WALAILAK JOURNAL
http://wjst.wu.ac.th Engineering and Physical Sciences

Impact of Chemical Reaction on MHD Mixed Convection Heat and Mass Transfer Flow with Thermophoresis

Prabir Kumar KUNDU[1], Kalidas DAS[2,*] and Subroto JANA[3]

[1]*Department of Mathematics, Jadavpur University, Kolkata, West Bengal, India*
[2]*Department of Mathematics, Kalyani Government Engineering College, Kalyani, Nadia, West Bengal, India*
[3]*Department of Mathematics, Jadavpur University, Kolkata, West Bengal, India*

(**[*]Corresponding author's e-mail: kd_kgec@rediffmail.com**)

Received: 5 March 2012, Revised: 9 August 2012, Accepted: 17 September 2013

Abstract

A mathematical model is analyzed in order to study the effects of chemical reaction and thermophoresis on MHD mixed convection boundary layer flow of an incompressible, electrically conducting fluid past a heated vertical permeable flat plate embedded in a uniform porous medium, by taking into account the radiative heat flux and variable suction. The governing partial differential equations are transformed into a set of coupled ordinary differential equations which are solved analytically using the regular perturbation technique. Numerical results for dimensionless velocity, temperature, concentration as well as the skin friction coefficient, Nusselt number and Sherwood number are presented through graphs and a table for pertinent parameters to show interesting aspects of the solution.

Keywords: Chemical reaction, thermal radiation, thermophoresis, mixed convection

Introduction

The present trend in the field of chemical reaction analysis is to give a mathematical model for a system to predict reactor performance. In particular, the study of heat and mass transfer with chemical reaction is of considerable importance in the chemical and hydrometallurgical industries. Chemical reactions can be codified as either heterogeneous or homogeneous processes. This codification depends on whether the reactions occur at an interface or as a single phase volume reaction. A few representative fields of interest in which combined heat and mass transfer with chemical reaction effect plays an important role are the design of chemical processing equipment, the formation and dispersion of fog, the distribution of temperature and moisture over agricultural fields and groves of fruit trees, the damage of crops due to freezing, food processing, and cooling towers. For example, the formation of smog is a first order homogeneous chemical reaction. Consider the emission of NO_2 from automobiles and other smoke-stacks; this NO_2 reacts chemically in the atmosphere with unburned hydrocarbons (aided by sunlight) and produces peroxyacetylnitrate, which forms an envelope of what is termed as photochemical smog. The study of heat and mass transfer of moving fluid is important from the point of view of some physical problems, such as fluids undergoing exothermic and endothermic chemical reaction. In addition, in many chemical engineering processes, chemical reaction takes place between a foreign mass and a working fluid which moves due to the stretching of the surface. The order of the chemical reaction depends on several factors, the simplest of which is the first order reaction, where the rate of reaction is directly proportional to the species concentration. A large amount of research work has been reported in this field [1-5]. Kandasamy *et al.* [6] studied the nonlinear MHD flow with heat and mass transfer of an incompressible, viscous, electrically conducting fluid on a vertical stretching surface with chemical reaction and thermal stratification effects. Patil and Kulkarni [7] considered the effects of chemical reaction on free convection flow of a polar fluid through a porous medium in the presence of internal heat generation. Cortell [8] studied flow and mass transfer with chemically reactive species for 2 classes of viscoelastic fluid over a porous stretching sheet. The problems involving chemical reactions can be found in the studies of Khedr *et al.* [9], Damseh *et al.* [10] and Magyari and Chamkha [11]. Recently Das [12] looked at the problem analytically to consider the effect of first order chemical reaction and thermal radiation on micropolar fluid in a rotating frame of reference.

However, the effect of thermal radiation on the flow and heat transfer has not been provided in the recent investigations. The effect of radiation on MHD flow and heat transfer problem has become more important industrially. At a high operating temperature, radiation effect can be quite significant. Many processes in engineering areas occur at high temperature, and knowledge of radiation heat transfer becomes very important for the design of reliable equipment, nuclear plants, gas turbines and various propulsion devices or aircraft, missiles, satellites and space vehicles. Based on these applications, Cogley *et al.* [13] showed that in the optically thin limit, the fluid does not absorb its own emitted radiation but the fluid does absorb radiation emitted by the boundaries. Makinde [14] examined the transient free convection interaction with thermal radiation of an absorbing emitting fluid along a moving vertical permeable plate. Ibrahim *et al.* 15] discussed the case of mixed convection flow of a micropolar fluid past a semi infinite, steady moving porous plate with a varying suction velocity normal to the plate in the presence of thermal radiation and viscous dissipation. Hayat *et al.* [16] studied a 2 dimensional mixed convection boundary layer MHD stagnation point flow through a porous medium bounded by a stretching vertical plate with thermal radiation. Das [17] discussed the effect of thermal radiation on MHD slip flow over a flat plate with variable fluid properties. Recently Olajuwon [18] examined convection heat and mass transfer in a hydromagnetic flow of a second grade fluid in the presence of thermal radiation and thermal diffusion.

Thermophoresis is a phenomenon which causes small particles to be driven away from a hot surface and towards a cold one. Small particles, such as dust, when suspended in a gas with a temperature gradient, experience a force in the direction of the temperature gradient. The velocity acquired by the particles is termed as thermophoretic velocity, and the force experienced by the suspended particles due to the temperature gradient is termed as thermophoretic force. The magnitudes of thermophoretic velocity and thermophoretic force are proportional to the temperature gradient and depend on thermal conductivity of aerosol particles, the carrier gas, the heat capacity of the gas, the thermophoretic coefficient and the Knudsen number. Due to thermophoresis, small micron sized particles are deposited on cold surfaces. In this process, the repulsion of particles from hot objects also takes place, and a particle-free layer is observed around hot bodies (see Goldsmith and May [19]). This phenomenon has many practical applications in removing small particles from gas particle trajectories from combustion devices and studying the particulate material deposition turbine blades. Goren [20] investigated the effect of thermophoresis on laminar flow over a horizontal flat plate which has been extended to a natural convection with variable properties by Jayaraj *et al.* [21]. Selim *et al.* [22] studied the effect of surface mass flux on mixed convection flow past a heated vertical flat plate with thermophoresis. Chamkha and Pop [23] considered the effect of thermophoresis particle deposition in a free convection boundary layer from a vertical plate embedded in a porous medium. Chamkha *et al*. [24] discussed the effect of thermophoresis of aerosol particles in a laminar boundary layer on a vertical plate. Kandasamy *et al*. [25] examined the effects of variable viscosity and thermophoresis on MHD mixed convective heat and mass transfer past a porous wedge. Zucco *et al.* [26] investigated the effect of thermophoresis particle deposition and of the thermal conductivity in a porous plate with dissipative heat and mass transfer. Recently, Singh *et al.* [27] considered the effects of thermophoresis on hydromagnetic mixed convection and mass transfer flow with variable suction and thermal radiation.

The aim of the present work is to study the effect of chemical reaction and thermophoresis on MHD mixed convection heat and mass transfer flow past a vertical permeable plate in the presence of thermal radiation and a heat source/sink. The resulting governing equations are transformed into a system of non-linear ordinary differential equations by applying suitable similarity transformations which are solved numerically using the perturbation technique. The numerical results are discussed through graphs and a table.

Mathematical formulation of the problem

Let us consider a steady MHD two dimensional laminar mixed convection flow of an electrically conducting incompressible fluid along a semi-infinite vertical permeable plate embedded in a uniform porous medium under the influence of a transverse magnetic field, B_0, in the presence of thermal radiation, thermophoresis, a first order chemical reaction and non-uniform heat source/sink. The magnetic Reynolds number of the flow is taken to be small enough so that induced magnetic field is assumed to be negligible in comparison with the applied magnetic field. The flow is assumed to be in the x-direction which is taken along the plate and the y-axis is normal to it. There is a constant suction/injection velocity v_w normal to the plate. The pressure gradient, body forces, viscous dissipation and Joule heating effects are neglected compared with the effect of internal heat source/sink. The wall is maintained at a constant temperature T_w and the concentration $C_{w,}$ is higher than the ambient temperature T_∞ and the concentration C_∞ respectively. Also, it is assumed that there exists a homogeneous first-order chemical reaction with a constant rate k_r between the diffusing species and the fluid. It is assumed that the porous medium is homogeneous and present everywhere in local thermodynamic equilibrium. It is to be mentioned that the hole size of the porous plate is taken to be constant. The rest of the properties of the fluid and the porous medium are assumed to be constant.

Under the foregoing assumptions, the governing equations [27] that describe the physical situation can be written as;

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\,, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)+g\beta^*(C-C_\infty)-\frac{\sigma B_0^2}{\rho}u-\frac{\gamma u}{k}\,, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y}-\frac{Q}{\rho C_p}(T-T_\infty), \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_T\frac{\partial^2 C}{\partial y^2}-\frac{\partial}{\partial y}(\mathrm{V_T}C)-k_r(C-C_\infty), \tag{4}$$

where u, v are the velocity components along the x, y-axis respectively, ν is the kinematic viscosity, ρ is the constant fluid density, σ is the electrical conductivity of the fluid, g is the acceleration due to gravity, β is the coefficient of thermal expansion, β^* is the coefficient of volumetric expansion, k is the permeability of the porous medium, κ is the thermal conductivity of the fluid, C_p is the specific heat at constant pressure p, q_r is the radiative heat flux, Q is the constant heat source/sink. D_T is the molecular diffusivity, V_T is the thermophoretic velocity, and k_r is the chemical reaction parameter.

The appropriate boundary conditions for the present problem are;

$$\begin{gathered} u=U_0,\ v=v_w(x),\ T=T_w,\ C=C_w \text{ at } y=0 \\ u\to 0,\ T\to T_\infty,\ C\to C_\infty \text{ as } y\to\infty \end{gathered} \tag{5}$$

The radiative heat flux q_r under Rosseland approximation is given by;

$$q_r = -\frac{4\sigma^*}{3\mathrm{k}^*}\frac{\partial T^4}{\partial y}. \tag{6}$$

where σ^* is the Stefan-Boltzmann constant and k^* is the mean absorption coefficient. Assuming that the differences in temperature within the flow are such that T^4 can be expressed as a linear combination of the temperature, T^4 in Taylor's series about T_∞ is expanded as follows;

$$\mathrm{T}^4 = \mathrm{T}_\infty{}^4 + 4\mathrm{T}_\infty^3(\mathrm{T}\text{-}\mathrm{T}_\infty) + 6\mathrm{T}_\infty^2(\mathrm{T}\text{-}\mathrm{T}_\infty)^2 + \cdots \tag{7}$$

and neglecting the higher order term beyond the first degree in $(T\text{-}T_\infty)$, results in;

$$\mathrm{T}^4 \cong 4\mathrm{T}_\infty^3\mathrm{T} - 3\mathrm{T}_\infty^4 \tag{8}$$

Thus, Eq. (3) reduces to;

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{16\sigma_1 T_\infty^3}{3\rho C_p K_1}\frac{\partial^2 T}{\partial y^2} - \frac{Q}{\rho C_p}(T - T_\infty), \tag{9}$$

The thermophoretic velocity V_T, which appears in Eq. (4), can be written as (Talbot *et al.* [28]);

$$\mathrm{V}_\mathrm{T} = -\kappa_1 \nu \frac{\nabla \mathrm{T}}{\mathrm{T}_\mathrm{r}} = -\frac{\kappa_1 \nu}{\mathrm{T}_\mathrm{r}}\frac{\partial \mathrm{T}}{\partial y}, \tag{10}$$

where T_r is some reference temperature and κ_1 is the thermophoretic coefficient which ranges in value from 0.2 to1.2 (Batchelor and Shen [29]).

In order to obtain similarity solutions of the problem, the following non-dimensional variables are introduced;

$$\eta = y\sqrt{\frac{U_0}{2\nu x}}, \psi = \sqrt{2\nu x U_0}\, f(n), \ \theta(n) = \frac{T - T_\infty}{T_w - T_\infty}, \ \phi(n) = \frac{C - C_\infty}{C_w - C_\infty} \tag{11}$$

where ψ is the stream function that satisfies the continuity Eq. (1).

Since $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$, therefore $u = U_0 f'$ and $v = -\sqrt{\frac{\nu U_0}{2x}}(f - \eta f')$. (12)

Here the prime denotes the ordinary differentiation with respect to the similarity variables η.

Using (11) and (12) in (2), (4) and (9) results in the following ordinary differential equations;

$$\frac{d^3 f}{d\eta^3} + f\frac{d^2 f}{d\eta^2} + \gamma\theta + \delta\phi - (M^2 + \frac{1}{K})\frac{df}{d\eta} = 0, \tag{13}$$

$$(3R + 4)\frac{d^2\theta}{d\eta^2} + 3R\Pr f\frac{d\theta}{d\eta} - 3R\Pr S\theta = 0, \tag{14}$$

$$\frac{d^2\phi}{d\eta^2}+Scf\frac{d\phi}{d\eta}-Sc\tau_p\frac{d\theta}{d\eta}\frac{d\phi}{d\eta}-Sc\tau_p\phi\frac{d^2\theta}{d\eta^2}-Sc\tau_p\mathrm{A}\frac{d^2\theta}{d\eta^2}+ScK_r\phi=0 \tag{15}$$

The boundary conditions (5) then turn into;

$$f=f_w,\ \frac{df}{d\eta}=1,\ \theta=1,\ \phi=1 \text{ at } \eta=0$$
$$\frac{df}{d\eta}\to 0,\ \theta\to 0,\ \phi\to 0 \text{ as } \eta\to\infty, \tag{16}$$

where $f_w=-v_w(x)\sqrt{\frac{2x}{\nu U_0}}$ is the well suction velocity at the permeable plate. Here $f_w>0$ denotes the suction.

The non-dimensional parameters are;

$R=\frac{\kappa\kappa_1}{4\sigma_1 T_\infty^3}$ (radiation parameter), $\mathrm{Re}_x=\frac{U_0 2x}{\nu}$ (local Reynolds number),

$\mathrm{Pr}=\frac{\nu\rho C_p}{\kappa}$ (Prandtl number), $M=B_o\sqrt{\frac{2x\sigma}{\rho U_0}}$ (local magnetic field parameter),

$Sc=\frac{\nu}{D_T}$ (Schmidt number), $\tau_p=\frac{-\kappa_1(T_w-T_\infty)f_w^2}{T_r}$ (thermophoretic parameter),

$\gamma=\frac{Gr_x}{\mathrm{Re}_x^2}$ (local buoyancy parameter), $Gr_x=\frac{g\beta(T_w-T_\infty)(2x)^3}{\nu^2}$ (local Grashof number),

$\delta=\frac{Gm_x}{\mathrm{Re}_x^2}$ (local modified buoyancy parameter), $N=\frac{3R}{3R+4}$ (thermal radiation parameter)

$Gm_x=\frac{g\beta^*(T_w-T_\infty)(2x)^3}{\nu^2}$ (local modified Grashof number), $S=\frac{Q}{\rho C_p U_0}$ (heat source parameter)

and $K_r=\frac{\mathrm{k_r}\nu}{U_0}$ (chemical reaction parameter).

Method and solution

Here the following transformation is used to obtain the closed form solution for velocity, temperature and concentration;

$$\xi=\eta f_w,\ f(\eta)=f_w\Theta(\xi),\ \theta(\eta)=f_w{}^2\Phi(\xi),\ \phi(\eta)=f_w{}^2\Psi(\xi) \tag{17}$$

Substituting Eq. (17) into Eqs. (13) - (15) results in a set of ordinary differential equations;

$$\frac{d^3\Theta}{d\xi^3}+\Theta(\xi)\frac{d^2\Theta}{d\xi^2}=\varepsilon\left\{\left(M^2+\frac{1}{K}\right)\frac{d\Theta}{d\xi}-\gamma\Phi-\delta\Psi\right\} \tag{18}$$

$$\frac{d^2\Phi}{d\xi^2} + N\Pr\Theta(\xi)\frac{d\Phi}{d\xi} - N\Pr S\varepsilon\phi(\xi) = 0 \tag{19}$$

$$Sc.\tau_p\left(\frac{d^2\Phi}{d\xi^2}\Psi(\xi) + \frac{d\Phi}{d\xi}\frac{d\Psi}{d\xi}\right) = \varepsilon\left(\frac{d^2\Psi}{d\xi^2} + Sc\Theta(\xi)\frac{d\Psi}{d\xi} - ASc\tau_p\frac{d^2\Phi}{d\xi^2}\right) \tag{20}$$

The corresponding boundary conditions (16) become;

$$\begin{aligned} &\Theta(\xi) = 1,\ \frac{d\Theta}{d\xi} = \varepsilon,\ \Phi(\xi) = \varepsilon,\ \Psi(\xi) = \varepsilon \ \text{ at } \xi = 0 \\ &\frac{d\Theta}{d\xi} \to 0,\ \Phi(\xi) \to 0,\ \Psi(\xi) \to 0 \ \text{ as } \xi \to \infty \end{aligned} \tag{21}$$

where ε ($=1/f_w^2$) is very small. Therefore, for large suction, Θ(ξ), Φ(ξ) and Ψ(ξ) can be represented in terms of ε as follows;

$$\Theta(\xi) = 1 + \varepsilon\Theta_1(\xi) + \varepsilon^2\Theta_2(\xi) + \varepsilon^3\Theta_3(\xi) + ... \tag{22}$$

$$\Phi(\xi) = \varepsilon\Phi_1(\xi) + \varepsilon^2\Phi_2(\xi) + \varepsilon^3\Phi_3(\xi) + ... \tag{23}$$

$$\Psi(\xi) = \varepsilon\Psi_1(\xi) + \varepsilon^2\Psi_2(\xi) + \varepsilon^3\Psi_3(\xi) + ... \tag{24}$$

Substituting Eqs. (22) - (24) into Eqs. (18) - (20) and neglect the higher order of $0(\varepsilon^4)$ result in the following three sets of ordinary differential equations and corresponding boundary conditions;
First order $o(\varepsilon)$;

$$\frac{d^3\Theta_1}{d\xi^3} + \frac{d^2\Theta_1}{d\xi^2} = 0, \tag{25}$$

$$\frac{d^2\Phi_1}{d\xi^2} + N\Pr\frac{d\Phi_1}{d\xi} = 0, \tag{26}$$

$$\frac{d^2\Psi_1}{d\xi^2} + Sc\frac{d\Psi_1}{d\xi} = ASc\tau_p\frac{d^2\Phi_1}{d\xi^2}, \tag{27}$$

with the following boundary conditions;

$$\begin{aligned} &\Theta_1(\xi) = 0,\ \frac{d\Theta_1}{d\xi} = 1,\ \Phi_1(\xi) = 1,\ \Psi_1(\xi) = 1 \ \text{ at } \xi = 0 \\ &\frac{d\Theta_1}{d\xi} \to 0,\ \Phi_1(\xi) \to 0,\ \Psi_1(\xi) \to 0 \ \text{ as } \xi \to \infty \end{aligned} \tag{28}$$

Second order $o(\varepsilon^2)$;

$$\frac{d^3\Theta_2}{d\xi^3}+\frac{d^2\Theta_2}{d\xi^2}+\Theta_1(\xi)\frac{d^2\Theta_1}{d\xi^2}=\left(M^2+\frac{1}{K}\right)\frac{d\Theta_1}{d\xi}-\gamma\Phi_1(\xi)-\delta\Psi_1(\xi), \tag{29}$$

$$\frac{d^2\Phi_2}{d\xi^2}+N\Pr\frac{d\Phi_2}{d\xi}=-N\Pr\Theta_1(\xi)\frac{d\Phi_1}{d\xi}+N\Pr S\Phi_1(\xi), \tag{30}$$

$$\frac{d^2\Psi_2}{d\xi^2}+Sc\frac{d\Psi_2}{d\xi}=-Sc\Theta_1(\xi)\frac{d\Psi_1}{d\xi}+Sc\tau_p\frac{d^2\Phi_1}{d\xi^2}\Psi_1(\xi)+Sc\tau_p\frac{d\Phi_1}{d\xi}\frac{d\Psi_1}{d\xi}+Sc\tau_p\mathrm{A}\frac{d^2\Phi_2}{d\xi^2}-ScK_r\Psi_1(\xi) \tag{31}$$

with the following boundary conditions;

$$\begin{gathered}\Theta_2(\xi)=0,\ \frac{d\Theta_2}{d\xi}=0,\ \Phi_2(\xi)=0,\ \Psi_2(\xi)=0\ \text{ at }\xi=0\\ \frac{d\Theta_2}{d\xi}\to 0,\ \Phi_2(\xi)\to 0,\ \Psi_2(\xi)\to 0\ \text{ as }\ \xi\to\infty\end{gathered} \tag{32}$$

Third order $o(\varepsilon^3)$;

$$\frac{d^3\Theta_3}{d\xi^3}+\frac{d^2\Theta_3}{d\xi^2}=\left(M^2+\frac{1}{K}\right)\frac{d\Theta_2}{d\xi}-\gamma\Phi_2(\xi)-\delta\Psi_2(\xi)-\Theta_1(\xi)\frac{d^2\Theta_2}{d\xi^2}-\Theta_2(\xi)\frac{d^2\Theta_1}{d\xi^2}, \tag{33}$$

$$\frac{d^2\Phi_3}{d\xi^2}+N\Pr\frac{d\Phi_3}{d\xi}=-N\Pr\Theta_1(\xi)\frac{d\Phi_2}{d\xi}-N\Pr\Theta_2(\xi)\frac{d\Phi_1}{d\xi}+N\Pr S\Phi_2(\xi), \tag{34}$$

$$\begin{aligned}\frac{d^2\Psi_3}{d\xi^2}+Sc\frac{d\Psi_3}{d\xi}&=Sc.\tau_p\left(\frac{d^2\Phi_1}{d\xi^2}\Psi_2+\frac{d^2\Phi_2}{d\xi^2}\Psi_1+\frac{d\Phi_1}{d\xi}\frac{d\Psi_2}{d\xi}+\frac{d\Phi_2}{d\xi}\frac{d\Psi_1}{d\xi}\right)\\ &\quad -Sc\left(\Theta_1(\xi)\frac{d\Psi_2}{d\xi}+\Theta_2(\xi)\frac{d\Psi_1}{d\xi}\right)+Sc\tau_p\mathrm{A}\frac{d^2\Phi_3}{d\xi^2}-ScK_r\Psi_2(\xi),\end{aligned} \tag{35}$$

with the following boundary conditions;

$$\begin{gathered}\Theta_3(\xi)=0,\ \frac{d\Theta_3}{d\xi}=0,\ \Phi_3(\xi)=0,\ \Psi_3(\xi)=0\ \text{ at }\ \xi=0\\ \frac{d\Theta_3}{d\xi}\to 0,\ \Phi_3(\xi)\to 0,\ \Psi_3(\xi)\to 0\ \text{ as }\ \xi\to\infty\end{gathered} \tag{36}$$

Without going into details, the solutions of Eqs. (25) - (27), (29) - (31), (33) - (35) with the help of boundary conditions (28), (32) and (36) are;

$$\Theta_1(\xi)=1-e^{-\xi}, \tag{37}$$

$$\Phi_1(\xi)=e^{-\mathrm{N}\Pr\xi}, \tag{38}$$

$$\Psi_1(\xi) = (1-\mathrm{k}_1)e^{-Sc\xi} + \mathrm{k}_1 e^{-\mathrm{N\,Pr}\,\xi}, \tag{39}$$

$$\Theta_2(\xi) = \mathrm{k}_4 + \mathrm{k}_5 e^{-\xi} + \left\{\left(M^2 + \frac{1}{K}\right) + 1\right\}\xi e^{-\xi} + \frac{1}{4}e^{-2\xi} + \mathrm{k}_2 e^{-\mathrm{N\,Pr}\,\xi} + \mathrm{k}_3 e^{-Sc\xi}, \tag{40}$$

$$\Phi_2(\xi) = \mathrm{k}_6 e^{-\mathrm{N\,Pr}\,\xi} - (\mathrm{N\,Pr} + S)\xi e^{-\mathrm{N\,Pr}\,\xi} - \mathrm{k}_6 e^{-(1+\mathrm{N\,Pr})\xi}, \tag{41}$$

$$\Psi_2(\xi) = \mathrm{k}_{14} e^{-Sc\xi} + \mathrm{k}_7 \xi e^{-Sc\xi} + \mathrm{k}_8 e^{-\mathrm{N\,Pr}\,\xi} + \mathrm{k}_9 e^{-2\mathrm{N\,Pr}\,\xi} + \mathrm{k}_{10} e^{-(1+\mathrm{N\,Pr})\xi} + \mathrm{k}_{11} e^{-(1+Sc)\xi} + \mathrm{k}_{12} e^{-(Sc+\mathrm{N\,Pr})\xi} + \mathrm{k}_{13} \xi e^{-\mathrm{N\,Pr}\,\xi}, \tag{42}$$

$$\begin{aligned}\Theta_3(\xi) &= (\mathrm{k}_{27} - \mathrm{k}_{28}) + \mathrm{k}_{28} e^{-\xi} + \mathrm{k}_{15} \xi e^{-\xi} + \mathrm{k}_{16}\left(\frac{\xi^2}{2} + 2\xi\right)e^{-\xi} - \frac{\mathrm{k}_{17}}{4}e^{-2\xi} + \mathrm{k}_{18} e^{-\mathrm{N\,Pr}\,\xi} + \mathrm{k}_{19} e^{-Sc\xi} \\ &- \frac{\mathrm{k}_{20}}{4}(\xi+2)e^{-2\xi} - \frac{5}{72}e^{-3\xi} + \mathrm{k}_{21} e^{-(1+\mathrm{N\,Pr})\xi} + \mathrm{k}_{22} e^{-(1+Sc)\xi} + \mathrm{k}_{23}\left(\xi + \frac{2}{\mathrm{N\,Pr}} + \frac{1}{\mathrm{N\,Pr}-1}\right)e^{-\mathrm{N\,Pr}\,\xi} \\ &+ \mathrm{k}_{24}\left(\xi + \frac{2}{Sc} + \frac{1}{Sc-1}\right)e^{-Sc\xi} + \mathrm{k}_{25} e^{-2\mathrm{N\,Pr}\,\xi} + \mathrm{k}_{26} e^{-(Sc+\mathrm{N\,Pr})\xi},\end{aligned} \tag{43}$$

$$\begin{aligned}\Phi_3(\xi) &= \mathrm{k}_{36} e^{-\mathrm{N\,Pr}\,\xi} + \mathrm{k}_{29} \xi e^{-\mathrm{N\,Pr}\,\xi} + \mathrm{k}_{30}\left(\frac{\xi^2}{2} + \frac{\xi}{\mathrm{N\,Pr}}\right)e^{-\mathrm{N\,Pr}\,\xi} + \mathrm{k}_{31} e^{-(1+\mathrm{N\,Pr})\xi} + \mathrm{k}_{32}\left(\xi + \frac{2+\mathrm{N\,Pr}}{1+\mathrm{N\,Pr}}\right)e^{-(1+\mathrm{N\,Pr})\xi} \\ &+ \mathrm{k}_{33} e^{-(2+\mathrm{N\,Pr})\xi} + \mathrm{k}_{34} e^{-2\mathrm{N\,Pr}\,\xi} + \mathrm{k}_{35} e^{-(Sc+\mathrm{N\,Pr})\xi},\end{aligned} \tag{44}$$

$$\begin{aligned}\Psi_3(\xi) &= \mathrm{k}_{57} e^{-Sc\xi} + \mathrm{k}_{37} e^{-(Sc+\mathrm{N\,Pr})\xi} + \mathrm{k}_{38} \xi e^{-(Sc+\mathrm{N\,Pr})\xi} + \mathrm{k}_{39} e^{-2\mathrm{N\,Pr}\,\xi} + \mathrm{k}_{40} e^{-3\mathrm{N\,Pr}\,\xi} + \mathrm{k}_{41} e^{-(1+2\mathrm{N\,Pr})\xi} \\ &+ \mathrm{k}_{42} e^{-(1+Sc+\mathrm{N\,Pr})\xi} + \mathrm{k}_{43} e^{-(Sc+2\mathrm{N\,Pr})\xi} + \mathrm{k}_{44} \xi e^{-2\mathrm{N\,Pr}\,\xi} + \mathrm{k}_{45} e^{-\mathrm{N\,Pr}\,\xi} + \mathrm{k}_{46} \xi e^{-\mathrm{N\,Pr}\,\xi} \\ &+ \mathrm{k}_{47}\left\{\xi^2 + \frac{2(2\mathrm{N\,Pr} - Sc)}{\mathrm{N\,Pr}(\mathrm{N\,Pr} - Sc)}\xi\right\}e^{-\mathrm{N\,Pr}\,\xi} + \mathrm{k}_{48} e^{-(1+\mathrm{N\,Pr})\xi} + \mathrm{k}_{49} \xi e^{-(1+\mathrm{N\,Pr})\xi} + \mathrm{k}_{50} e^{-(2+\mathrm{N\,Pr})\xi} \\ &+ \mathrm{k}_{51} \xi e^{-Sc\xi} + \mathrm{k}_{52}\left(\xi + \frac{1}{Sc}\right)e^{-Sc\xi} + \mathrm{k}_{53} e^{-(1+Sc)\xi} + \mathrm{k}_{54}\left(\xi + \frac{2+Sc}{1+Sc}\right)e^{-(1+Sc)\xi} + \mathrm{k}_{55} e^{-(2+Sc)\xi} \\ &+ \mathrm{k}_{56} e^{-2Sc\xi} - \frac{K_r \mathrm{k}_7}{2}\xi^2 e^{-Sc\xi},\end{aligned} \tag{45}$$

where k_i's are given in the appendix.

The quantities of main physical interest are the skin friction coefficient (rate of shear stress), the Nusselt number (rate of heat transfer) and the Sherwood number (rate of mass transfer). The equation defining the wall shear stress is;

$$\tau = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

Thus, the local skin friction coefficient C_f is;

$$C_f = 2(\mathrm{Re}_x)^{-1/2} f''(0)$$

$$\text{or, } C_f^{\ *} = f''(0) = -1 + \varepsilon\left[\mathrm{k}_5 + Sc^2\mathrm{k}_3 + \mathrm{N}^2\,\mathrm{Pr}^2\,\mathrm{k}_2 - 2\left\{\left(M^2 + \frac{1}{K}\right) + 1\right\} + 1\right] \tag{46}$$
$$+\varepsilon^2(\mathrm{A}_{20}+\mathrm{A}_{21}+\mathrm{A}_{22}+\mathrm{A}_{23}).$$

where $C_f^{\ *} = \frac{1}{2}(\mathrm{Re}_x)^{1/2} C_f$

Knowing the temperature field, the effect of the thermal radiation on the rate of heat transfer q_w is given by;

$$q_w = \left(\frac{\partial T}{\partial y}\right)_{y=0} - \left(\frac{4\sigma^*}{3\mathrm{k}^*}\frac{\partial T^4}{\partial y}\right)_{y=0}$$

So the rate of heat transfer in terms of the dimensionless Nusselt number is defined as follows;

$$Nu = -\frac{1}{2}\left(\mathrm{Re}_x\right)^{1/2}(1+N)\theta'(0)$$

$$\text{or, } Nu^* = (1+N)\theta'(0) = (1+N)\{\mathrm{N\,Pr} + \varepsilon(\mathrm{N\,Pr} + S - \mathrm{k}_6) + \varepsilon^2(\mathrm{A}_{24}+\mathrm{A}_{25})\} \tag{47}$$

where $Nu^* = 2(\mathrm{Re}_x)^{-1/2} Nu$

Similarly, the rate of mass transfer in terms of local Sherwood number is given by;

$$Sh^* = \phi'(0) = Sc(1-\mathrm{k}_1) + \mathrm{N\,Pr\,k}_1 + \varepsilon(\mathrm{A}_{26}+\mathrm{A}_{27}) + \varepsilon^2(\mathrm{A}_{28}+\mathrm{A}_{29}+\mathrm{A}_{30}) \tag{48}$$

It is worth mentioning that the present results are in excellent agreement with the results of Singh *et al.* [27] in the absence of permeability parameter and chemical reaction.

Results and discussion

In order to have an insight into the effects of the parameters on the steady hydromagnetic mixed convection heat and mass transfer flow past a vertical permeable plate in the presence of thermal radiation and thermophoresis with first order chemical reaction, the numerical results have been presented graphically in **Figures 1 - 9** and **Table 1** for several sets of values of the pertinent parameters, such as thermal radiation parameter N, thermophoretic parameter τ_p, magnetic field parameter M, suction parameter f_w and chemical parameter K_r. In the simulation the default values of the parameters are considered as M = 0.4, Sc = 0.42, δ = 0.2, γ = 1.0, Pr = 0.71, N = 0.2 and τ_p = 0.1 unless otherwise specified. From **Table 1**, it is noteworthy that the heat transfer rate at the plate increases with increasing values of N, f_w but the effect is opposite for K, Kr. It is also seen that the effect of increasing values of N, f_w and M is to increase the absolute values of skin friction, whereas it decreases slightly with increasing values of K. Further, it is found that an increase in Kr leads to a decrease in the values of the rate of mass transfer, while the effect is reversed for τ_p and N.

Table 1 Effects of different parameters on C_f^*, Nu^* and Sh^*.

N	f_w	M	K	K_r	τ_p	C_f^*	Nu^*	Sh^*
0.4	3.0	0.3	5	0.2	0.1	–0.7730	0.0738	0.3313
0.5						–0.8059	0.2299	0.3806
0.6						–0.8331	0.3467	2.1874
0.4	4.0	0.3	5	0.2	0.1	–0.8410	0.2296	
	5.0					–0.8890	0.2681	
	6.0					–0.9194	0.2801	
0.4	3.0	0.6	5	0.2	0.1	–0.8478		
		0.8				–0.8987		
		1.0				–0.9487		
0.4	3.0	0.3	1	0.2	0.1	–0.7918	0.0798	
			2			–0.7819	0.0760	
			∞			–0.7660	0.0723	
0.4	3.0	0.3	5	0.0	0.1		0.0765	0.3548
				0.4			0.0711	0.3092
				0.8			0.0656	0.2691
0.4	3.0	0.3	5	0.2	0.0			0.3290
					0.4			0.3387
					0.8			0.3492

Velocity profiles

In **Figures 1 - 3**, the behavior of the non-dimensional fluid velocity for various material parameters is presented. As the parameter value of M increases in the presence of thermal radiation and thermophoresis at the plate surface, the flow rate retards and thereby gives rise to a decrease in the velocity profiles, as shown in **Figure 1**. The momentum boundary layer thickness generally decreases with increasing values of M. The reason behind this phenomenon is that application of a magnetic field to an electrically conducting fluid gives rise to a resistive type force called the Lorentz force. This force has the tendency to slow down the motion of the fluid in the boundary layer. These results are in agreement with the results obtained by Singh *et al*. [27]. **Figure 2** depicts the effect of permeability parameter K on the fluid velocity. These graphs reveal that an increase in the values of K results in an increase of the velocity distribution. It can be easily seen from **Figure 3** that the velocity decreases as η increases for a fixed value of thermal radiation parameter N. For a non-zero fixed value of η, velocity distribution across the boundary layer decreases with increasing values of N. The physics behind this reason is that the increased radiation decreases the thickness of momentum boundary layer, which ultimately diminishes the velocity.

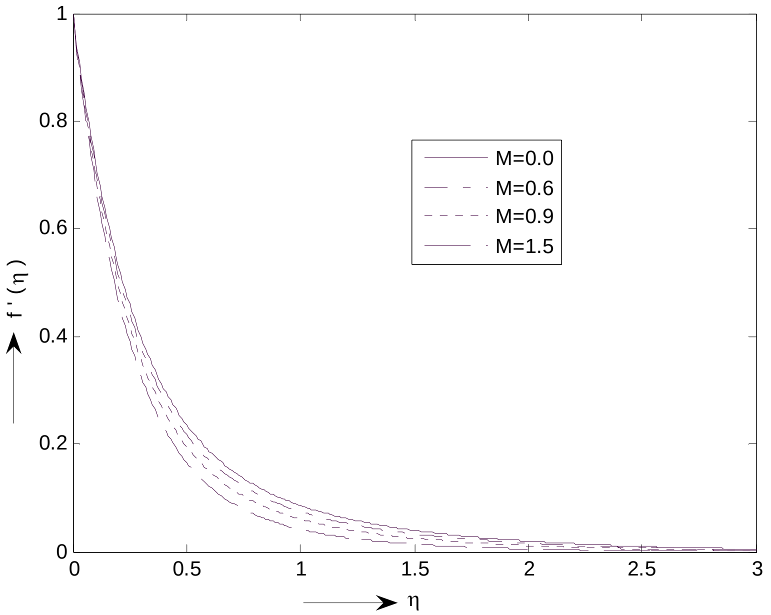

Figure 1 Non-dimensional velocity profiles for different values of M.

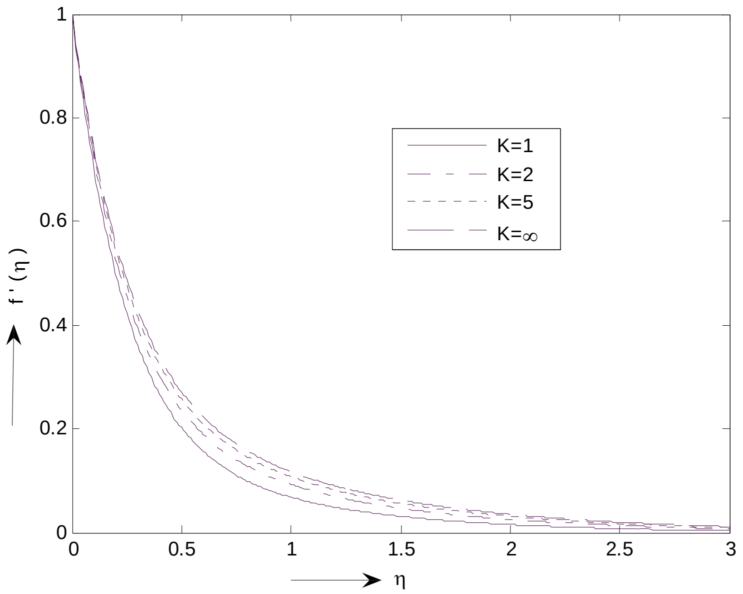

Figure 2 Non-dimensional velocity profiles for different values of K.

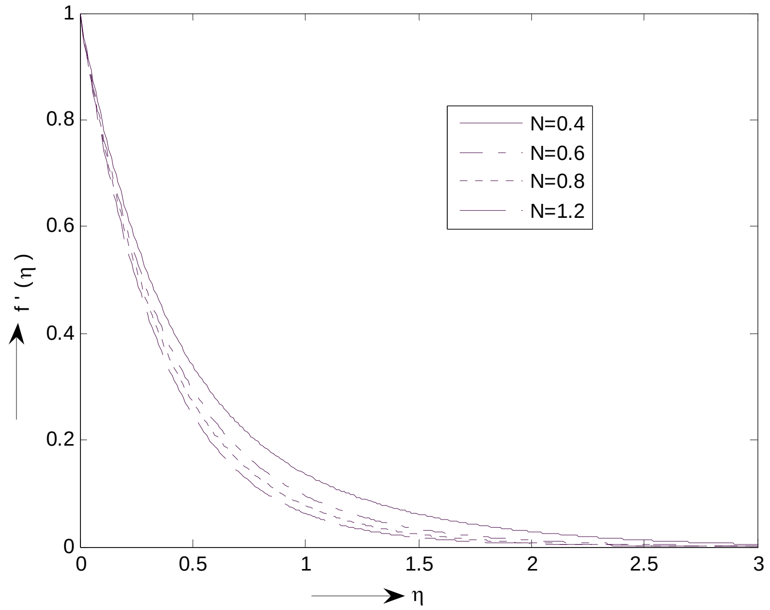

Figure 3 Non-dimensional velocity profiles for different values of N.

Temperature profiles

The effects of thermal radiation parameter and suction parameter on the fluid temperature are illustrated in **Figures 4** and **5** respectively. Typical variations of the temperature profiles against η are shown in **Figure 4** for various values of suction parameter f_w. The results show that with an increase in the suction parameter f_w the temperature profiles decrease adjacent to the surface of the plate; the effect is not significant far away from the plate and hence there is a decrease in the thermal boundary layer thickness. **Figure 5** shows the effect of thermal radiation parameter N on temperature distribution. From this figure, it can be seen that the temperature distribution decreases uniformly with increasing thermal radiation parameter and hence increases the thickness of the thermal boundary layer.

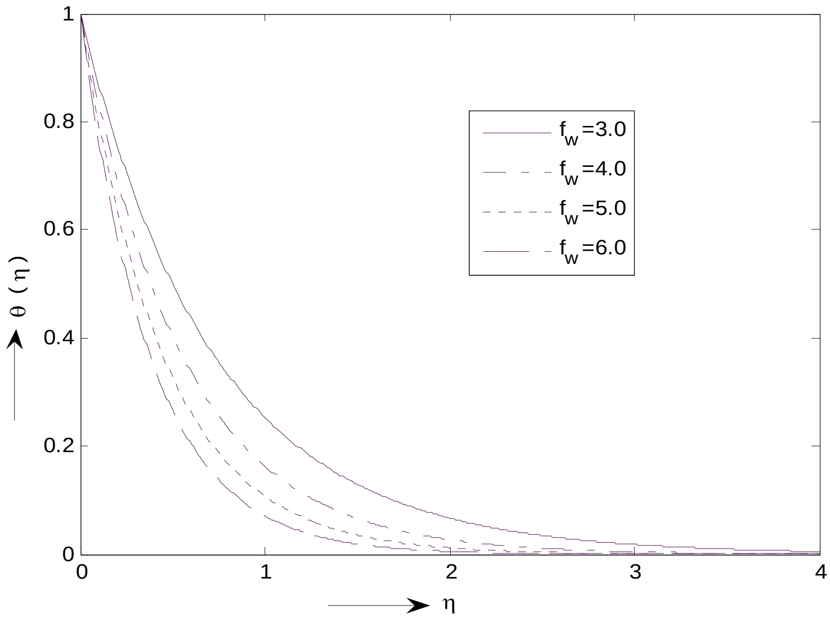

Figure 4 Non-dimensional temperature profiles for different values of f_w.

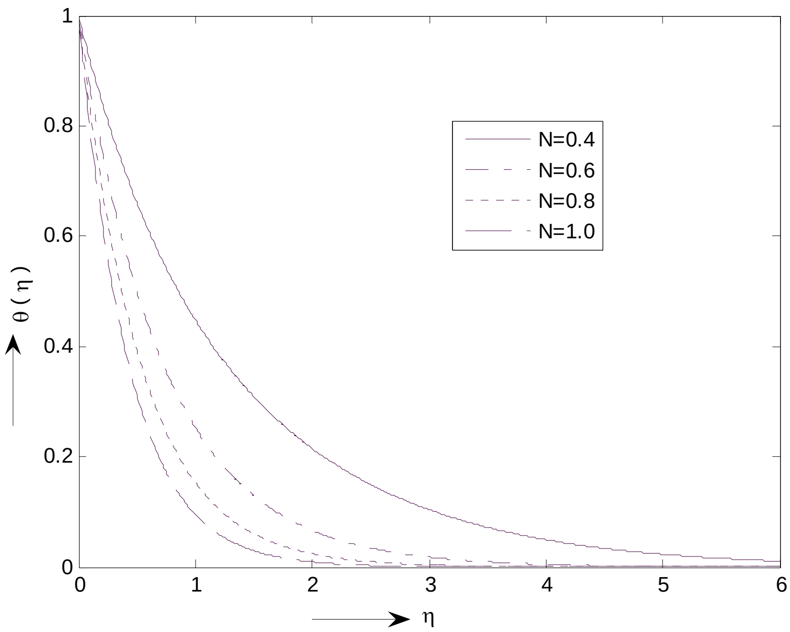

Figure 5 Non-dimensional temperature profiles for different values of N.

Concentration profiles

Figures 6 - 9 depict chemical species concentration profiles against η for various values of thermophysical parameters in the boundary layer. Generally, whenever the species concentration at the plate surface is higher than that of the free stream, a gradual decrease in concentration profile towards the free stream is observed. The trend is reversed whenever the species concentration at the plate surface is lower than the free stream concentration. **Figure 6** shows the variation of concentration distribution across the boundary layer for different values of f_w. It is observed from this figure that the concentration of the fluid decreases with an increase in the suction parameter f_w. **Figure 7** demonstrates the effects of N on the concentration profiles in the presence of thermophoresis. It is observed from the figure that concentration decreases on increasing the thermal radiation parameter N in the boundary layer region and is at a maximum in the vicinity of the surface of the plate. **Figure 8** shows the variation of the concentration distribution across the boundary layer for various values of the chemical reaction parameter Kr. It is seen that the effect of increasing values of the chemical reaction parameter results in increasing the concentration distribution across the boundary layer. The reason behind this phenomenon is that chemical reaction increases the rate of interfacial mass transfer. **Figure 9** illustrates the influence of the thermophoretic parameter on the concentration profiles. It is seen that concentration of the fluid increases with an increase of the thermophoretic parameter τ_p. So, thermophoretic parameter is expected to alter the concentration boundary layer significantly. These results fall in line with the results obtained by Singh *et al.* [27].

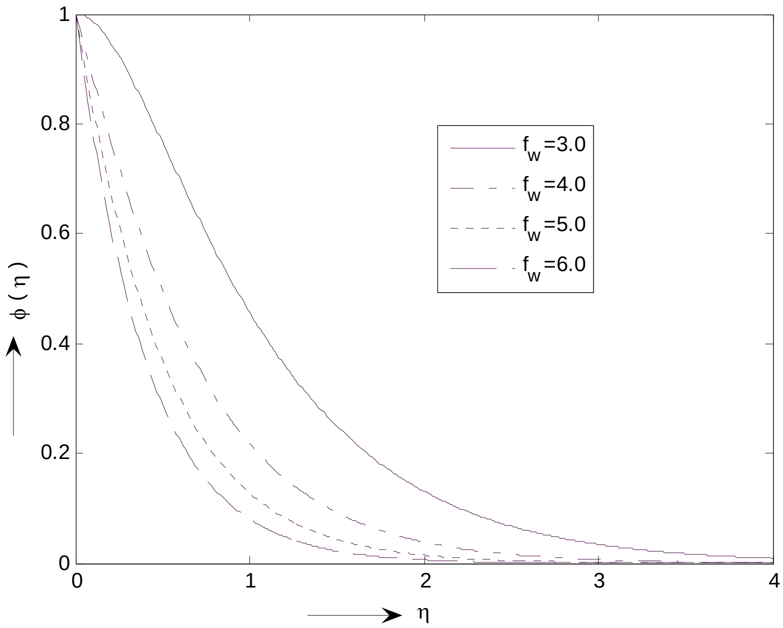

Figure 6 Non-dimensional concentration profiles for different values of f_w.

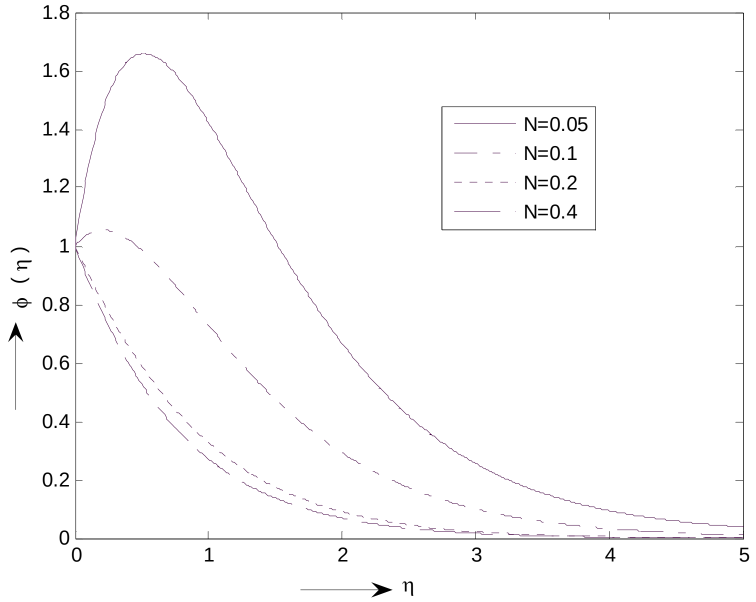

Figure 7 Non-dimensional concentration profiles for different values of N.

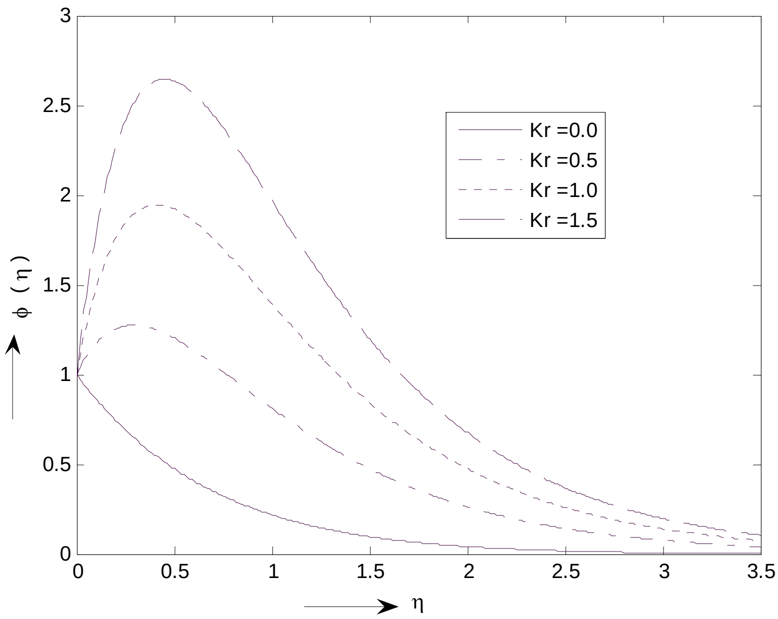

Figure 8 Non-dimensional concentration profiles for different values of Kr.

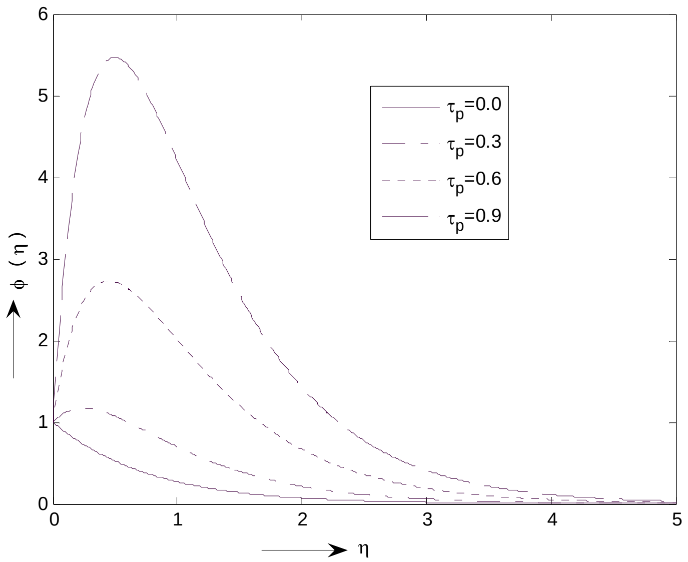

Figure 9 Non-dimensional concentration profiles for different values of τ_p.

Conclusions

The effects of thermophoresis and chemical reaction on steady mixed convection hydromagnetic flow of an incompressible electrically conducting fluid past a heated vertical permeable flat plate in the presence of thermal radiation is studied. Using the similarity transformation, the governing system of partial differential equations is transformed into ordinary differential equations and is solved analytically using the perturbation method. Numerical results are presented to illustrate the details of the flow, heat and mass transfer characteristics and their dependence on material parameters. It can be concluded the following results from our investigation:

(i) The velocity distribution decrease for increasing values of the magnetic field parameter M and the thermal radiation parameter N but the effect is reversed for permeability parameter K.

(ii) The temperature profiles decrease with an increase in both thermal radiation parameter N and suction parameter f_w.

(iii) The species concentration profiles increase for increasing chemical reaction parameter Kr and thermophoretic parameter τ_p but the effect is reversed for thermal radiation parameter N and suction parameter f_w.

(iv) The skin friction coefficient (absolute value) C_f increases with an increase of thermal radiation parameter N, suction parameter f_w and magnetic field parameter M, but the effect is reversed for permeability parameter K.

(v) The Nusselt number Nu decreases with increasing values of K, Kr. It is also noticeable that the rate of heat transfer increases due to the presence of thermal radiation and suction.

(vi) An increase in Kr leads to a decrease in the values of the Sherwood number, while the effect is reversed for τ_p and N.

Acknowledgements

The authors wish to express their very sincere thanks to the honorable referees for their valuable comments and suggestions for improving the quality of the paper.

References

[1] A Apelblat. Mass transfer with a chemical reaction of the first order. Effects of axial diffusion. *Chem. Eng. J.* 1982; **23**, 193-9.
[2] KF Jensen, EQ Einset and DI Fotiadis. Flow phenomena in chemical vapor deposition of thin films. *Ann. Rev. Fluid Mech.* 1991; **23**, 197-232.
[3] PG Siddheshwar and S Manjunath. Unsteady convective diffusion with heterogeneous chemical reaction in a plane-Poiseuille flow of a micropolar fluid. *Int. J. Eng. Sci.* 2000; **38**, 765-83.
[4] BM Mitrovic and DV Papavassiliou. Effects of a first-order chemical reaction on turbulent mass transfer. *Int. J. Heat Mass Tran.* 2004; **47**, 34-61.
[5] AJ Chamkha. MHD flow of a uniformly stretched vertical permeable surface in the presence of heat generation/absorption and a chemical reaction. *Int. Comm. Heat Mass Tran.* 2003; **30**, 413-22.
[6] R Kandasamy, K Periasamy and KKS Prabhu. Chemical reaction, heat and mass transfer on MHD flow over a vertical stretching surface with heat source and thermal stratification effects. *Int. J. Heat Mass Tran.* 2005; **48**, 4557-61.
[7] PM Patil and PS Kulkarni. Effects of chemical reaction on free convective flow of a polar fluid through a porous medium in the presence of internal heat generation. *Int. J. Therm. Sci.* 2008; **47**, 1043-54.
[8] R Cortell. Toward an understanding of the motion and mass transfer with chemically reactive species for two classes of viscoelastic fluid over a porous stretching sheet. *Chem. Eng. Process.* 2007; **46**, 982-9.
[9] MEM Khedr, AJ Chamkha and M Bayomi. MHD flow of a micropolar fluid past a stretched permeable surface with heat generation or absorption. *Nonlinear Anal. Model. Contr.* 2009; **14**, 27-40.
[10] RA Damseh, MQ Al-Odat, AJ Chamkha and BA Shannak. Combined effect of heat generation or absorption and first-order chemical reaction on micropolar fluid flows over a uniformly stretched permeable surface. *Int. J. Therm. Sci.* 2009; **48**, 1658-63.
[11] E Magyari and AJ Chamkha. Combined effect of heat generation or absorption and first-order chemical reaction on micropolar fluid flows over a uniformly stretched permeable surface: The full analytical solution. *Int. J. Therm. Sci.* 2010; **49**, 1821-8.
[12] K Das. Effect of chemical reaction and thermal radiation on heat and mass transfer flow of MHD micropolar fluid in a rotating frame of reference. *Int. J. Heat Mass Tran.* 2011; **54**, 3505-13.
[13] AC Cogley, WE Vincenty and SE Gilles. Differential approximation for radiation in a non-gray gas near equilibrium. *Am. Inst. Aeronaut. Astronaut. J.* 1968; **6**, 551-3.
[14] OD Makinde. Free convection flow with thermal radiation and mass transfer past a moving vertical porous plate. *Int. Comm. Heat Mass Tran*. 2005; **32**, 1411-9.
[15] FS Ibrahim, AM Elaiw and AA Bakr. Influence of viscous dissipation and radiation on unsteady MHD mixed convection flow of micropolar fluids. *Appl. Math. Inform. Sci.* 2008; **2**, 143-62.
[16] T Hayat, Z Abbas, I Pop and S Asghar. Effects of radiation and magnetic field on the mixed convection stagnation-point flow over a vertical stretching sheet in a porous medium. *Int. J. Heat Mass Tran.* 2010; **53**, 466-74.
[17] K Das. Impact of thermal radiation on MHD slip flow over a flat plate with variable fluid properties. *Heat Mass Tran.* 2012; **48**, 767-78.
[18] BI Olajuwon. Convection heat and mass transfer in a hydromagnetic flow of a second grade fluid in the presence of thermal radiation and thermal diffusion. *Int. Comm. Heat Mass Tran.* 2011; **38**, 377-82.
[19] P Goldsmith and FG May. *Diffusiophoresis and Thermophoresis in Water Vapour Systems*. *In*: CN Davies (ed.). Aerosol, Science, Academic Press, London, 1966, p. 163-94.
[20] SL Goren. Thermophoresis of aerosol particles in laminar boundary layer on flat plate. *J. Colloid Interf. Sci.* 1977; **61**, 77-85.
[21] S Jayaraj, KK Dinesh and KL Pallai. Thermophoresis in natural convection with variable properties. *Heat Mass Tran.* 1999; **34**, 469-75.

[22] A Selim, MA Hossain and DAS Rees. The effect of surface mass transfer on mixed convection flow past a heated vertical flat permeable plate with thermophoresis. *Int. J. Therm. Sci.* 2003; **42**, 973-82.
[23] AJ Chamkha and I Pop. Effects of thermophoresis particle deposition in free convection boundary layer from a vertical flat plate embedded in a porous medium. *Int. Comm. Heat Mass Tran*. 2004; **31**, 421-30.
[24] AJ Chamkha, AF Al-Mudhaf and I Pop. Effects of heat generation or absorption on thermophoretic free convection boundary layer from a vertical flat plate embedded in a porous medium. *Int. Comm. Heat Mass Tran.* 2006; **33**, 1096-102.
[25] R Kandasamy, I Muhaimin and AB Khamis. Thermophoresis and variable viscosity effects on MHD mixed convective heat and mass transfer past a porous wedge in the presence of chemical reaction. *Heat Mass Tran*. 2009; **45**, 703-12.
[26] J Zueco, OA Beg and LM Lopez-Ochoa. Effects of thermophoresis particle deposition and of the thermal conductivity in a porous plate with dissipative heat and mass transfer. *Acta Mech. Sin.* 2011; **27**, 389-98.
[27] NP Singh, AK Singh, AK Singh and P Agnihotri. Effects of thermophoresis on hydromagnetic mixed convection and mass transfer flow past a vertical permeable plate with variable suction and thermal radiation. *Comm. Nonlinear Sci. Numer. Simulat.* 2011; **16**, 2519-34.
[28] L Talbot, RK Chang, AW Schefer and DR Wills. Thermophoresis of particles in a heated boundary layer. *J. Fluid Mech.* 1980; **101**, 737-58.
[29] GK Batchelor and C Shen. Thermophoretic deposition of particles in gas flowing over gold surface. *J. Colloid Interf. Sci.* 1985; **107**, 21-37.

Appendix

$$\mathrm{k}_1=\frac{\mathrm{A}\tau_{\mathrm{p}}ScN\,\mathrm{Pr}}{N\,\mathrm{Pr}-Sc},\quad \mathrm{k}_2=\frac{\gamma+\delta\mathrm{k}_1}{N^2\,\mathrm{Pr}^2(N\,\mathrm{Pr}-1)},\quad \mathrm{k}_3=\frac{\delta(1-\mathrm{k}_1)}{Sc^2(Sc-1)},$$

$$\mathrm{k}_4=\mathrm{k}_2(N\,\mathrm{Pr}-1)+\mathrm{k}_3(Sc-1)-\frac{3}{4}-\left(M^2+\frac{1}{K}\right),$$

$$\mathrm{k}_5=\left(M^2+\frac{1}{K}\right)+\frac{1}{2}-N\,\mathrm{Pr}\,\mathrm{k}_2-Sc\mathrm{k}_3,$$

$$\mathrm{k}_6=\frac{N^2\,\mathrm{Pr}^2}{1+N\,\mathrm{Pr}},\quad \mathrm{k}_7=Sc(\mathrm{k}_1-1)\left(1-\frac{K_r}{Sc}\right),$$

$$\mathrm{k}_8=\frac{Sc}{N\,\mathrm{Pr}-Sc}\left\{\mathrm{k}_1+\mathrm{A}\tau_{\mathrm{p}}\mathrm{k}_6N\,\mathrm{Pr}-2\mathrm{A}\tau_{\mathrm{p}}N\,\mathrm{Pr}(N\,\mathrm{Pr}+Sc)-\frac{K_r\mathrm{k}_1}{N\,\mathrm{Pr}}\right\},$$

$$\mathrm{k}_9=\frac{Sc\tau_{\mathrm{p}}\mathrm{k}_1N\,\mathrm{Pr}}{2N\,\mathrm{Pr}-Sc},\quad \mathrm{k}_{10}=-\frac{Sc[\mathrm{k}_1N\,\mathrm{Pr}+\mathrm{A}\tau_{\mathrm{p}}\mathrm{k}_6(1+N\,\mathrm{Pr})^2]}{(1+N\,\mathrm{Pr})(1+N\,\mathrm{Pr}-Sc)},$$

$$\mathrm{k}_{11}=\frac{Sc^2(\mathrm{k}_1-1)}{1+Sc},\quad \mathrm{k}_{12}=Sc\tau_{\mathrm{p}}(1-\mathrm{k}_1),\quad \mathrm{k}_{13}=-\frac{\mathrm{A}\tau_{\mathrm{p}}N\,\mathrm{Pr}(N\,\mathrm{Pr}+Sc)}{N\,\mathrm{Pr}-Sc},$$

$$\mathrm{k}_{14}=-(\mathrm{k}_8+\mathrm{k}_9+\mathrm{k}_{10}+\mathrm{k}_{11}+\mathrm{k}_{12}),$$

$$\mathrm{k}_{15}=\left[\mathrm{k}_4-\mathrm{k}_5\left\{\left(M^2+\frac{1}{K}\right)+1\right\}+\left\{\left(M^2+\frac{1}{K}\right)+1\right\}\left\{\left(M^2+\frac{1}{K}\right)+2\right\}\right],$$

$$\mathrm{k}_{16}=-\left\{\left(M^2+\frac{1}{K}\right)+1\right\}^2,\quad \mathrm{k}_{17}=\left\{2\mathrm{k}_5-2\left\{\left(M^2+\frac{1}{K}\right)+1\right\}-\frac{1}{2}\left(M^2+\frac{1}{K}\right)-1\right\},$$

$$\mathrm{k}_{18}=-\frac{\left[N^2\,\mathrm{Pr}^2\,\mathrm{k}_2+\left(M^2+\frac{1}{K}\right)N\,\mathrm{Pr}\,\mathrm{k}_2+\gamma\mathrm{k}_6+\delta\mathrm{k}_8\right]}{N^2\,\mathrm{Pr}^2(1-N\,\mathrm{Pr})},$$

$$\mathrm{k}_{19}=-\frac{\left[Sc^2\mathrm{k}_3+\left(M^2+\frac{1}{K}\right)Sc\mathrm{k}_3+\delta\mathrm{k}_{14}\right]}{Sc^2(1-Sc)},\ \mathrm{k}_{20}=2\left\{\left(M^2+\frac{1}{K}\right)+1\right\},$$

$$\mathrm{k}_{21}=-\frac{(N^2\,\mathrm{Pr}^2\,\mathrm{k}_2+\mathrm{k}_2+\gamma\mathrm{k}_6-\delta\mathrm{k}_{10})}{N\,\mathrm{Pr}(1+N\,\mathrm{Pr})^2},\ \mathrm{k}_{22}=-\frac{(Sc^2\mathrm{k}_3+\mathrm{k}_3-\delta\mathrm{k}_{11})}{Sc(Sc+1)^2},$$

$$\mathrm{k}_{23}=\frac{\{\gamma(N\,\mathrm{Pr}+S)-\delta\mathrm{k}_{13}\}}{N^2\,\mathrm{Pr}^2(1-N\,\mathrm{Pr})},\ \mathrm{k}_{24}=\frac{\delta\mathrm{k}_7}{Sc^2(Sc-1)},$$

$$\mathrm{k}_{25}=-\frac{\delta\mathrm{k}_9}{4N^2\,\mathrm{Pr}^2(1-2N\,\mathrm{Pr})},\ \mathrm{k}_{26}=-\frac{\delta\mathrm{k}_{12}}{(N\,\mathrm{Pr}+Sc)^2(1-N\,\mathrm{Pr}-Sc)},$$

$$k_{27}=\frac{k_{17}}{4}-k_{18}-k_{19}+\frac{k_{20}}{2}+\frac{5}{72}-k_{21}-k_{22}-k_{23}\left(\frac{2}{N\Pr}+\frac{1}{N\Pr-1}\right)$$
$$-k_{24}\left(\frac{2}{Sc}+\frac{1}{Sc-1}\right)-k_{25}-k_{26},$$
$$k_{28}=k_{15}+2k_{16}+\frac{k_{17}}{2}-N\Pr k_{18}-Sck_{19}+\frac{3}{4}k_{20}+\frac{5}{24}-(1+N\Pr)k_{21}-(1+Sc)k_{22}$$
$$-N\Pr\left(\frac{2}{N\Pr}+\frac{1}{N\Pr-1}\right)k_{23}+k_{23}-Sc\left(\frac{2}{Sc}+\frac{1}{Sc-1}\right)k_{24}+k_{24}$$
$$-2N\Pr k_{25}-(N\Pr+Sc)k_{26},\ k_{29}=-\frac{(N^2\Pr^2 k_6-N\Pr k_4-Sk_6)}{N\Pr},$$
$$k_{30}=\frac{(N\Pr+S)(N^2\Pr^2-S)}{N\Pr},\ k_{31}=\frac{\{Sk_6-k_6N\Pr(1+N\Pr)-N^2\Pr^2k_6-N\Pr k_5\}}{(1+N\Pr)},$$
$$k_{32}=-\frac{N\Pr\left[\left\{\left(M^2+\frac{1}{K}\right)+1\right\}+N\Pr(N\Pr+S)\right]}{(1+N\Pr)},\ k_{33}=\frac{N\Pr\left\{k_6(1+N\Pr)-\frac{1}{4}\right\}}{2(2+N\Pr)},$$
$$k_{34}=-\frac{k_2}{2N\Pr},\ k_{35}=-\frac{k_3N\Pr}{Sc(N\Pr+Sc)},\ k_{36}=-k_{30}-\left(\frac{2+N\Pr}{1+N\Pr}\right)k_{31}-k_{32}-k_{33}-k_{34}-k_{35},$$
$$k_{37}=\frac{Sc}{N\Pr(N\Pr+Sc)}[\tau_p N\Pr A_1+A_2+\tau_p(2N\Pr+Sc)\{k_7-(1-k_1)(N\Pr+S)\}]+A_3-\frac{ScK_rk_{12}}{N^2\Pr^2+N\Pr Sc},$$
$$k_{38}=Sc\tau_p\{k_7-(1-k_1)(N\Pr+S)\},$$
$$k_{39}=\frac{Sc}{2(2N\Pr-Sc)}\{\tau_pA_4+A_5+\tau_pA_6\}-\frac{ScK_rk_9}{4N^2\Pr^2-2ScN\Pr},\ k_{40}=\frac{Sc\tau_pN\Pr}{3N\Pr-Sc},$$
$$k_{41}=\frac{Sc}{(1+2N\Pr-Sc)}\{\tau_pA_{14}-A_{15}\},\ k_{42}=\frac{Sc}{(1+N\Pr)}\{\tau_pA_{16}-A_{17}\},\ k_{43}=\frac{k_{12}Sc\tau_p}{2},$$
$$k_{44}=\frac{k_{13}Sc\tau_pN\Pr}{2N\Pr-Sc},\ k_{45}=\frac{Sc}{N\Pr(N\Pr-Sc)}\{A\tau_pA_7+A_8+A_9-A_{10}\}-\frac{ScK_rk_8}{N^2\Pr^2-ScN\Pr},$$
$$k_{46}=\frac{Sc}{(N\Pr-Sc)}\{A\tau_p(k_{29}N\Pr+k_{30})+k_{13}\}-\frac{ScK_rk_{13}}{N^2\Pr^2-ScN\Pr},\ k_{47}=\frac{k_{30}ASc\tau_pN\Pr}{2(N\Pr-Sc)},$$
$$k_{48}=\frac{Sc}{(1+N\Pr-Sc)}\{A\tau_pA_{11}+A_{12}\}+A_{13}-\frac{ScK_rk_{10}}{(1+N\Pr)^2-Sc(1+N\Pr)},$$
$$k_{49}=\frac{k_{32}ASc\tau_p(1+N\Pr)}{N\Pr}-\frac{Sc(k_{13}-k_1M-k_1)}{1+N\Pr},$$
$$k_{50}=\frac{k_{33}ASc\tau_p(2+N\Pr)}{2+N\Pr-Sc}-\frac{Sc\left(k_{10}+k_{10}N\Pr-\frac{k_1}{4}N\Pr\right)}{(2+N\Pr)(2+N\Pr-Sc)},$$

$\mathrm{k}_{51} = Sc(\mathrm{k}_7 - \mathrm{k}_{14}Sc + \mathrm{k}_1\mathrm{k}_4 Sc - \mathrm{k}_4 Sc) + K_r\mathrm{k}_{14} - K_r\mathrm{k}_7$, $\mathrm{k}_{52} = -Sc\mathrm{k}_7$,

$\mathrm{k}_{53} = \dfrac{Sc}{1+Sc}\{\mathrm{k}_{11}(1+Sc) - Sc\mathrm{k}_{14} + \mathrm{k}_7 + Sc\mathrm{k}_5(1-\mathrm{k}_1)\} - \dfrac{ScK_r\mathrm{k}_{11}}{1+Sc}$,

$\mathrm{k}_{54} = -\dfrac{Sc}{1+Sc}\left[Sc\mathrm{k}_7 - Sc(1-\mathrm{k}_1)\left\{\left(M^2+\dfrac{1}{K}\right)+1\right\}\right]$,

$\mathrm{k}_{55} = -\dfrac{Sc}{2(2+Sc)}\left\{\mathrm{k}_{11}(1+Sc) - \dfrac{Sc}{4}(1-\mathrm{k}_1)\right\}$, $\mathrm{k}_{56} = \dfrac{\mathrm{k}_3(1-\mathrm{k}_1)}{2}$, $\mathrm{k}_{57} = -(\mathrm{A}_{18}+\mathrm{A}_{19})$,

$\mathrm{A}_1 = (N\,\mathrm{Pr}+Sc)(\mathrm{k}_{14}+\mathrm{k}_6-\mathrm{k}_1\mathrm{k}_6) - \mathrm{k}_7$, $\mathrm{A}_2 = \mathrm{k}_{12}(N\,\mathrm{Pr}+Sc) + Sc\mathrm{k}_2(1-\mathrm{k}_1) + N\,\mathrm{Pr}\,\mathrm{k}_1\mathrm{k}_3$,

$\mathrm{A}_3 = \dfrac{\mathrm{k}_{35}\mathrm{A}\tau_\mathrm{p}Sc(N\,\mathrm{Pr}+Sc)}{N\,\mathrm{Pr}}$, $\mathrm{A}_4 = 2\mathrm{k}_8 N\,\mathrm{Pr} + 2N\,\mathrm{Pr}\,\mathrm{k}_1\mathrm{k}_6 - \mathrm{k}_{13}$,

$\mathrm{A}_5 = 2\mathrm{A}\tau_\mathrm{p}N\,\mathrm{Pr}\,\mathrm{k}_{34} + 2\mathrm{k}_9 + \mathrm{k}_1\mathrm{k}_2$, $\mathrm{A}_6 = \dfrac{(4N\,\mathrm{Pr}-Sc)(\mathrm{k}_{13}-\mathrm{k}_1N\,\mathrm{Pr}-\mathrm{k}_1S)}{(2N\,\mathrm{Pr}-Sc)}$,

$\mathrm{A}_7 = N^2\,\mathrm{Pr}^2\,\mathrm{k}_{36} - 2N\,\mathrm{Pr}\,\mathrm{k}_{29} - \mathrm{k}_{30}$, $\mathrm{A}_8 = N\,\mathrm{Pr}\,\mathrm{k}_8 - \mathrm{k}_{13} + N\,\mathrm{Pr}\,\mathrm{k}_1\mathrm{k}_4$,

$\mathrm{A}_9 = \dfrac{(2N\,\mathrm{Pr}-Sc)\{\mathrm{A}\tau_\mathrm{p}(N\,\mathrm{Pr}\,\mathrm{k}_{29}+\mathrm{k}_{30})+\mathrm{k}_{13}\}}{(N\,\mathrm{Pr}-Sc)}$, $\mathrm{A}_{10} = \dfrac{\mathrm{k}_{30}\mathrm{A}\tau_\mathrm{p}N\,\mathrm{Pr}}{(N\,\mathrm{Pr}-Sc)}$,

$\mathrm{A}_{11} = \mathrm{k}_{31}(1+N\,\mathrm{Pr}) - \mathrm{k}_{32}(4+N\,\mathrm{Pr})$, $\mathrm{A}_{12} = \dfrac{1}{1+N\,\mathrm{Pr}}\{\mathrm{k}_{10}(1+N\,\mathrm{Pr}) + N\,\mathrm{Pr}(\mathrm{k}_1\mathrm{k}_5-\mathrm{k}_8)+\mathrm{k}_{13}\}$,

$\mathrm{A}_{13} = \dfrac{Sc(1+2N\,\mathrm{Pr})}{N^2\,\mathrm{Pr}^2(1+N\,\mathrm{Pr})^2}\left[\mathrm{A}\tau_\mathrm{p}\mathrm{k}_{32}(1+N\,\mathrm{Pr})^2 - N\,\mathrm{Pr}\,\mathrm{k}_{13} + \mathrm{k}_1N\,\mathrm{Pr}\left\{\left(M^2+\dfrac{1}{K}\right)+1\right\}\right]$,

$\mathrm{A}_{14} = \mathrm{k}_{10}N\,\mathrm{Pr} - \mathrm{k}_1\mathrm{k}_6(1+N\,\mathrm{Pr})$, $\mathrm{A}_{15} = \dfrac{2\mathrm{k}_9N\,\mathrm{Pr}}{1+2N\,\mathrm{Pr}}$, $\mathrm{A}_{16} = \mathrm{k}_{11}N\,\mathrm{Pr} - \mathrm{k}_6(1-\mathrm{k}_1)(1+N\,\mathrm{Pr})$,

$\mathrm{A}_{17} = \dfrac{N\,\mathrm{Pr}+Sc}{1+N\,\mathrm{Pr}+Sc}$, $\mathrm{A}_{18} = \mathrm{k}_{37}+\mathrm{k}_{39}+\mathrm{k}_{40}+\mathrm{k}_{41}+\mathrm{k}_{42}+\mathrm{k}_{43}+\mathrm{k}_{45}$,

$\mathrm{A}_{19} = \mathrm{k}_{48}+\mathrm{k}_{50}+\dfrac{\mathrm{k}_{52}}{Sc}+\mathrm{k}_{53}+\mathrm{k}_{54}\left(\dfrac{2+Sc}{1+Sc}\right)+\mathrm{k}_{55}+\mathrm{k}_{56}$,

$\mathrm{A}_{20} = \mathrm{k}_{28} - 2\mathrm{k}_{15} - 3\mathrm{k}_{16} - \mathrm{k}_{17} + N^2\,\mathrm{Pr}^2\,\mathrm{k}_{18} + Sc^2\mathrm{k}_{19} - \mathrm{k}_{20}$,

$\mathrm{A}_{21} = (1+N\,\mathrm{Pr})^2\mathrm{k}_{21} + (1+Sc)^2\mathrm{k}_{22} - \dfrac{5}{8}$,

$\mathrm{A}_{22} = N^2\,\mathrm{Pr}^2\,\mathrm{k}_{23}\left(\dfrac{2}{N\,\mathrm{Pr}}+\dfrac{1}{N\,\mathrm{Pr}-1}\right) - 2N\,\mathrm{Pr}\,\mathrm{k}_{23} + Sc^2\mathrm{k}_{24}\left(\dfrac{2}{Sc}+\dfrac{1}{Sc-1}\right) - 2Sc\mathrm{k}_{24}$,

$\mathrm{A}_{23} = 4N^2\,\mathrm{Pr}^2\,\mathrm{k}_{25} + (N\,\mathrm{Pr}+Sc)^2\mathrm{k}_{26}$,

$\mathrm{A}_{24} = N\,\mathrm{Pr}\,\mathrm{k}_{36} - \mathrm{k}_{29} - \dfrac{\mathrm{k}_{30}}{N\,\mathrm{Pr}} + (1+N\,\mathrm{Pr})(\mathrm{k}_{31}+\mathrm{k}_{32})$,

$\mathrm{A}_{25} = (2+N\,\mathrm{Pr})\mathrm{k}_{33} + 2N\,\mathrm{Pr}\,\mathrm{k}_{34} + (N\,\mathrm{Pr}+Sc)\mathrm{k}_{35}$,

$\mathrm{A}_{26} = Sc\mathrm{k}_{14} - \mathrm{k}_7 + N\,\mathrm{Pr}(\mathrm{k}_8+2\mathrm{k}_9) + (1+N\,\mathrm{Pr})\mathrm{k}_{10}$,

$$A_{27} = (1+Sc)k_{11} + (N\,\Pr + Sc)k_{12} - k_{13},$$

$$A_{28} = Sc(k_{57} + k_{37}) + N\,\Pr\, k_{37} - k_{38} - N\,\Pr(2k_{39} + 3k_{40}) + (1+N\,\Pr)k_{41} + (1+N\,\Pr + Sc)k_{42},$$

$$A_{29} = (2N\,\Pr + Sc)k_{43} - k_{44} + N\,\Pr\, k_{45} - k_{46} - \frac{2}{N\,\Pr}\left(\frac{2N\,\Pr - Sc}{N\,\Pr - Sc}\right)k_{47} + (1+N\,\Pr)k_{48} - k_{49},$$

$$A_{30} = (2+N\,\Pr)k_{50} - k_{51} + (1+Sc)k_{53} + (1+Sc)k_{54} + (2+Sc)k_{55} + 2Sc\,k_{56}.$$