Development and Optimization of Hybrid Friction Materials Consisting of Nanoclay and Carbon Nanotubes by using Analytical Hierarchy Process (AHP) and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) under Fuzzy Atmosphere

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Abstract

The tribo-performance of nanoclay and multi-walled carbon nanotube (MWNT) filled and graphite lubricated phenolic composites, reinforced with a combination of lapinus and kevlar fibers, have been evaluated on a Kraus friction testing machine. The combined fuzzy analytical hierarchy process (FAHP) and fuzzy technique for order preference by similarity to ideal solution (FTOPSIS) approach, taking into account performance defining attributes (PDAs) such as friction performance, wear, friction-fade, friction-recovery, stability coefficient, variability coefficient, friction fluctuations and temperature rise of the disc, was used for the performance assessment of fabricated friction composite materials. The weight of different PDAs were evaluated by FAHP; μ-performance (0.144, 0.255, 0.435), wear (0.144, 0.255, 0.435), fade-% (0.073, 0.15, 0.307), recovery-% (0.063, 0.126, 0.268), stability coefficient (0.037, 0.075, 0.156), variability coefficient (0.032, 0.063, 0.136), frictional fluctuations (0.023, 0.037, 0.069), and DTR (0.023, 0.037, 0.069) respectively. FTOPSIS was employed to determine the optimal ranking of the friction composite materials as NC-7>NC-8>NC-6>NC-5>NC-3>NC-4>NC-2>NC-1. The alternative with kevlar: lapinus, 2.5:27.5 wt-% and graphite: nanoclay: carbon nanotube, 2.25:2.75 wt-% exhibits the optimal properties.

Keywords: Friction composites, FAHP, FTOPSIS

Introduction

Composite friction materials in automotive braking application are one of the most complicated areas of composite materials. They contain a polymer matrix as a binder, inside which are a number of reinforcing fibers, fillers and property modifiers which are distributed to achieve the desired stringent level of performance defining attributes (PDAs) like stable and high friction coefficient, low wear and low noise under varying operating speeds and applied loads [1,2]. Evaluation of such PDAs and the optimal selection of desired formulations have multi-level and multi-factor aspects to consider; therefore, such difficulties can be regarded as multiple criteria decision-making (MCDM) [3]. Hence, selection of appropriate ingredients and their justifiable volumetric loading for successful friction material design is often complex and involves tedious tasks of fabrication, characterization and performance evaluation of a large number of composites. In general, several conflicting tangible and intangible factors exist for evaluating friction composite materials. Identification of these evaluation criteria, their impact on each other, assessing their importance, and choosing a best alternative among many alternatives is a well designed MCDM [4-8].

Recently, formulation designers have tried to exploit size scale advantages in friction materials. Jang *et al*. [9] concluded that the addition of MWNT into friction composition leads to an improved and observable stable coefficient of friction, as well as to improved wear resistance. Also, the incorporation of MWNT enhances the fade and recovery performances of friction composites [10]. Literature suggests that the nature of friction film has a great influence on the performance of friction material [11-13] and it is expected that inclusion of nano-fillers is significant in enhancing the friction film that theoretically leads to improvement in braking performance. The stochastic nature of this friction film affects the performance of friction composite materials [14-16].

The complications involved in performance evaluation of composite friction materials are usually not easy to overcome, not only because of the complex mechanical characteristics, but also because of their compositional variations that comprise of distinct materials properties. The use of distinct materials eventually affects braking performance due to severe interfacial interactions at the braking interfaces. Complications in performance evaluations arise more as the same composition of the friction composite materials yield different results with different manufacturing conditions [17]. The intrinsically non-deterministic nature of friction and wear processes further add complexities in the evaluation of friction composite materials. The PDAs, which are fundamental expressions of some other materials, procedures and operational mechanisms, induced sub-attributes/variables whose actual influential modes are highly complex, and hence the predictive accuracy regarding the performance trends of such multi-phase composites becomes fuzzy. The problems involve vagueness and fuzziness and the formulation designers have the difficult task of choosing among many alternatives to specify the best alternative. The imprecision comes from a variety of sources, such as the presence of multiple attributes, which may be assigned crisp or fuzzy valuations. The fuzzy approach is used to explain the evaluation of available alternatives for the selection of the best solution when criteria have complicated and non-deterministic perceptions. Therefore, the evaluation process must be conducted in a fuzzy environment by taking into account the linguistic variables. According to the above highlighted complexities of the problem, FTOPSIS is used to rank the alternatives, which is strengthened by FAHP by the estimation of weight of different PDAs. FAHP and FTOPSIS are two popular methods due to their wide applicability and simple computational procedures. The FAHP is a simple MCDM technique that copes with complicated PDAs problems and has been successfully employed in many areas [18-22]. FAHP integrates human opinions and evaluation scores, and reduces s complex problems into simple problems. The relative importance of each weight criteria with respect to the goal of the problem is determined by using a typical pair-wise comparison matrix in which all the attributes are compared with each other and scores are given according to a rational scale. Meanwhile, the FTOPSIS is a ranking method based on the philosophy that the best selected alternative should have the shortest distance from the fuzzy positive ideal solution and the longest distance from the fuzzy negative ideal solution. Chen and Hwang [23] first applied fuzzy numbers to establish FTOPSIS. As a result FTOPSIS is used in many optimization problems [24-30]. The hybrid approach of fuzzy AHP and fuzzy TOPSIS was used in solving many industrial and engineering problems viz. Rostamzadeh and Sofian [31] implemented it for improving production systems performance; Zouggari and Benyoucef [32] made use of fuzzy AHP-TOPSIS technique, for a supplier selection problem; Sun [33] implemented it to develop an evaluation model for a systematic decision support tool. Fatemeh *et al*. [34] explored the use of AHP/TOPSIS and FAHP/FTOPSIS in solving a MCDM problem. Recently, Büyüközkan and Gizem [35] used this technique for a quality improvement in the healthcare industry to evaluate a set of hospital web site alternatives to select the best alternatives that satisfied the needs of customers. Zouggari and Benyoucef [36] used a hybrid fuzzy AHP/TOPSIS approach in a supplier selection problem. When the weight and performance evaluation criteria are unclear and inaccurate, then the FAHP and FTOPSIS are the preferred techniques in solving the MCDM problem. In this research paper the optimal friction formulation is regarded as an MCDM problem and is solved by a hybrid approach of FAHP and FTOPSIS.

This research paper is structured firstly with an experimental part, followed by an evaluation methodology which includes FAHP and FTOPSIS, and how FAHP and FTOPSIS are used for optimal friction formulation.

Experimental

Fabrication of composites

Friction composite materials based on straight phenolic resin of Novolac type (JA 10), Kevlar fiber (IF 258; Twaron, Teijin-Germany), Lapinus fiber (RB 220), barites (inert filler), graphite (Graphite India Limited), nanoclay and multiwalled carbon nanotubes amounting to 100 % by weight were fabricated. The compositional variations and the designation of the composites are given in **Table 1**. The ingredients were mixed in a plough shear type of mixer to ensure mechanical isotropy of the composites. The curing of the mixture of ingredients was done in a compression molding machine at a temperature of 155 °C for 10 min under a pressure of 15 MPa, followed by post-curing in a standard oven at 150 °C for about 5 h. The friction surfaces were then polished to wipe off the resinous skin. Finally, the fabricated composites were characterized for their tribological properties.

Table 1 Details of composite designation and composition.

	Composite designation							
Composition (wt. %)	**NC-1**	**NC-2**	**NC-3**	**NC-4**	**NC-5**	**NC-6**	**NC-7**	**NC-8**
PF Resin	15	15	15	15	15	15	15	15
$BaSO_4$	50	50	50	50	50	50	50	50
Kevlar Fibre	10	10	7.5	7.5	5	5	2.5	2.5
Lapinus Fibre	20	20	22.5	22.5	25	25	27.5	27.5
Nanoclay	2	1	2.25	1.125	2.5	1.25	2.75	1.375
MWNT	0	1	0	1.125	0	1.25	0	1.375
Graphite	3	3	2.75	2.75	2.5	2.50	2.25	2.25

Tribo-performance evaluation methodology

The fade and recovery tests were conducted using a Krauss type RWDC 100C (450 V/50 Hz) machine. The Krauss machine was computer controlled and had data acquisition capabilities. The disc was connected through an interchangeable flange to a shaft that generated a moment of inertia of 2.5 kgm^2. Two brake pads with a total area of 30 cm^2 were press-fit into a pressure-actuated sliding caliper assembly. Pads were forced against opposite sides of the rotor disc at a mean contact radius of 95 mm. The load on the pads was adjusted to keep the applied contact pressure at 2 MPa. The friction force and temperature rise on the disc surface were recorded after every cycle of braking in a synchronized manner. In order to evaluate the cold friction-fade-recovery characteristics of the friction materials, a standard regulatory test PVW-3212, as per the European norms conforming to Economic Commission for Europe (ECE) regulations, has been adopted. The details of the performance evaluation procedure on this machine and the protocol behind the PVW-3212 standard are reported elsewhere [37].

Result and discussion

Effect of composition on various performance defining attributes

The descriptions of eight selected performance attributes and their experimental data of the investigated composites are given in **Tables 1 - 2**. Whereas, **Figure 1** shows the effect of compositions on the PDA-1 and PDA-2 of the investigated composites. The PDA-1 (μp) and PDA-2 (wear) are shown the improvement deterioration in prosperities respectively as the composition changed from NC-1 to NC-8. This increment in μ_p may be attributed to the increased contents of nanoclay, MWNT and lapinus. The highest wear loss in the case of NC-8 may be attributed to the presence of hard ingredients that make the abrasive component dominant; while braking, this brings larger wear losses in comparison to other composites.

Table 2 Description of the different performance defining attributes.

Performance defining attributes (PDAs)	Description of the individual PDAs	Performance implications of different PDAs
Friction performance (μ_p)	The average coefficient of friction in cold, fade and recovery cycles and denoted as μ_P.	PDA-1 higher-the-better
Wear (gm)	The progressive removal of the material from the surface of the brake pad due to thermo-mechanical and shear stresses caused by frictional interactions.	PDA-2 lower-the-better
Fade performance (%)	The difference between the performance friction coefficient and the minimum coefficient of friction (μ_F) in the fade cycle after 270 °C. It is calculated as $\frac{\mu_P - \mu_F}{\mu_P} \times 100$, and should be as minimal as possible.	PDA-3 lower-the-better
Recovery performance (%)	The revival of the braking efficiency in terms of attaining the same performance after the friction material is cooled down to a lower temperature. It is calculated as $\frac{\mu_R}{\mu_P} \times 100$ (μ_R is the highest coefficient of friction in the recovery cycle) and should be in the range of 100 - 120 % for a good friction material.	PDA-4 higher-the-better
Stability coefficient, α (%)	The ratio of performance friction to maximum friction i.e. μ_P/μ_{max} and denoted by α.	PDA-5 higher-the-better
Variability coefficient, γ (%)	The ratio of minimum friction to maximum friction, i.e. μ_{min}/μ_{max} and denoted by γ.	PDA-6 lower-the-better
Friction fluctuation (μ_{max}-μ_{min})	The frictional fluctuation (μ_{max}-μ_{min}), defined as the difference between the maximum and minimum friction in the three cycles (cold, fade and recovery). The μ_{max}-μ_{min} should be at a minimum, as it indicates unsteadiness during braking.	PDA-7 lower-the-better
Disc temperature rise (DTR) °C	The temperature rise of the rotor disc due to friction braking, irrespective of all the runs. The lower the disc temperature, the better the performance of the friction material.	PDA-8 lower-the-better

These wear particles get entrapped in the mating zone during braking and act as hard abrasives in the form of third bodies, leading to third body abrasion, which consequently enhances the friction level of the composites. The presence of a higher proportion of kevlar and graphite readily forms a good quality transfer film that adheres nicely to the rubbing surfaces, thereby minimizing wear loss.

Table 3 Experimental data of the PDAs as evaluated on a Krauss testing machine.

Composite designation	PDA-1 (μ_P)	PDA-2 (wear)	PDA-3 (fade)	PDA-4 (recovery)	PDA-5 (stability)	PDA-6 (variability)	PDA-7 (μ_{max}-μ_{min})	PDA-8 (DTR)
NC-1	0.289	1.07	47.4	130.8	0.74	0.216	0.305	459
NC-2	0.301	1.3	47.51	133.55	0.75	0.224	0.318	487
NC-3	0.32	2.57	46.56	139.06	0.72	0.225	0.345	484
NC-4	0.327	3	43.12	138.83	0.70	0.247	0.347	514
NC-5	0.323	4.5	49.23	145.51	0.65	0.166	0.413	505
NC-6	0.341	5.3	30.49	140.18	0.68	0.231	0.384	539
NC-7	0.359	7.36	40.39	139.55	0.68	0.086	0.486	531
NC-8	0.386	7.8	22.80	130.37	0.77	0.266	0.372	558

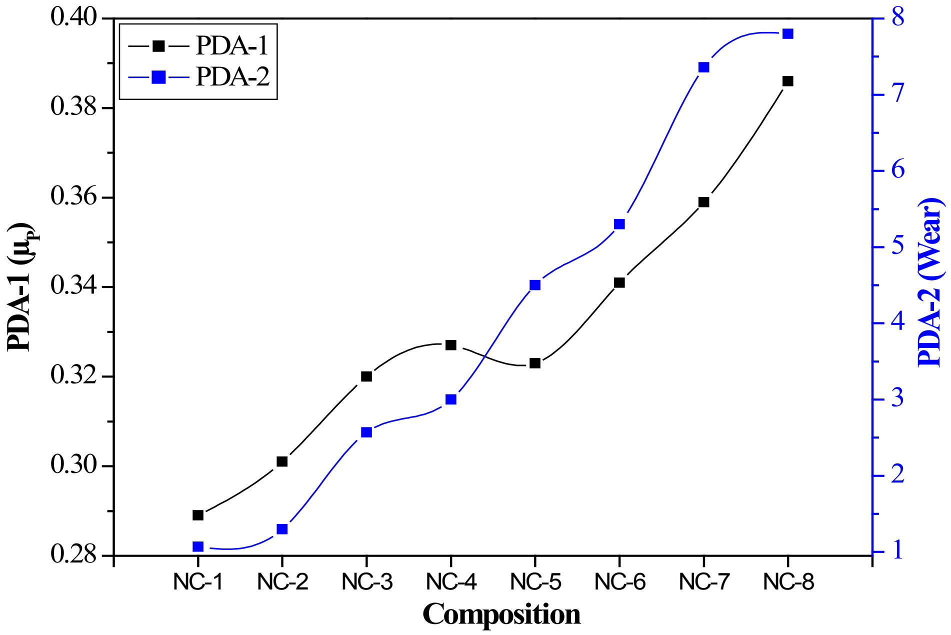

Figure 1 Variations in the PDA-1 and PDA-2 with composition.

Figure 2 shows the effect of composition on the PDA-3 (%-fade) and PDA-4 (% recovery) of the investigated composites. The temporary loss of braking effectiveness at higher temperature due to the loss of friction between the rubbing surfaces as a result of the frictional heat is known as fade, whereas the revival of the same when the friction surface is cooled down is called recovery. A higher extent of fade is undesirable since it deteriorates the braking efficiency of vehicles. The extent of μ-fade remains in the range of ~40 - 49 % for the composites having nanoclay from 2 - 2.75 wt. % and for the composites having combination of nanoclay and MWNT ≤ 2.25 wt. %.

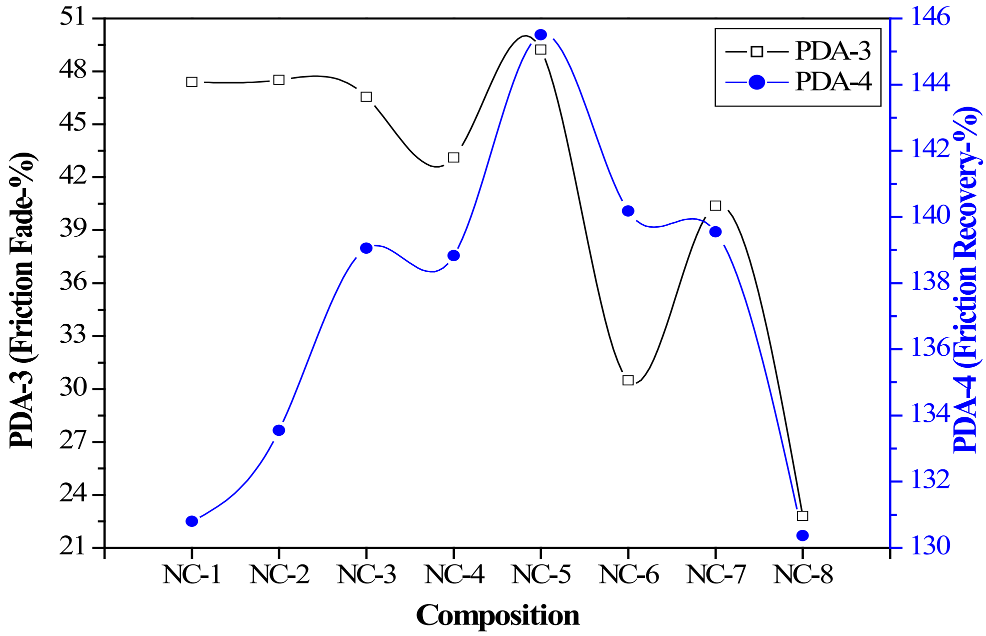

Figure 2 Variations in the PDA-3 and PDA-4 with composition.

This range is beyond most of the prescribed industrial standards and is attributed to the increased content of kevlar fiber. The composites which have NC-6 and NC-8 fade remain in the range of 22 - 30 %, which may be attributed to the abrasive nature of metallic silicate content present in nanoclay and lapinus fiber and to thermal resistance property of MWNT, and continue to enhance the friction/wear mechanisms even at elevated temperatures. The recovery performance (~130 - 145 %) of all the compositions remains high as prescribed by the standard. As the surface temperature of the pads gets lower, the friction film consisting of loosely attached wear-debris particles disintegrates and the surface underneath the friction film hardens. During the recovery cycle, this wear debris gets entrapped in the mating zone and acts as a hard abrasive in the form of third bodies which leads to third body abrasion. Consequently, the third body abrasive action enhances the friction level and the frictional response of the composites gets restored. **Figure 3** shows the effect of composition on the PDA-5 and PDA-6 (stability coefficient and variability coefficient). The stability coefficient remains lowest for NC-5 (0.65) and highest for NC-8 (0.77). Similarly, the variability coefficient remains lowest for NC-7 (0.08) and highest for NC-8 (0.266). It is required that the stability coefficient should be as high as possible and the variability coefficient should be as low as possible for the most efficient frictional response while braking. **Figure 4** shows the effect of composition on the PDA-7 (friction fluctuations) and PDA-8 (temperature rise of the disc). Among all the investigated composites, NC-8 exhibited a very high temperature rise of the disc. The temperature rise of the disc remains in the range of 460 - 510 °C for the composites, with a combination of 2 - 2.5 wt-% of nanoclay and 2 - 2.25 wt-% of nanoclay and MWNT combination, whereas it remains at 530 - 560 °C for the friction composites with 2.5 wt-% of nanoclay or combination of nanoclay and MWNT.

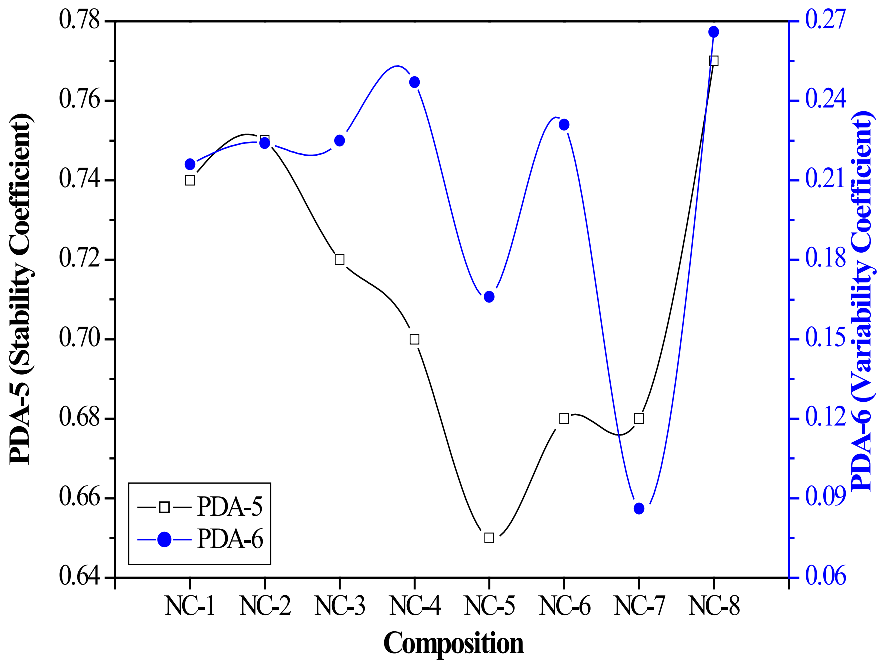

Figure 3 Variations in the PDA-5 and PDA-6 with composition.

These two classes of composites (NC-1 to NC-5) and (NC-6 to NC-8) with different compositions have not only lead to two distinct levels of temperature rise but also into two classes of friction performance (0.30 - 0.34 and 0.37 - 0.48), wear performance (1 - 4 gram and 5 - 8 gram), and fade performance (46 ± 3 and 31 ± 9 %).

It is clearly observed from **Figures 1 - 4** that the composition variation has caused a distinct transition in performance of fabricated composites. Thus, the combinational influence has proved to have played a pivotal role as a performance determinant in friction materials. Performance analysis as a function of compositional variations has clearly revealed that the various PDAs are the fundamental expressions of other materials, procedures and operational mechanisms. Hence the predictive accuracy in the performance trends of such multiphase frictional composites becomes fuzzy. In this scenario, the combined approach of FAHP and FTOPSIS has been adopted for the selection of optimal formulation.

Evaluation methodology

Several performance defining criteria or attributes must be satisfied in order to obtain an optimal solution for many decision making problems. However, sometimes these criteria or attributes, which must be satisfied, conflict. MCDM methods are commonly used to solve these types of problems. The main goal of this paper is to rank the best friction composite material from a group of alternatives. For this, FAHP is used to determine the weight of different criteria, and the best alternative is selected by the FTOPSIS method. The evaluation methodology consists of three main phases: **Phase I:** Identification of the performance criteria or attributes, **Phase II:** Determination of weight criteria using the FAHP method, and **Phase III:** Ranking of the alternatives using FTOPSIS.

In the first phase, the different alternatives and PDAs which will be used in the evaluation of optimal friction formulation are determined and a decision hierarchy is constructed. In the next phase weight is assigned to each PDA by using FAHP technique. In the last phase the best alternate is determined by using the FTOPSIS method by use of linguistic values and triangular fuzzy numbers.

Phase I: Identification of PDAs

In the first phase the various PDAs or criteria used in the performance evaluation of friction composite materials are identified. The eight identified PDAs are friction performance, wear performance, fade performance, recovery performance, stability performance, variability performance, friction fluctuations and disc temperature rise, and are briefly described in **Table 1**.

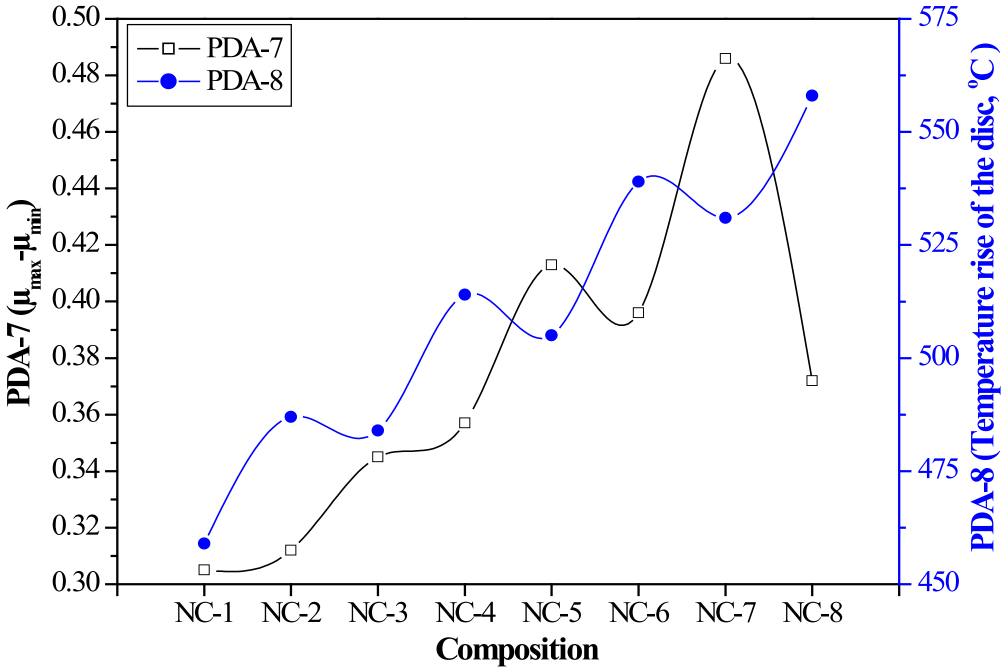

Figure 4 Variations in the PDA-7 and PDA-8 with composition.

Phase II: Determination of criteria weights

Due to the diverse significance of the PDAs in the performance evaluation of friction composite materials, one cannot assume that each PDA is of equal importance. There are many methods viz. the eigen-vector method, the entropy method, AHP etc. that can be employed to judge weights [38]. The selection of the method depends on the nature of the problems. The AHP is a structured method invented by Saaty [39] and provides a comprehensive framework for structuring a system of objectives, criteria and alternatives. The AHP consists of four main steps, including construction of hierarchy, weight analysis, and consistency verification. The AHP is a simple MCDM technique that copes with complicated PDA problems and is successfully employed in many areas of engineering and management [40-42]. Buckley extended the AHP method by using fuzzy numbers to calculate fuzzy weights by a geometric mean method [43]. In this section, we briefly review concepts of fuzzy set theory. Fuzzy set theory can be used to present linguistic values, which allows the decision makers to incorporate incomplete information.

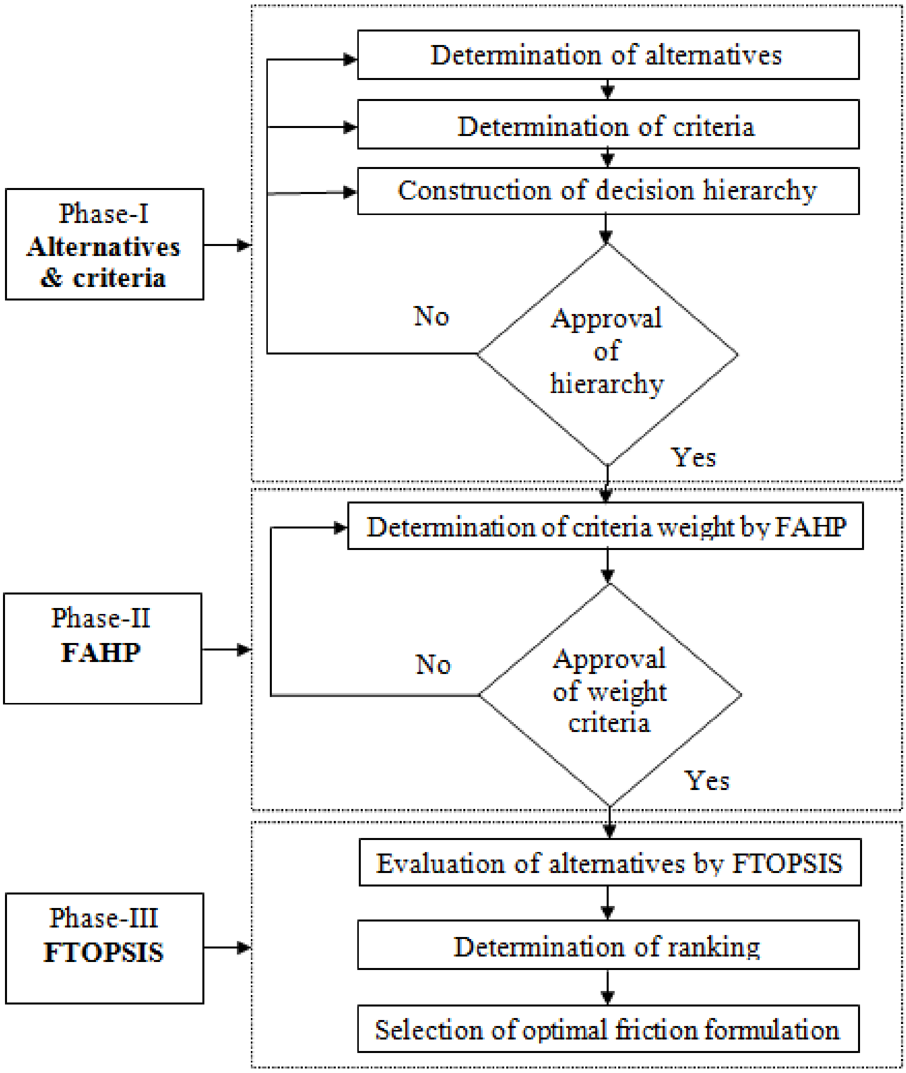

Figure 5 The evaluation methodology.

Fuzzy set theory, fuzzy numbers and Linguistic variables

Fuzzy set theory is a mathematical theory given by Zadeh [44]. The key idea of fuzzy set theory is that an element has a degree of membership in a fuzzy set, ranging between 0 and 1 [45]. A triangular fuzzy number (TFN) is defined by a triplet (l, m, n). The membership function of this fuzzy number $\mu_{\tilde{A}}(X): R \rightarrow [0,1]$ is given in Eq. 1.

Let $\tilde{A} = (l_1, m_1, n_1)$ and $\tilde{B} = (l_2, m_2, n_2)$ are two TFNs then the operational laws of these TFNs are shown in Table 4 [46]. Assuming that $\tilde{A} = (l_1, m_1, n_1)$ and $\tilde{B} = (l_2, m_2, n_2)$ are real numbers then the distance between $\tilde{A}$ and $\tilde{B}$ is equal to the Euclidean distance given by the vertex method as in Eq. 2.

$$\mu_{\tilde{A}}(X) = \begin{cases} 0 & x < l \\ \dfrac{x-l}{m-l} & l \le x \le m \\ \dfrac{x-n}{m-n} & m \le x \le n \\ 0 & x > n \end{cases} \quad (1)$$

$$D(\tilde{A}, \tilde{B}) = \sqrt{\frac{1}{3}\left[(l_1 - l_2)^2 + (m_1 - m_2)^2 + (n_1 - n_2)^2\right]} \quad (2)$$

Table 4 Operational laws of triangular fuzzy numbers.

Operational Laws	Description
Addition	$\tilde{A} + \tilde{B} = (l_1, m_1, n_1) + (l_2, m_2, n_2) = (l_1 + l_2, m_1 + m_2, n_1 + n_2)$
Subtraction	$\tilde{A} - \tilde{B} = (l_1, m_1, n_1) - (l_2, m_2, n_2) = (l_1 - n_2, m_1 - m_2, n_1 - l_2)$
Multiplication	$\tilde{A} \times \tilde{B} = (l_1, m_1, n_1) \times (l_2, m_2, n_2) = (l_1 \times l_2, m_1 \times m_2, n_1 \times n_2)$
Division	$\tilde{A} / \tilde{B} = (l_1, m_1, n_1) / (l_2, m_2, n_2) = (l_1 / n_2, m_1 / m_2, n_1 / l_2)$
Inverse	$(\tilde{A})^{-1} = (l_1, m_1, n_1)^{-1} = \left(\frac{1}{l_1}, \frac{1}{m_1}, \frac{1}{n_1}\right)$

It is difficult for formulation designers to assign a precise performance rating to an alternate under different PDAs. Decision makers usually use linguistic variables to evaluate the importance of the criteria and to rate the alternate with respect to various criteria. A linguistic variable is a variable whose values are expressed in linguistic terms and are very useful in dealing with situations which are too conflicted to evaluate. The concept of a linguistic variable is very useful in dealing with situations, which are too complex or not well defined, to be reasonably described in conventional quantitative expressions [47,48]. The linguistic values that are utilized in the optimal friction formulation selection can be expressed in positive triangular fuzzy numbers (TFNs). The criterion of each alternative is defined by a linguistic value like very high (VH), high (H), medium (M), low (L) and very low (VL) etc., and the matching TFNs to these linguistic values are given in **Table 6**.

Fuzzy analytic hierarchy process (FAHP)

The procedure for determining the evaluation criteria weights by FAHP can be summarized in the following steps:

Step-I The hierarchy is constructed in such a way that the overall goal is at the top, PDAs are in the middle and various alternatives at the bottom, as shown in **Figure 6**.

Step-II The relative importance of each criteria with respect to the goal of the problem is determined by using a typical pair-wise comparison matrix in which all the attributes are compared with each other, and scores are given using a nine-point scale as shown in **Table 7**. For N criteria the size of the comparison matrix (C) will be N×N and the entry c_{ij} donates the relative importance of criterion i with respect to criterion j.

$$C = \begin{bmatrix} c_{11} & \cdots & c_{1N} \\ \vdots & \ddots & \vdots \\ c_{N1} & \cdots & c_{NN} \end{bmatrix}, \quad c_{ii} = 1, c_{ji} = \frac{1}{c_{ij}}, c_{ij} \neq 0 \tag{3}$$

Linguistic terms are applied to the pairwise comparisons as;

$$\tilde{C} = \begin{bmatrix} \tilde{c}_{11} & \cdots & \tilde{c}_{1N} \\ \vdots & \ddots & \vdots \\ \tilde{c}_{N1} & \cdots & \tilde{c}_{NN} \end{bmatrix}, \quad \tilde{c}_{ii} = 1, \tilde{c}_{ji} = \frac{1}{\tilde{c}_{ij}}, \tilde{c}_{ij} \neq 0 \tag{4}$$

Step-III. The geometric mean method is used for fuzzy weights evaluation [49]. The fuzzy geometric mean and fuzzy weights of each criterion is calculated by using Eq. 5; the fuzzy weight of the i^{th} attribute, indicated by a triangular fuzzy number, is given by Eq. 6.

$$\tilde{r}_i = \left[\tilde{c}_{i1} \times \tilde{c}_{i2} \times \ldots \times \tilde{c}_{iN}\right]^{1/N} \tag{5}$$

$$\tilde{W}_i = \tilde{r}_i \times \left[\tilde{r}_1 + \tilde{r}_2 + \ldots + \tilde{r}_i + \ldots + \tilde{r}_N\right]^{-1} \tag{6}$$

where $\tilde{c}_{iN}$ is the fuzzy comparison value of the i^{th} criterion to criterion N, $\tilde{r}_i$ is the fuzzy geometric mean of the fuzzy comparison value of criterion i to each criterion and $\tilde{W}_i$ is the fuzzy weight of the i^{th} criterion which can be indicated by a TFN, $\tilde{W}_i = (lw_i, mw_i, nw_i)$, where lw_i, mw_i and nw_i are the lower, middle and upper values of the fuzzy weight of the i^{th} criterion respectively.

Phase III: The FTOPSIS method for ranking of the alternatives

The TOPSIS (technique for order preference by similarity to ideal solution) has wide applicability and is used for tackling ranking problems due to its simplicity. TOPSIS was developed by Hwang and Yoon [38]. Due to the presence of ambiguous and vague issues in the performance evaluation of friction composite materials, FTOPSIS is employed for performance evaluation which use linguistic values rather than numerical values, which means that the rankings in the performance evaluation are evaluated by linguistic variables. Linguistic value can deal with ambiguities, uncertainties and vagueness. The FTOPSIS consist of the following steps:

Step I: A decision matrix is created after identifying the performance defining criterion and alternatives of the problem. If the number of alternatives is M and the number of performance defining criterion are N then the decision matrix having an order of M × N is represented according to Eq. 7.

$$D_{M\times N} = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1N} \\ a_{21} & a_{22} & \ldots & a_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ a_{M1} & a_{M2} & \ldots & a_{MN} \end{bmatrix} \tag{7}$$

where an element a_{ij} of the decision matrix $D_{M\times N}$ represents the actual value of the i^{th} alternative in term of j^{th} PDA.

Step II: In order to transform the performance values to fuzzy linguistic variables, the decision matrix is converted to a normalized decision matrix (a_{ij}) by converting the performance values of the decision matrix into a range of [0, 1]. The normalized values of each element in the normalized decision matrix can be calculated by using Eq. 8.

$$r_{ij} = \frac{a_{ij} - \min\{a_{ij}\}}{\max\{a_{ij}\} - \min\{a_{ij}\}}, \text{ for benefit criteria, and}$$

$$r_{ij} = \frac{\max\{a_{ij}\} - a_{ij}}{\max\{a_{ij}\} - \min\{a_{ij}\}}, \text{ for cost criteria} \tag{8}$$

Step III: The linguistic values ($\tilde{a}_{ij}$, i= 1, 2... M, j = 1, 2... N) are chosen for M alternatives with respect to N criteria. These fuzzy linguistic values preserve the properties that the range of fuzzy numbers belongs to [0, 1].

Step IV: A weighted normalized fuzzy decision matrix is calculated by using Eq. 9.

Step V: A determination of fuzzy positive ideal solution (FPIS, $\tilde{A}^{+}$) and a fuzzy negative ideal solution (FNIS, $\tilde{A}^{-}$) are made by using Eq. 10 and 11.

$$\tilde{V}_{ij} = \tilde{a}_{ij} \times \tilde{W}_{i} \tag{9}$$

$$\tilde{A}^{+} = \left(\tilde{V}_1^{+}, \tilde{V}_2^{+}, \ldots, \tilde{V}_N^{+}\right), \text{ and } \tilde{A}^{-} = \left(\tilde{V}_1^{-}, \tilde{V}_2^{-}, \ldots, \tilde{V}_N^{-}\right) \tag{10}$$

Where

$$\tilde{V}_j^{+} = \begin{Bmatrix} \left(\max_i \tilde{V}_{ij}\right) \text{if j is benefit criteria} \\ \left(\min_i \tilde{V}_{ij}\right) \text{if j is cos t criteria} \end{Bmatrix}, \text{ and}$$

$$\tilde{V}_j^{-} = \begin{Bmatrix} \left(\min_i \tilde{V}_{ij}\right) \text{if j is benefit criteria} \\ \left(\max_i \tilde{V}_{ij}\right) \text{if j is cos t criteria} \end{Bmatrix}, \text{ for j = 1, 2. . .N} \tag{11}$$

Step VI: The Euclidian distances between each of the alternatives and the fuzzy positive ideal solution and the fuzzy negative ideal solution are calculated by using Eq. 12.

Step VII: Finally, the overall preference or fuzzy closeness index ($\tilde{CI}_i$) of the alternatives is calculated with the help of Eq. 13.

$$\tilde{D}_i^{+} = \sqrt{\sum_{j=1}^{N} D\left(\tilde{V}_i^{+} - \tilde{V}_{ij}\right)^2}, \text{ and}$$

$$\tilde{D}_i^{-} = \sqrt{\sum_{j=1}^{N} D\left(\tilde{V}_{ij} - \tilde{V}_i^{-}\right)^2}, \text{ for i = 1, 2. . . M} \tag{12}$$

$$\tilde{C}\tilde{I}_i = \frac{\tilde{D}_i^+}{\tilde{D}_i^+ + \tilde{D}_i^-}, \text{ for } i = 1, 2 \ldots M \tag{13}$$

Criteria weight calculation and ranking of the alternatives

The decision matrix from Eq. 7 is used for the fuzzy TOPSIS analysis. In order to transform the performance values to fuzzy linguistic variables, the performance values in **Table 3** are normalized into a range of [0, 1] by using Eq. 8. The normalized decision matrix is given in **Table 5**. Linguistic values are used to evaluate the importance of the different PDAs. To illustrate the idea of fuzzy analysis, the actual values of the decision matrix are converted to the fuzzy linguistic variables by using **Table 6**.

Table 5 The normalized decision matrix.

	PDA-1 (μ_P)	PDA-2 (wear)	PDA-3 (fade)	PDA-4 (recovery)	PDA-5 (stability)	PDA-6 (variability)	PDA-7 (μ_{max}-μ_{min})	PDA-8 (DTR)
NC-1	0.000	1.000	0.069	0.028	0.750	0.278	1.000	1.000
NC-2	0.124	0.966	0.065	0.210	0.833	0.233	0.928	0.717
NC-3	0.320	0.777	0.101	0.574	0.583	0.228	0.779	0.747
NC-4	0.392	0.713	0.231	0.559	0.417	0.106	0.768	0.444
NC-5	0.351	0.490	0.000	1.000	0.000	0.556	0.403	0.535
NC-6	0.536	0.371	0.709	0.648	0.250	0.194	0.564	0.192
NC-7	0.722	0.065	0.334	0.606	0.250	1.000	0.000	0.273
NC-8	1.000	0.000	1.000	0.000	1.000	0.000	0.630	0.000

Table 6 Linguistic values and fuzzy numbers.

Linguistic values	Fuzzy numbers
Very low (VL)	(0, 0.10, 0.25)
Low (L)	(0.15, 0.30, 0.45)
Medium (M)	(0.35,0.50,0.65)
High (H)	(0.55, 0.70, 0.85)
Very high (VH)	(0.75, 0.90, 1)

Table 7 The fundamental relational scale for pair-wise comparisons.

Intensity of importance on an absolute scale	Definition	Explanation	Scale of FUZZY numbers
1	Equal importance	Two activities contribute equally to the objective	(1,1,1)
2	Weak importance	Experience and judgment slightly favour one activity over another	(1,2,3)
3	Moderate importance	Experience and judgment moderately favour one activity over another	(2,3,4)
4	Preferable	Experience and judgment strongly favour one activity over another	(3,4,5)

Intensity of importance on an absolute scale	Definition	Explanation	Scale of FUZZY numbers
5	Essential or strong importance	Experience and judgment strongly favour one activity over another	(4,5,6)
6	Fairly good importance	Experience and judgment strongly favour one activity over another	(5,6,7)
7	Very strong importance	An activity is very strongly favored and its dominance is demonstrated in practice	(6,7,8)
8	Absolute	An activity is absolutely favored and its dominance is demonstrated in practice	(7,8,9)
9	Extreme importance	The evidence favoring one activity over another is of the highest possible order of affirmation	(8,9,10)

Calculation of weight criteria of different PDAs

The weights of the different PDAs used in this ranking process are calculated by using the FAHP method. For this a pair-wise comparison matrix is formed by using the scale given in **Table 7**. The constructed pair-wise comparison matrix is given in **Table 8** and the corresponding pair-wise comparison matrix of PDAs in terms of fuzzy numbers is given in **Table 9**. The results obtained from comparison matrix are given in **Table 10**.

Table 8 The pair-wise comparison matrix.

	PDA-1 (μ_P)	**PDA-2 (wear)**	**PDA-3 (fade)**	**PDA-4 (recovery)**	**PDA-5 (stability)**	**PDA-6 (variability)**	**PDA-7 (μ_{max}-μ_{min})**	**PDA-8 (DTR)**
PDA-1 (μ_P)	1	1	2	2	4	4	6	6
PDA-2 (wear)	1	1	2	2	4	4	6	6
PDA-3 (fade)	1/2	1/2	1	2	2	2	4	4
PDA-4 (recovery)	1/2	1/2	1/2	1	2	2	4	4
PDA-5 (stability)	1/4	1/4	1/2	1/2	1	2	2	2
PDA-6 (variability)	1/4	1/4	1/2	1/2	1/2	1	2	2
PDA-7 (μ_{max}-μ_{min})	1/6	1/6	1/4	1/4	1/2	1/2	1	1
PDA-8 (DTR)	1/6	1/6	1/4	1/4	1/2	1/2	1	1

Table 9 Pair-wise comparison matrix of PDAs in terms of fuzzy numbers.

	PDA-1 (μ_P)	**PDA-2 (wear)**	**PDA-3 (fade)**	**PDA-4 (recovery)**	**PDA-5 (stability)**	**PDA-6 (variability)**	**PDA-7 (μ_{max}-μ_{min})**	**PDA-8 (DTR)**
PDA-1 (μ_P)	(1,1,1)	(1,1,1)	(1,2,3)	(1,2,3)	(3,4,5)	(3,4,5)	(5,6,7)	(5,6,7)
PDA-2 (wear)	(1,1,1)	(1,1,1)	(1,2,3)	(1,2,3)	(3,4,5)	(3,4,5)	(5,6,7)	(5,6,7)
PDA-3 (fade)	(0.33,0.5, 1.00)	(0.33,0.50, 1.00)	(1,1,1)	(1,2,3)	(1,2,3)	(1,2,3)	(3,4,5)	(3,4,5)
PDA-4 (recovery)	(0.33,0.5, 1.00)	(0.33,0.50, 1.00)	(0.33,0.50, 1.00)	(1,1,1)	(1,2,3)	(1,2,3)	(3,4,5)	(3,4,5)
PDA-5 (stability)	(0.2,0.25, 0.33)	(0.2,0.25, 0.33)	(0.33,0.50, 1.00)	(0.33,0.50, 1.00)	(1,1,1)	(1,2,3)	(1,2,3)	(1,2,3)
PDA-6 Variability)	(0.2,0.25, 0.33)	(0.2,0.25, 0.33)	(0.33,0.50, 1.00)	(0.33,0.50, 1.00)	(0.33,0.50, 1.00)	(1,1,1)	(1,2,3)	(1,2,3)
PDA-7 (μ_{max}-μ_{min})	(0.14,0.17 ,0.20)	(0.14,0.17, 0.20)	(0.2,0.25, 0.33)	(0.2,0.25,0 .33)	(0.33,0.50, 1.00)	(0.33,0.50, 1.00)	(1,1,1)	(1,1,1)
PDA-8 (DTR)	(0.14,0.17 ,0.20)	(0.14,0.17, 0.2)	(0.2,0.25, 0.33)	(0.2,0.25,0 .33)	(0.33,0.50, 1.00)	(0.33,0.50, 1.00)	(1,1,1)	(1,1,1)

Table 10 Results of comparison matrix by using FAHP.

PDAs	**Fuzzy weight criteria**
PDA-1 (μ_P)	(0.144, 0.255, 0.435)
PDA-2 (wear)	(0.144, 0.255, 0.435)
PDA-3 (fade)	(0.073, 0.15, 0.307)
PDA-4 (recovery)	(0.063, 0.126, 0.268)
PDA-5 (stability)	(0.037, 0.075, 0.156)
PDA-6 (variability)	(0.032, 0.063, 0.136)
PDA-7 (μ_{max}-μ_{min})	(0.023, 0.037, 0.069)
PDA-8 (DTR)	(0.023, 0.037, 0.069)

Evaluation of ranking of the alternatives

A fuzzy evaluation matrix is created, after normalizing the decision matrix defining PDAs, alternatives and data obtained after krauss testing, by using **Table 5** and **Table 6** and is given in **Table 11** After the determination of a fuzzy evaluation matrix, a weighted fuzzy evaluation matrix is calculated by using **Table 11** and Eq. 9, as given in **Table 12**.

Table 11 The linguistic fuzzy evaluation matrix for the ranking of alternatives.

	PDA-1 (μ_P)	PDA-2 (wear)	PDA-3 (fade)	PDA-4 (recovery)	PDA-5 (stability)	PDA-6 (variability)	PDA-7 (μ_{max}-μ_{min})	PDA-8 (DTR)
NC-1	VL	VH	VL	VL	H	L	VH	VH
NC-2	VL	VH	VL	L	VH	L	VH	H
NC-3	L	H	VL	M	M	L	H	H
NC-4	L	H	L	M	M	VL	H	M
NC-5	L	M	VL	VH	VL	M	M	M
NC-6	M	L	H	H	L	L	M	L
NC-7	H	VL	L	H	L	VH	VL	L
NC-8	VH	VL	VH	VL	VH	VL	H	VL
NC-1	(0.00, 0.10, 0.25)	(0.75, 0.90, 1.00)	(0.00, 0.10, 0.25)	(0, 0.10, 0.25)	(0.55, 0.70, 0.85)	(0.15, 0.30, 0.45)	(0.75, 0.90, 1.00)	(0.75, 0.90, 1.00)
NC-2	(0.00, 0.10, 0.25)	(0.75, 0.90, 1.00)	(0.00, 0.10, 0.25)	(0.15, 0.30, 0.45)	(0.75, 0.90, 1.00)	(0.15, 0.30, 0.45)	(0.75, 0.90, 1.00)	(0.55, 0.70, 0.85)
NC-3	(0.15, 0.30, 0.45)	(0.55, 0.70, 0.85)	(0.00, 0.10, 0.25)	(0.35,0.50,0.65)	(0.35,0.50, 0.65)	(0.15, 0.30, 0.45)	(0.55, 0.70, 0.85)	(0.55, 0.70, 0.85)
NC-4	(0.15, 0.30, 0.45)	(0.55, 0.70, 0.85)	(0.15, 0.30, 0.45)	(0.35,0.50,0.65)	(0.35,0.50, 0.65)	(0.00, 0.10, 0.25)	(0.55, 0.70, 0.85)	(0.35,0.50, 0.65)
NC-5	(0.15, 0.30, 0.45)	(0.35,0.50, 0.65)	(0.00, 0.10, 0.25)	(0.75, 0.90, 1.00)	(0.00, 0.10, 0.25)	(0.35,0.50, 0.65)	(0.35,0.50,0.65)	(0.35,0.50, 0.65)
NC-6	(0.35,0.50, 0.65)	(0.15, 0.30, 0.45)	(0.55, 0.70, 0.85)	(0.55, 0.70, 0.85)	(0.15, 0.30, 0.45)	(0.15, 0.30, 0.45)	(0.35,0.50,0.65)	(0.15, 0.30, 0.45)
NC-7	(0.55, 0.70, 0.85)	(0.00, 0.10, 0.25)	(0.15, 0.30, 0.45)	0.55, 0.70, 0.85)	(0.15, 0.30, 0.45)	(0.75, 0.90, 1.00)	(0.00, 0.10, 0.25)	(0.15, 0.30, 0.45)
NC-8	(0.75, 0.90, 1.00)	(0.00, 0.10, 0.25)	(0.75, 0.90, 1.00)	(0.00, 0.10, 0.25)	(0.75, 0.90, 1.00)	(0.00, 0.10, 0.25)	(0.55, 0.70, 0.85)	(0.00, 0.10, 0.25)
Weight criteria	(0.144,0.255, 0.435)	(0.144,0.255, 0.435)	(0.073,0.15, 0.307)	(0.063,0.126, 0.268)	(0.037,0.075, 0.156)	(0.032,0.063, 0.136)	(0.023,0.037, 0.069)	(0.023,0.037, 0.069)

Table 12 The fuzzy weighted evaluation matrix.

	PDA-1 (μ_P)	PDA-2 (wear)	PDA-3 (fade)	PDA-4 (recovery)	PDA-5 (stability)	PDA-6 (variability)	PDA-7 (μ_{max}-μ_{min})	PDA-8 (DTR)
NC-1	(0.00,0.026, 0.109)	(0.108,0.23, 0.435)	(0.00,0.015, 0.077)	(0.00, 0.013, 0.067)	(0.02, 0.053, 0.133)	(0.005, 0.019, 0.061)	(0.017, 0.033, 0.069)	(0.017, 0.033, 0.069)
NC-2	(0.00,0.026, 0.109)	(0.108,0.23, 0.435)	(0.00,0.015, 0.077)	(0.009, 0.038, 0.121)	(0.028, 0.068, 0.156)	(0.005, 0.019, 0.061)	(0.017, 0.033, 0.069)	(0.013, 0.026, 0.059)
NC-3	(0.022,0.077, 0.196)	(0.079,0.179, 0.37)	(0.00,0.015, 0.077)	(0.022, 0.063, 0.174)	(0.013, 0.038, 0.086)	(0.005, 0.019, 0.061)	(0.013, 0.026, 0.059)	(0.013, 0.026, 0.059)
NC-4	(0.022,0.077, 0.196)	(0.079,0.179, 0.37)	(0.011,0.045, 0.138)	(0.022, 0.063, 0.174)	(0.013, 0.038, 0.086)	(0.00, 0.006, 0.034)	(0.013, 0.026, 0.059)	(0.008, 0.019, 0.045)
NC-5	(0.022,0.077, 0.196)	(0.05,0.128, 0.283)	(0.00,0.015, 0.077)	(0.047, 0.113, 0.268)	(0.00, 0.008, 0.039)	(0.011, 0.032, 0.088)	(0.003, 0.011, 0.031)	(0.008, 0.019, 0.045)
NC-6	(0.05,0.128, 0.283)	(0.022,0.077, 0.196)	(0.04,0.105, 0.261)	(0.035, 0.088, 0.228)	(0.006, 0.023, 0.07)	(0.005, 0.019, 0.061)	(0.008, 0.019, 0.045)	(0.003, 0.011, 0.031)
NC-7	(0.079,0.179, 0.37)	(0,0.026, 0.109)	(0.011,0.045, 0.138)	(0.035, 0.088, 0.228)	(0.006, 0.023, 0.07)	(0.024, 0.057, 0.136)	(0, 0.004, 0.017)	(0.003, 0.011, 0.031)

	PDA-1 (μ_P)	PDA-2 (wear)	PDA-3 (fade)	PDA-4 (recovery)	PDA-5 (stability)	PDA-6 (variability)	PDA-7 (μ_{max}-μ_{min})	PDA-8 (DTR)
NC-8	(0.108,0.23, 0.435)	(0,0.026, 0.109)	(0.055,0.135, 0.307)	(0.00, 0.013, 0.067)	(0.028, 0.068, 0.156)	(0.00, 0.006, 0.034)	(0.013, 0.026, 0.059)	(0.00, 0.004, 0.017)
$\tilde{A}^+$	$\tilde{V}_1^+$=(1,1,1)	$\tilde{V}_2^+$=(0,0,0)	$\tilde{V}_3^+$ =(0,0,0)	$\tilde{V}_4^+$=(1,1,1)	$\tilde{V}_5^-$=(1,1,1)	$\tilde{V}_6^+$=(0,0,0)	$\tilde{V}_7^+$=(0,0,0)	$\tilde{V}_8^+$=(0,0,0)
$\tilde{A}^-$	$\tilde{V}_1^-$=(0,0,0)	$\tilde{V}_2^-$ =(1,1,1)	$\tilde{V}_3^-$ =(1,1,1)	$\tilde{V}_4^-$=(0,0,0)	$\tilde{V}_5^-$=(0,0,0)	$\tilde{V}_6^-$=(1,1,1)	$\tilde{V}_7^-$=(1,1,1)	$\tilde{V}_8^-$=(1,1,1)

As the element in **Table 12** is normalized to TFN and their ranges are associated to the closed interval [0, 1], we can define the fuzzy positive ideal solution (FPIS, $\tilde{A}^+$) as $\tilde{V}_j^+$= (1,1,1) and $\tilde{V}_j^-$ = (0,0,0) for benefit criteria and fuzzy negative ideal solution (FPIS, $\tilde{A}^-$) as $\tilde{V}_j^+$ =(0,0,0) and $\tilde{V}_j^-$= (1,1,1) for cost criteria as according to Eq. 10 and 11; then, the distance between each alternative from $\tilde{D}_i^+$ and $\tilde{D}_i^-$ is calculated by using Eq. 12. Finally, the overall preference or fuzzy closeness index ($\tilde{C}\tilde{I}_i$) of the alternatives is calculated by using Eq. 13. All the alternatives are then arranged in descending order according to the value of their closeness index. The alternative at the top of the list is the most preferred one, which gives the ranking of the eight friction materials with respect to their overall performance. The results are summarized in **Table 13** and **Figure 7**. According to $\tilde{C}\tilde{I}_i$ values, the friction composite ranking in descending order is NC-7, NC-8, NC-6, NC-5, NC-3, NC-4, NC-2, and NC-1.

Table 13 The fuzzy closeness index and ranking of the alternatives.

Composite Designation	$\tilde{D}_i^+$	$\tilde{D}_i^-$	$\tilde{C}\tilde{I}_i$	Ranking
NC-1	3.326	4.805	0.591	8
NC-2	3.275	4.862	0.598	7
NC-3	3.175	4.961	0.610	5
NC-4	3.187	4.952	0.608	6
NC-5	3.09	5.046	0.620	4
NC-6	3.077	5.074	0.623	3
NC-7	2.918	5.227	0.642	1
NC-8	2.975	5.174	0.635	2

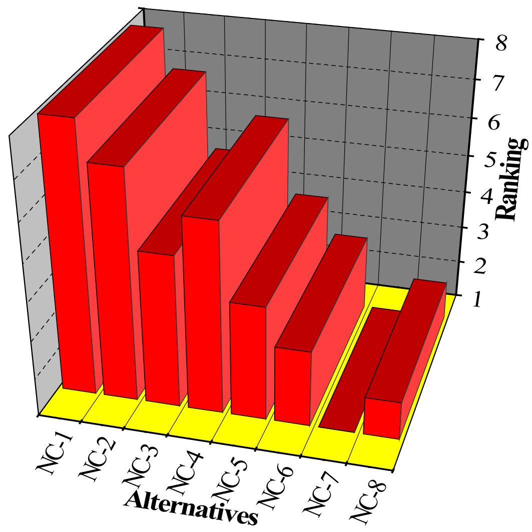

Figure 7 Ranking of the alternatives.

Conclusions

Nanoclay and MWNT filled hybrid friction materials, lubricated with graphite and reinforced with a combination of lapinus and aramid fibers, have been developed and evaluated on a Krauss type tester for braking efficiency. The change in the combination of reinforcing phase from kevlar-lapinus-nanoclay-graphite to kevlar-lapinus-nanoclay-graphite-MWNT has led to substantial differences in terms of their performance trends. The addition of MWNTs clearly affects the various PDAs. The tribological results obtained from the Krauss type tester were considered as criterions in the performance optimization of friction materials. The weight criteria of the PDAs as evaluated with FAHP is: μ-performance (0.144, 0.255, 0.435), wear (0.144, 0.255, 0.435), fade-% (0.073, 0.15, 0.307), recovery-% (0.063, 0.126, 0.268), stability coefficient (0.037, 0.075, 0.156), variability coefficient (0.032, 0.063, 0.136), frictional fluctuations (0.023, 0.037, 0.069), and DTR (0.023, 0.037, 0.069). FTOPSIS is used to rank the alternatives; the order of alternatives could be obtained as NC-7>NC-8>NC-6>NC-5>NC-3>NC-4>NC-2>NC-1. The alternative NC-7, which has kevlar: lapinus, 2.5:27.5 wt-% and graphite: nanoclay, 2.25:2.75 wt-% exhibits the optimal properties. The FTOPSIS method strengthened by FAHP is an effective tool for the ranking or selection of friction materials and should be helpful in optimal friction formulation selection without performing long and costly laboratory experiments. Thus, a performance evaluation of composite friction materials having various PDAs may be predicted with appreciable accuracy for designing friction material formulations.

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