{u}_n(\tau) - g(\tau) \right) . d\tau \tag{11}$$

where λ is a general Lagrangian multiplier [20,21], which can be identified optimally via the variational theory. The subscript n indicates the nth approximation and $\tilde{u}_n$ is considered as a restricted variation [20,21], i.e., $\delta\tilde{u}_n = 0$.

Adomian decomposition method

To clarify the basic ideas of ADM, the following differential equation is considered;

$$Lu + Ru + Nu = g \tag{12}$$

where L is the highest order derivative which is assumed to be easily invertible, R the linear differential operator of less order than L, Nu presents the nonlinear terms, and g is the source term. Applying the inverse operator L-1 to the both sides of Eq. (12), and using the given conditions;

$$u = f(x) - L^{-1}(Ru) - L^{-1}(Nu) \tag{13}$$

is obtained, where the function f(x) represents the terms arising from integration of the source term g(x), using given conditions. For nonlinear differential equations, the nonlinear operator $Nu = F(u)$ is represented by an infinite series of the so-called Adomian polynomials.

$$F(u) = \sum_{m=0}^{\infty} A_m \tag{14}$$

The polynomials A_m are generated for all kind of nonlinearity so that A_0 depends only on u_0, A_1 depends on u_0 and u_1, and so on. The Adomian method defines the solution $u(x)$ by the series;

$$u_0 = u_1 + u_2 + u_3 + \cdots \tag{15}$$

In the case of $F(u)$, the infinite series is a Taylor expansion about u_0. In other words,

$$F(u) = F(u_0) + F'(u_0)(u - u_0) + F''(u_0)\frac{(u - u_0)^2}{2!} + F'''(u_0)\frac{(u - u_0)^3}{3!} + \cdots \tag{16}$$

By rewriting Eq. (15) as $u_0 = u_1 + u_2 + u_3 + \cdots$, substituting it into Eq. (16), and then equating 2 expressions for F(u) found in Eq. (16) and Eq. (14), defining formulas for the Adomian polynomials are obtained.

$$F(u) = A_1 + A_2 + A_3 + \cdots = F(u_0) + F'(u_0)(u_1 + u_2 + \cdots) + F''(u_0)\frac{(u_1 + u_2 + \cdots)^2}{2!} + \cdots \tag{17}$$

By equating terms in Eq. (17), the first few Adomian's polynomials A_0, A_1, A_2, A_3 and A_4 are given;

$$A_0 = F(u_0) \tag{18}$$

$$A_1 = u_1 F'(u_0) \tag{19}$$

$$A_2 = u_2 F'(u_0) + \frac{1}{2!}u_1^2 F''(u_0) \tag{20}$$

$$A_3 = u_3 F'(u_0) + u_1 u_2 F''(u_0) + \frac{1}{3!}u_1^3 F'''(u_0) \tag{21}$$

$$\vdots$$

Now that the A_k are known, Eq. (15) can be substituted in Eq. (14) to specify the terms in the expansion for the solution of Eq. (16).

Applications

The steady three-dimensional flow of a Walter's B fluid in vertical channel

There are numerous models of viscoelastic fluids suggested in the literature. To get some insight into their flow behavior, it is preferable to restrict investigation to a model with a minimum number of parameters in the constitutive equations. The model of Walter's B fluid has been chosen for this study as it involves only one non-Newtonian parameter. The Cauchy stress tensor T in such a fluid is related to the motion in the following manner (Beard and Walters, 1964);

$$T = -pI + 2\eta_0 e - 2k_0 \frac{\delta e}{\delta t} \tag{22}$$

In this equation, p is the pressure, I is the identity tensor, and the rate of strain tensor e is defined by;

$$2e = \nabla v + (\nabla v)^T \tag{23}$$

where v is the velocity vector, ∇ is the gradient operator, and $\delta / \delta t$ denotes the convected differentiation of a tensor quantity in relation to the material in motion. The convected differentiation of the rate of strain tensor is given by;

$$\frac{\delta e}{\delta t} = \frac{\delta e}{\delta t} + v.\nabla e - e.\nabla v - (\nabla v)^T.e \tag{24}$$

Finally η_0 and k_0 are, respectively, the limiting viscosity at small rate of shear and the short memory coefficient, which are defined through;

$$\eta_0 = \int_0^\infty N(\tau)d\tau, k_0 = \int_0^\infty \tau N(\tau)d\tau \tag{25}$$

where N (τ) is the distribution function with relaxation time τ. This idealized model is a valid approximation of Walter's B fluid, taking very short memory into account so that terms involving;

$$\int_0^\infty \tau^n N(\tau)d\tau, n \geq 2 \tag{26}$$

have been neglected. For a detailed description of this model, the reader should consult Beard and Walters (1964). **Figure 1** shows the physical model and coordinate system. A fluid is injected through a vertical porous plate at $y = d$ with uniform velocity U. The fluid strikes another vertical impermeable plate at $y = 0$. It flows out through the opening of the plates, due to the action of gravity along the z-axis. The distance between the walls, d, has been assumed to be small compared to the dimensions of the plates, $i.e. L >> B >> d$. Due to this assumption, the edge effects can be ignored, and the isobars are parallel to the z-axis.

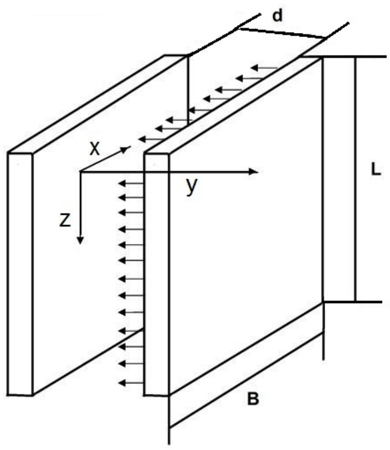

Figure 1 Sketch of flow geometry and coordinate system.

In addition to Eq. (22), the basic equations of the problem are the following;

Continuity equation

$$\nabla . v = 0, \tag{27}$$

Equations of motion

$$\rho(v.\nabla v) = \nabla T + \rho g, \tag{28}$$

Energy equation

$$\rho c_p (v.\nabla T) = k\,\Delta T\,, \tag{29}$$

where ρ is the density, g the gravitational acceleration vector, T the temperature, cp the specific heat at constant pressure, k the thermal conductivity, and Δ the Laplacian operator. The assumptions made in the above equations are as follows: (a) the flow is steady and laminar; (b) the fluid is incompressible; (c) the body force per unit mass is taken to be equal to the gravitational acceleration; (d) all the physical properties, e.g. viscosity, specific heat, and thermal conductivity of the fluid, remain invariable throughout the fluid; (e) the heat flux vector can be represented by Fourier's law, and (f) the effects of radiant heating and viscous dissipation are negligible. Substituting the Cauchy stress tensor from Eq. (22) into equations of motion (28), with the aid of Eqs. (23) and (3),

$$\rho(v.\nabla v) = -\nabla p + \rho g + \eta_0 \nabla^2 v - 2k_0 v.\nabla\nabla^2 v + k_0 \nabla^2 (v.\nabla v) \tag{30}$$

is obtained. The velocity components corresponding to the x, y and z directions are respectively denoted by u, v and w. Following Wang and Skalak (1974), a solution is sought, compatible with the continuity Eq. (27), of the form;

$$u = \frac{U_x}{d} f'(\eta), v = -Uf(\eta), w = \frac{d^2 g\rho h(\eta)}{\eta_0}, \tag{31}$$

where η = y/d and the prime denotes the differentiation with respect to η.

The boundary conditions for the velocity field are;

$$\begin{aligned} &\eta = 0 : f(0) = 0, f'(0) = 0, h(0) = 0, \\ &\eta = 1 : f(1) = 1, f'(1) = 0, h(1) = 0, \end{aligned} \tag{32}$$

It follows from Eq. (31) and the equation of motion (30) that;

$$\frac{\partial p}{\partial x} = \frac{U_x}{d^2}\left(U\rho f'^2 + U\rho f f'' + \frac{Uk_0 f''^2}{d^2} + \frac{\eta_0 f'''}{d} - \frac{2Uk_0 f f'''}{d^2} + \frac{Uk_0 f f^{(IV)}}{d^2}\right) \tag{33}$$

$$\frac{\partial p}{\partial \eta} = -U^2 \rho f f' - \frac{U\eta_0 f''}{d} + \frac{3U^2 k_0 f' f''}{d^2} - \frac{U^2 k_0 f f'''}{d^2}, \tag{34}$$

$$h'' + \mathrm{Re} f\, h' + \mathrm{Re}\, S\,(f\, h''' - f'' h' - 2f' h'') + 1 = 0, \tag{35}$$

where the cross-flow Reynolds number, Re, and the elastic number, S, are defined through, respectively,

$$\mathrm{Re} = \frac{Ud\rho}{\eta_0}, S = \frac{k_0}{\rho d^2} \tag{36}$$

Integrating Eq. (34) with respect to η,

$$p(x,\eta) = -\frac{1}{2}\rho U^2 f^2 - \frac{U\eta_0 f'}{d} + \frac{2U^2 k_0 f'^2}{d^2} - \frac{U^2 k_0 f f''}{d^2} + \phi(x), \tag{37}$$

is obtained, where φ(x) is an arbitrary function of x. Differentiation of the above equation with respect to x yields;

$$\frac{\partial p}{\partial x} = \frac{d\phi}{dx} \tag{38}$$

Combining Eqs. (33) and (38) gives;

$$\frac{d\phi}{dx} = \frac{U_x \eta_0}{d^3}\{f''' + \mathrm{Re}(ff'' - f'^2) + \mathrm{Re}\,S\,(ff^{(IV)} + f''^2 - 2ff''')\} \tag{39}$$

It is apparent that the quantity in parentheses in Eq. (39) must be independent of η. Hence, the following equation for f is given by;

$$f''' + \mathrm{Re}(ff'' - f'^2) + \mathrm{Re}\,S\,(ff^{(IV)} + f''^2 - 2ff''') = C \tag{40}$$

where C is an arbitrary constant which takes the value;

$$C = f'''(0) + \mathrm{Re}\,Sf''^2(0), \tag{41}$$

In this example, the Newtonian character of the fluid is considered. Thus, the equation converts to;

$$f^{(iv)} + R\,(f'''f - f'f'') = 0 \tag{42}$$

where R is the cross-flow Reynolds number.

The boundary conditions for the velocity field are;

$$f(0) = 0, f(1) = 1, f'(0) = 0, f'(1) = 0 \tag{43}$$

Variational iteration method

In order to solve Eq. (42) with boundary conditions (43) using VIM, a correction functional is constructed as follows;

$$f_{n+1} = f_n + \int_0^{\eta} \lambda.\left\{f_n^{(iv)} + R\,(f_n'''f_n - f_n'f_n'')\right\} d\tau \tag{44}$$

Its stationary conditions can be obtained as follows;

$$\lambda^{iv}(\tau) = 0\,,\quad 1 - \lambda'(\tau)\big|_{\tau=x} = 0\,,\quad \lambda(\tau)\big|_{\tau=x} = 0 \tag{45}$$

The Lagrangian multiplier can therefore be identified as;

$$\lambda(\tau)=\frac{(\tau-\eta)^3}{6} \tag{46}$$

As a result, the following iteration formula is obtained.

$$f_{n+1}=f_n+\int_0^\eta \frac{(\tau-\eta)^3}{6}.\left\{f_n^{(iv)}+R(f_n'''f_n-f_n'f_n'')\right\}d\tau \tag{47}$$

Now an arbitrary initial approximation is used to start, which satisfies the initial condition;

$$f_0(\eta)=-2\eta^3+3\eta^2 \tag{48}$$

Using the above Variational formula (44) gives;

$$f_1=f_0+\int_0^\eta \frac{(\tau-\eta)^3}{6}.\left\{f_0^{(iv)}+R(f_0'''f_0-f_0'f_0'')\right\}d\tau \tag{49}$$

Substituting Eq. (48) in to Eq. (49) and after simplification,

$$\begin{aligned} f(\eta)=&-2\eta^3+3\eta^2+0.000008372\eta^{15}-0.005357142857\eta^8-0.0000627943\eta^{14}\\ &+0.00021978\eta^{13}-0.000454545\eta^{12}+0.0011471\eta^{11}-0.00380952\eta^{10}\\ &+0.0047619\eta^9+0.057142\eta^7+0.3\eta^5-0.2\eta^6 \end{aligned} \tag{50}$$

is obtained, and so on. Similarly, in the same manner, the rest of the components of the iteration formula can be obtained.

Homotopy perturbation method

A HPM can be constructed as follows;

$$H(f,p)=(1-p)\left(f^{(iv)}\right)+p\left(f^{(iv)}+R(ff'''-ff'')\right) \tag{51}$$

One can now try to obtain a solution of Eq. (51) in the form of

$$v(f)=v_0(\eta)+p.v_1(\eta)+\ldots \tag{52}$$

where vi (η), i = 0, 1, 2,... are functions yet to be determined. According to Eq. (51) the initial approximation to satisfy initial condition is;

$$f_0(\eta)=-2\eta^3+3\eta^2 \tag{53}$$

Substituting Eqs. (51) And (52) into Eq. (50) yields;

$$\frac{d^4}{d\eta^4}f(\eta)-12R(-2\eta^3+3\eta^2)-R(-6\eta^2+6\eta)(-12\eta+6)=0 \tag{54}$$

The solution of Eq. (54) may be written as follows;

$$f_1(\eta)=(\frac{2}{35})\eta^7-(\frac{1}{5})\eta^6+(\frac{3}{10})\eta^5-(\frac{17}{70})\eta^3+(\frac{8}{35})\eta^2 \tag{55}$$

In the same manner, the rest of components can be obtained using the Maple package. According to the HPM, it can be concluded that;

$$f(\eta)=\lim_{p\to 1}v(\eta)=v_0(\eta)+v_1(\eta)+v_2(\eta)+\cdots \tag{56}$$

Therefore,

$$\begin{aligned}f(\eta)&=-2.03866\eta^3+3.0228453\eta^2-0.0000535\eta^8\\&+0.00000692\eta^{11}-0.0000380\eta^{10}+0.00004761\eta^9\\&+0.005934\eta^7+0.0304571\eta^5-0.020538\eta^6\end{aligned} \tag{57}$$

and so on. In the same manner, the rest of the components of the iteration formula can be obtained.

Adomian decomposition method

In order to apply ADM to a nonlinear equation in fluids problem, Eq. (42) is rewritten in the following operator form;

$$L_{\eta\eta\eta\eta}f^{(iv)}=-R(f'''f-f'f'') \tag{58}$$

where the notation;

$$L_{\eta\eta\eta\eta}=\frac{\partial^4}{\partial\eta^4} \tag{59}$$

is the linear operator. By using the inverse operator Eq.(58) can be rewritten in the following form;

$$f(\eta)=L^{-1}{}_{\eta\eta\eta\eta}[-R(f'''f-f'f'')] \tag{60}$$

where the inverse operator is defined by;

$$L^{-1}{}_{\eta\eta\eta\eta}(\ )=\int_0^\eta\int_0^\eta\int_0^\eta\int_0^\eta(\ )d\eta d\eta d\eta d\eta \tag{61}$$

where

$$N_1(f)=R(f'''f), N_2(f)=R(f'f'') \tag{62}$$

The nonlinear operators $N_1(f), N_2(f)$ are defined by the following infinite series;

$$N_i(f)=\sum_{n=0}^{\infty}A_{in}, i=1,2 \tag{63}$$

where A_{in} is called an Adomian polynomial and defined by;

$$A_{i,n}=\frac{1}{n!}\left[\frac{d^n}{d\lambda^n}N_i\left[\sum_{k=0}^{n}\lambda^k f_k\right]\right]_{\lambda=0} \tag{64}$$

Hence the components series solution is obtained by the following recursive relation;

$$f_{n+1}(\eta)=L^{-1}_{\eta\eta\eta\eta}[-R(f_n'''f_n-f_n'f_n'')] \tag{65}$$

where $n\geq 0$. It is easy to program a computer code with Adomian's polynomials formula, Eq. (64), , to get as many polynomials as needed in the calculation. The first few Adomian's polynomials of the A_{in} can be given as;

$$A_{1,0}=24\eta^3-36\eta^2, \tag{66}$$
$$A_{1,1}=(72\eta^2-72\eta)\,((\frac{2}{35})\,\eta^7-(\frac{1}{5})\eta^6+(\frac{3}{10})\eta^5)$$

$$A_{2,0}=(-6\eta^2+6\eta)(-12\eta a+6)$$
$$A_{2,1}=((-12\eta+6)^2+72\eta^2-72\eta)\times \tag{67}$$
$$R\,((\frac{2}{35})\eta^7-(\frac{1}{5})\eta^6+(\frac{3}{10})\eta^5)$$

and so on. The rest of the polynomials can be constructed in a similar manner. Using the recursive relation Eq. (65) and Adomian's polynomials formula Eq. (64) with the initial conditions Eq. (43) gives;

$$f_0(\eta)=-2\eta^3+3\eta^2$$
$$f_1(\eta)=(\frac{2}{35}\eta^7-\frac{1}{5}\eta^6+\frac{3}{10}\eta^5) \tag{68}$$
$$f_2(\eta)=(\frac{18}{25025}\eta^{13}-\frac{9}{1925}\eta^{12}+\frac{107}{7700}\eta^{11}$$
$$-\frac{1}{70}\eta^{10}+\frac{1}{280}\eta^9-\frac{6}{25025}\eta^8)$$

where

$$f(\eta) = \frac{18}{25025}\eta^{13} - \frac{9}{1925}\eta^{12} - \frac{1}{70}\eta^{10} + \frac{1}{280}\eta^{9} - \frac{6}{25025}\eta^{8} + \frac{2}{35}\eta^{7} - \frac{1}{5}\eta^{6} + \frac{3}{10}\eta^{5} - 2\eta^{3} + 3\eta^{2} \tag{69}$$

and so on. In the same manner, the rest of the components of the iteration formula can be attained.

Results and discussion

An approximate analytical solution for the steady three-dimensional flow of a Walter's B fluid in a vertical channel with porous wall was obtained. The effects of cross-flow Reynolds number on normal velocity (f) are shown in the following figures. In **Figure 5**, it can be seen that when the Reynolds number increases from 1 to 10, the normal velocity increases, which means Re has a direct relation with *f*. **Figure 6** displays the effect of a dimensionless variable (η) on a tangential velocity. The dimensional variable has a direct relationship with the tangential velocity (when η < 0.5), but it has the reverse relationship with the tangential velocity (when η > 0.5); also, this figure depicts the effect of the cross-flow Reynolds number on the tangential velocity. It is clear from this figure that, when the Reynolds number increases from 1 to 10, the tangential velocity increases (when η < 0.5), and it decreases when η > 0.5. It can be seen that if Re = 1, 5, 10, the maximum tangential velocity is observed at η = 0.5, 0.4, 0.3. That means with increase of Re, the maximum point of η decreases. Moreover, the cross-flow Reynolds number makes this point closer to the impermeable wall. It is interesting to note that the both of normal and tangential velocity has a direct relation with the cross-flow Reynolds number. The results are compared with the HPM, VIM and ADM. **Figure 2** depicts the steady three-dimensional flow of a Walter's B fluid in a vertical channel with porous wall for the 3 methods. It is observed that the Homotopy perturbation method approximant solution is more accurate than the VIM and ADM. Comparing **Figure 2** gives closer results to the numerical solution. It is interesting to note that the HPM is very close to the numerical results.

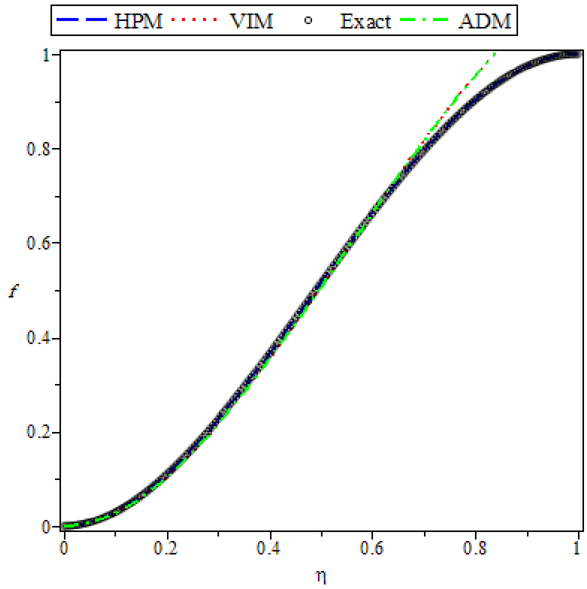

Figure 2 The comparison of the results of the 3 methods with exact solution, at Re = 1.

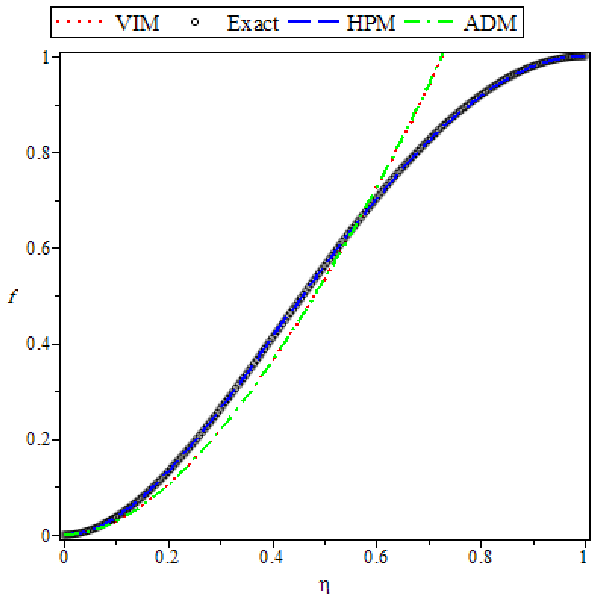

Figure 3 The comparison of the results of the 3 methods with exact solution, at Re = 5.

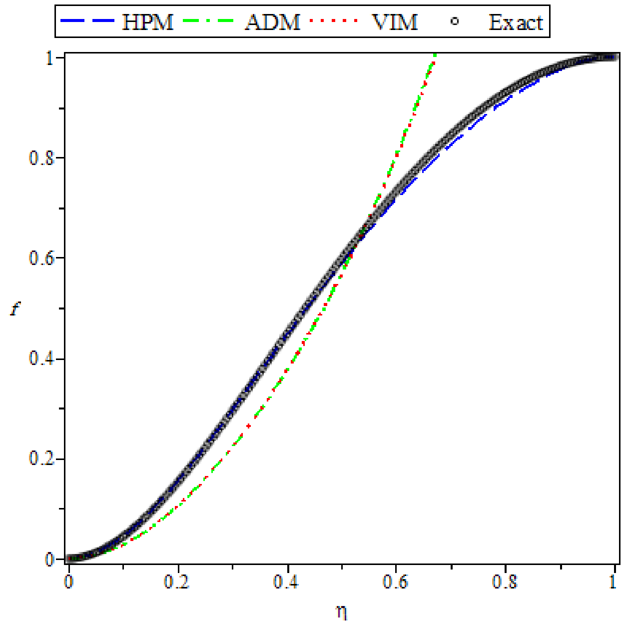

Figure 4 The comparison of the results of the 3 methods with exact solution, at Re = 10.

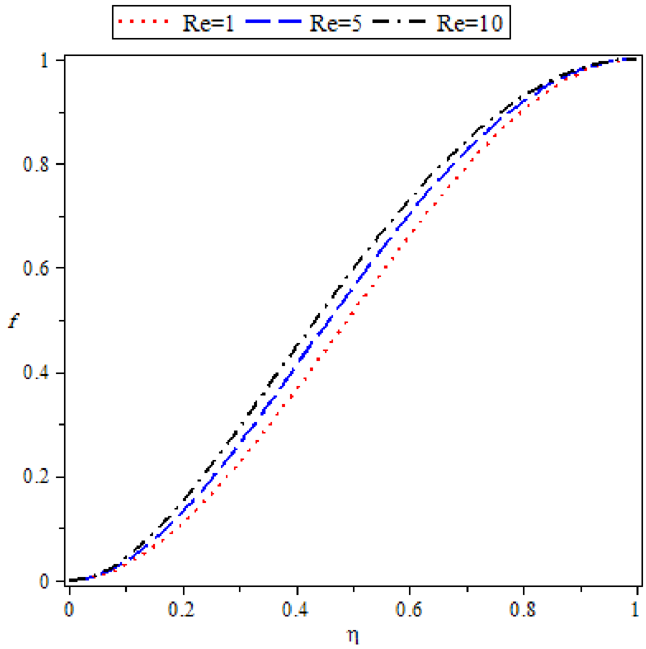

Figure 5 The comparison of the results of exact solution with different Reynolds numbers.

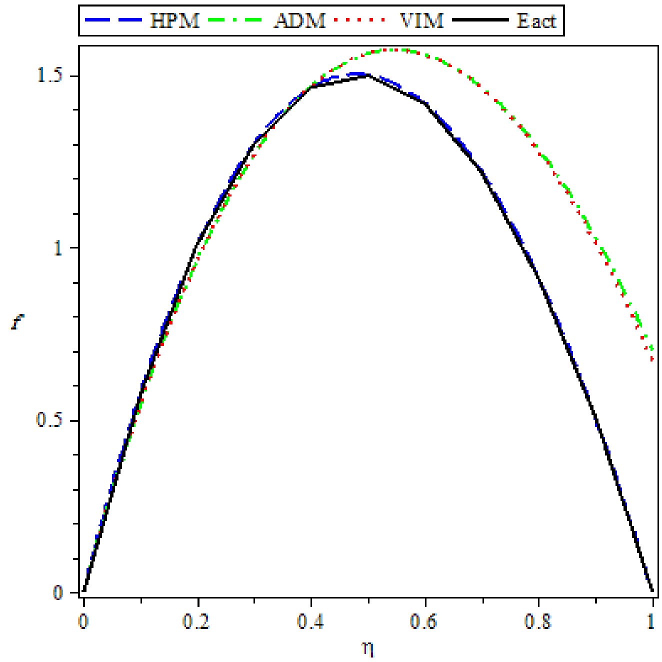

Figure 6 The comparison of the results of the 3 methods with exact solution (f'), at Re = 1.

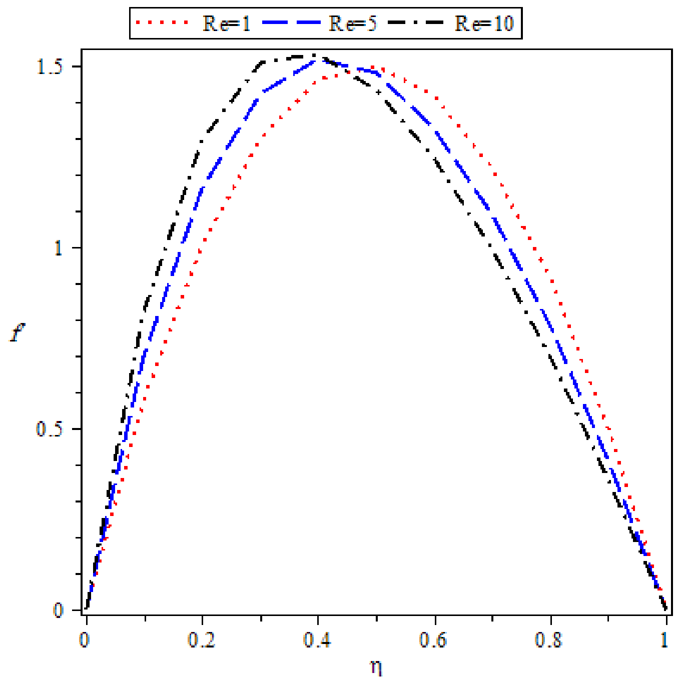

Figure 7 The comparison of the results of exact solution (f') with different Reynolds numbers.

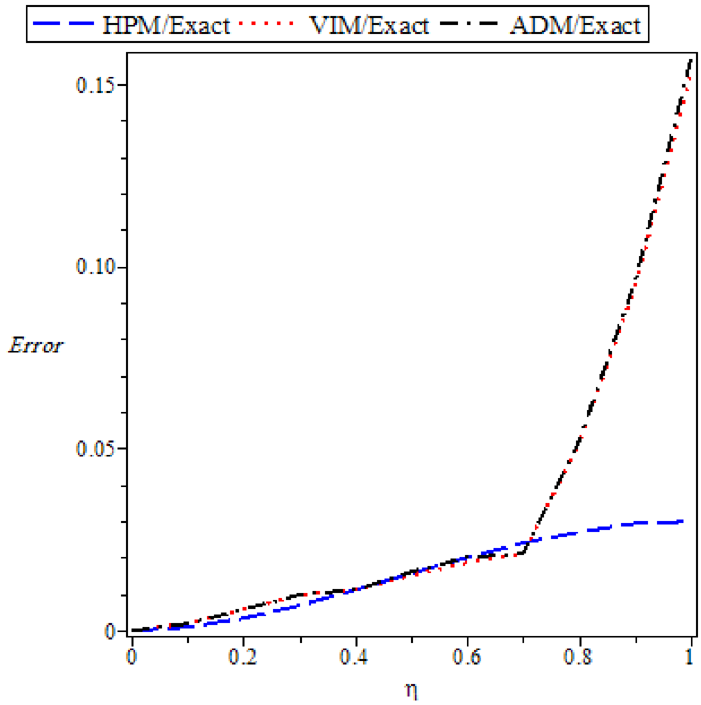

Figure 8 The comparison of the errors in answers results by VIM, HPM and ADM.

Table 1 The results of VIM, HPM and ADM methods and their errors.

η	VIM	HPM	ADM	Exact	Error of VIM	Error of HPM	Error of ADM
0	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000	0.0000000000
0.1	0.0280028056	0.0289859170	0.0280028057	0.0298823886	0.0018795830	0.0008964716	0.0018795829
0.2	0.1040839198	0.1067320215	0.1040839324	0.1100330119	0.0059490921	0.0033009904	0.0059490795
0.3	0.2165954187	0.2196844487	0.2165957234	0.2264788131	0.0098833944	0.0067943644	0.0098830897
0.4	0.3543438026	0.3547730720	0.3543466536	0.3657454351	0.0114016325	0.0109723631	0.0113987815
0.5	0.5066815551	0.4994954195	0.5066975016	0.5149437315	0.0082621764	0.0154483120	0.0082462299
0.6	0.6635348235	0.6419331054	0.6635997253	0.6617867066	0.0017481168	0.0198536012	0.0018130186
0.7	0.8153910171	0.7707341287	0.8156052020	0.7945712668	0.0208197503	0.0238371381	0.0210339352
0.8	0.9532671570	0.8750910374	0.9538799189	0.9021557087	0.0511114483	0.0270646712	0.0517242102
0.9	1.0686753836	0.9447400772	1.0702628337	0.9739588425	0.0947165411	0.0292187653	0.0963039912
1.0	1.1535960957	0.9700000000	1.1574275727	1.0000000000	0.1535960957	0.0300000000	0.1574275727

Conclusions

In the present work, the basic idea of the VIM, the HPM and the ADM are introduced and then applied to solve the steady three-dimensional flow of a Walter's B fluid in a vertical channel with porous wall, because a forth order non-linear ordinary differential equation has been derived as the governing equation for this problem. The result demonstrates the HPM is simple and offers superior accuracy compared with the VIM and the ADM. Also, it is found that these methods are powerful mathematical tools that they can be applied to a large class of linear and nonlinear problems arising in different fields of science and engineering, especially for some flow fluids equations.

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