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Application of 2-D Vertically Averaged Boundary-Fitted Coordinate Model of Tidal Circulation in Thale Sap Songkhla, Thailand

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ABSTRACT

A 2-D vertically averaged boundary-fitted coordinate hydrodynamic model was employed to simulate circulation in Thale Sap Songkhla due to tides in the Gulf of Thailand. The model was calibrated against a set of current velocity data collected between June and July 1997. The best fit for observations at Ko Yo and Pak Ro was achieved. To comprehend the hydrodynamic in the lake, the current vectors were illustrated for both the flood and ebb stages. Detailed analysis indicated that there existed a turning current at the northern tip of Ko Yo Island, which induced a significant current along its northern shoreline. The calculations show the current was stronger in the deep channel north of Ko Yo than in the southern circuit. The model also predicted a gyre near the deep channel of the lake entrance, which persisted for some time during the changing direction of the flood and ebb currents.

Keywords: Hydrodynamic, circulation, boundary-fitted coordinate model, Thale Sap Songkhla, Ko Yo

INTRODUCTION

Characteristics of Songkhla Lake

Songkhla Lake, the largest estuary in Thailand, is situated on the east coast of Southern Thailand (**Figure 1**). The lake is approximately 80 km long, and varies in width from 13 km in the south to a maximum of 20 km in the north. It opens to the Gulf of Thailand through the lake inlet in the lower lake in the south. The water body of the lake is characterized by three portions, a relatively salty area in the lower portion ("Thale Sap Songkhla"), brackish in the central portion ("Thale Sap"), and fresh in the upper portion ("Thale Luang"), **Figure 2** shows the longitudinal depth profile of the Songkhla Lake versus distance along the lake system [1,2]. The discharge into the lake comes from several small drainage areas with mean annual flow to sea approximately 5,679 million m^3 [3].

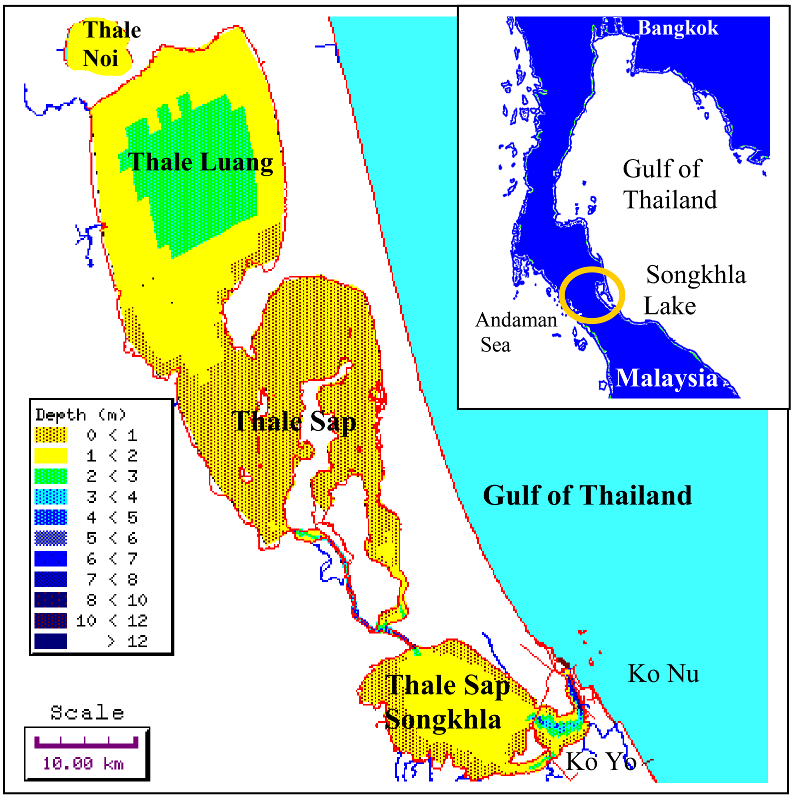


Figure 1 Songkhla Lake and its characteristics.

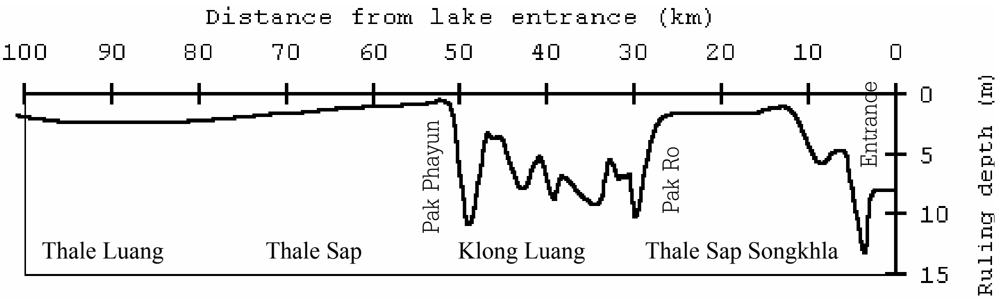


Figure 2 Longitudinal depth profile of Songkhla Lake.

Circulation in Thale Sap Songkhla

The comprehensive understanding of the hydrodynamics of the lake is crucial for explaining the changes in water quality around Ko Yo which affects the sustainable utilization of Thale Sap Songkhla. Although there has been extensive development in the area, little is known about the hydrodynamics and salinity mixing in Thale Sap Songkhla. The complexity of the lake system is governed by tide, runoff, wind and wave. Pornpinatepong reports that the tides in the Gulf at Ko Nu Island are mixed and well defined by five tidal constituents, K1, O1, M2, P1 and S2 [4]. Harmonic analysis reveals that the tides are strongly predominated by semi-diurnal constituent (~12.4 h period). The tidal ranges are approximately 30 and 60 cm for neap and spring tides, respectively.

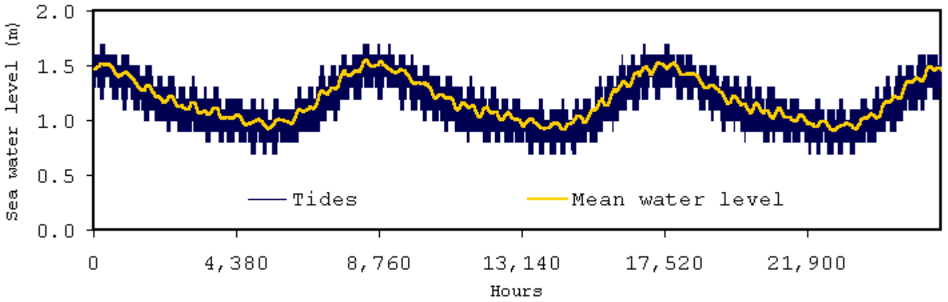


Figure 3 Tidal characteristics in the sea at Ko Nu.

Current measurements conducted in the dry season indicate that the water column is generally well-mixed [1]. A major portion of the flow in the lake is confined within the central deep area, where the maximum velocity increases with distance toward the lake entrance (km 0). The observed tidal current fluctuation north of Ko Yo Island (km 10) in June 1997, shows a maximum current speed of about 40 cm/s during flood tide. The current speed dramatically decreases moving upstream to Pak Ro (km 30), where the maximum speed is about 30 cm/s.

MATERIALS AND METHODS

Model selection

Based on the available data collected in the low flow, a 2-D vertically averaged boundary-fitted coordinate model of hydrodynamic model is appropriate to describe the circulation in Thale Sap Songkhla. The model first developed by Spaulding, and extended by Swanson, has been used in this study [5,6]. It employs shallow water hydrodynamic assumptions and the Boussinesq approximation to simplify the governing equations.

The advantages of the model include the use of a boundary-fitted coordinate grid to resolve complex geometry [7], which reduces the number of grids used for the lake. By solving a set of coupled quasi-linear elliptic transformation equations for an arbitrary horizontal region, a rectangular computational plane can be obtained. The governing equations of motion are then solved in this transformed space [8].

Model equation

The two-dimensional shallow water wave equations are derived from the governing equations in spherical coordinates (ϕ,θ,r), where ϕ is positive east, θ is positive north, and r is positive up [7]. The conservation of mass and momentum equations are vertically averaged over depth from the bottom to the free surface. The equations are transformed in the horizontal direction into a generalized, curvilinear, grid system (ξ,η) [5]. The boundary-fitted coordinate equations of motion written in terms of the vertically averaged contra-variant velocity components (U, V) are [9]:

Continuity Eq.

$$\frac{\partial \zeta}{\partial t}+\frac{1}{Jr\cos\theta}\frac{\partial}{\partial \xi}\left(UHJ\cos\theta\right)+\frac{1}{Jr\cos\theta}\frac{\partial}{\partial \eta}\left(VHJ\cos\theta\right)=0 \qquad (1)$$

Momentum in ξ-direction

$$\frac{\partial UH}{\partial t}+\frac{1}{J^2 r\cos^2\theta}\theta_\eta\left[\frac{\partial}{\partial \xi}\left(\cos^2\theta\left(UUHJ\phi_\xi+UVHJ\phi_\eta\right)\right)+\frac{\partial}{\partial \eta}\left(\cos^2\theta\left(UVHJ\phi_\xi+VVHJ\phi_\eta\right)\right)\right]$$
$$-\frac{1}{J^2 r\cos^2\theta}\phi_\eta\left[\frac{\partial}{\partial \xi}\left(\cos^2\theta\left(UUHJ\theta_\xi+UVHJ\theta_\eta\right)\right)+\frac{\partial}{\partial \eta}\left(\cos^2\theta\left(UVHJ\theta_\xi+VVHJ\theta_\eta\right)\right)\right]$$
$$=-\frac{gH\,G_{22}}{\rho_o J^2 r\cos^2\theta}\left(\zeta\frac{\partial \rho}{\partial \xi}\,\rho\frac{\partial \zeta}{\partial \xi}\right)+\frac{gH\,G_{12}}{\rho_o J^2 r\cos^2\theta}\left(\zeta\frac{\partial \rho}{\partial \eta}\,\rho\frac{\partial \zeta}{\partial \eta}\right)-\frac{\tau_{b\xi}}{\rho} \qquad (2)$$

Momentum in η-direction

$$\frac{\partial VH}{\partial t}-\frac{1}{J^2 r\cos^2\theta}\theta_\xi\left[\frac{\partial}{\partial \xi}\left(\cos^2\theta\left(UUHJ\phi_\xi+UVHJ\phi_\eta\right)\right)+\frac{\partial}{\partial \eta}\left(\cos^2\theta\left(UVHJ\phi_\xi+VVHJ\phi_\eta\right)\right)\right]$$
$$+\frac{1}{J^2 r\cos^2\theta}\phi_\xi\left[\frac{\partial}{\partial \xi}\left(\cos^2\theta\left(UUHJ\theta_\xi+UVHJ\theta_\eta\right)\right)+\frac{\partial}{\partial \eta}\left(\cos^2\theta\left(UVHJ\theta_\xi+VVHJ\theta_\eta\right)\right)\right]$$

$$=-\frac{gH\,G_{11}}{\rho_o J^2 r\cos^2\theta}\left(\zeta\frac{\partial \rho}{\partial \eta}\,\rho\frac{\partial \zeta}{\partial \eta}\right)+\frac{gH\,G_{12}}{\rho_o J^2 r\cos^2\theta}\left(\zeta\frac{\partial \rho}{\partial \xi}\,\rho\frac{\partial \zeta}{\partial \xi}\right)-\frac{\tau_{b\eta}}{\rho} \qquad (3)$$

where U,V = contra-variance velocities in ξ- and η-direction
ζ = water surface elevation
H = depth (MSL)
ρ = density of sea water.
$\tau_{b\xi}, \tau_{b\eta}$ = bottom shear stress in ξ- and η- direction
J = Jacobian $=\phi_\xi\theta_\eta-\phi_\eta\theta_\xi=\frac{\partial \phi}{\partial \xi}\frac{\partial \theta}{\partial \eta}-\frac{\partial \phi}{\partial \eta}\frac{\partial \theta}{\partial \xi}$ (4)
G_{11} = $\theta_\xi\theta_\xi+\phi_\xi\phi_\xi\cos^2\theta$
G_{22} = $\theta_\eta\theta_\eta+\phi_\eta\phi_\eta\cos^2\theta$ (5)
G_{12} = $\theta_\xi\theta_\eta+\phi_\xi\phi_\eta\cos^2\theta$

In the momentum equation, terms on the left hand side express the local and the convective terms, and the other terms on the right hand side include the pressure gradients and the bottom shear stress.

At the bottom, a quadratic drag law is applied:

$$\begin{aligned} \tau_{b\eta} &= \rho C_f V \sqrt{U^2 + V^2} \\ \tau_{b\xi} &= \rho C_f U \sqrt{U^2 + V^2} \end{aligned} \tag{6}$$

where C_f is the bottom drag coefficient.

The contra-variant velocities are related to the spherical coordinate velocities as follows:

$$\begin{aligned} V_\phi &= \cos\theta(U\,\phi_\xi + V\,\phi_\eta) \\ V_\theta &= (U\,\theta_\xi + V\,\theta_\eta) \end{aligned} \tag{7}$$

where (V_ϕ, V_θ) are the vertically averaged velocities in the ϕ and θ directions, respectively.

Model application

The model domain encompasses the three portions of the Songkhla Lake. The grid size is selected based on the complexity of the lake geometry, which varies between 100 - 1500 m to adequately resolve the geometric effects of the meandering lake. This results in a grid net of 1,717 active water cells. **Figure 4** illustrates the computational grid system in the lower lake (Thale Sap Songkhla). For each grid the mean water depth is given at the center of each cell.

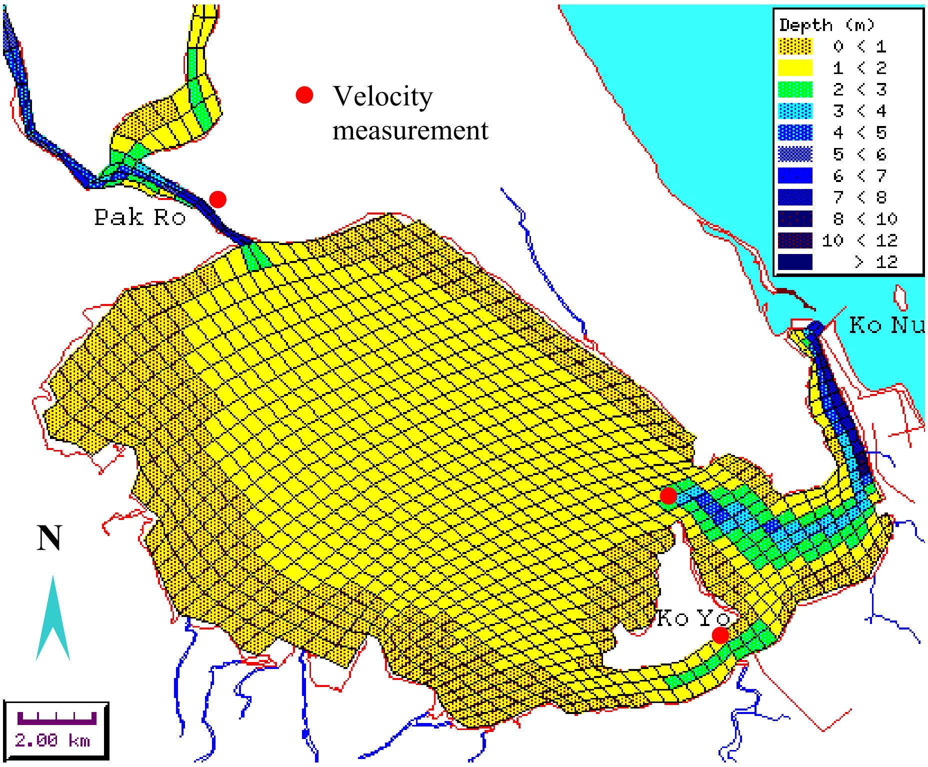


Figure 4 Computational grid system for Thale Sap Songkhla.

RESULTS AND DISCUSSION

Hydrodynamic model calibration

A systematic set of current velocity data collected from June 14 to July 2, 1997 was employed to calibrate the model. Water surface elevation at Ko Nu represented by amplitudes and phases of the five principal tidal constituents was applied at the ocean boundary. The model was spun up for two months (June-July, 1997) from the initial conditions at rest prior to comparison with observations at the selected locations in Thale Sap Songkhla.

A time step of 450 s was chosen to spin up the model to test the sensitivity of the tidal range and current velocity to the bottom friction coefficient. Based on an overall comparison, a bottom friction coefficient of 0.0011 yielded the optimal value for the predicted tidal range and current. **Figure 5** shows the comparison of the computed current velocity and the observed at Pak Ro and Ko Yo. The model overpredicted the maximum current speed at Pak Ro (**Figure 5a**) but underpredicted those at Ko Yo (**Figures 5b** and **5c**). In general, the prediction matched the observations fairly well (within 3 - 5 cm/s or 10 - 15 % on average).

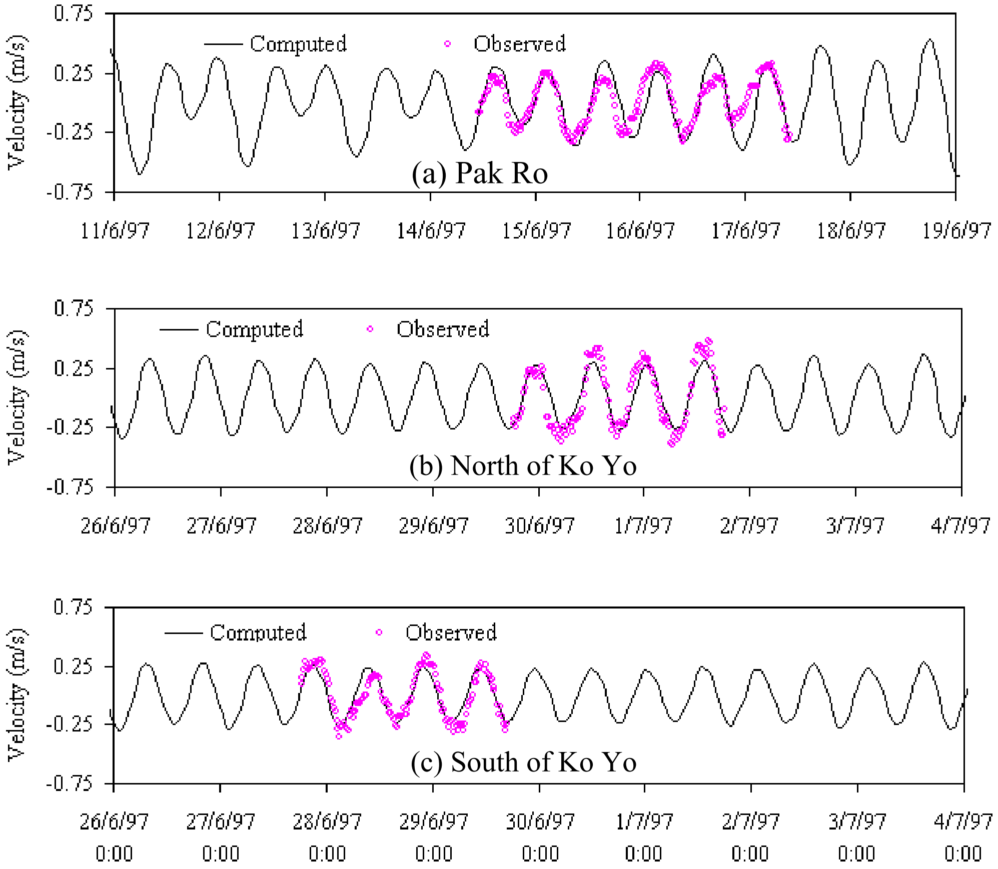


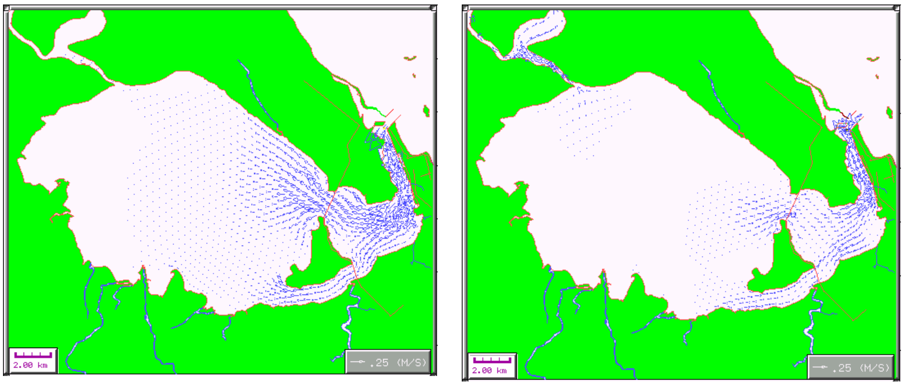
Figure 5 Comparison of observed and computed current velocity in deep channels at Pak Ro and Ko Yo from June 14 to July 2, 1997.

Hydrodynamic model predictions

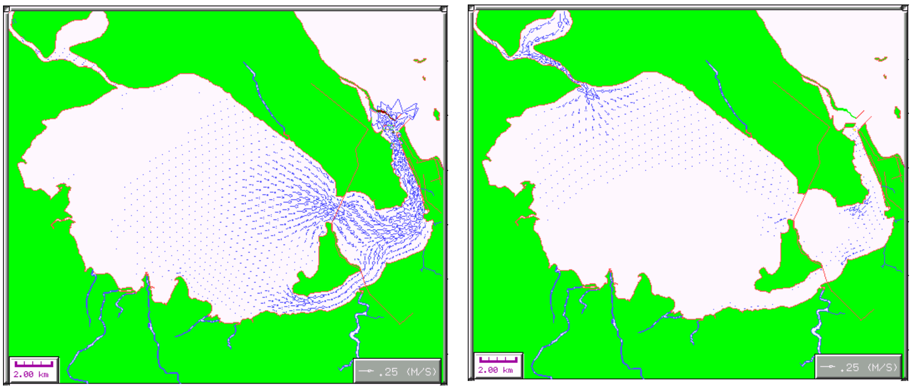
The model has been employed to simulate circulation pattern in Thale Sap Songkhla. **Figure 6** shows typical current vectors prepared at 3-h intervals starting from the maximum flood current (0 h). At this stage, the lake geometry and bathymetry play a crucial role in controlling circulation with the current prevailing in the deep channel. The maximum current speed of about 0.75 m/s is in the deep channel of the lake entrance. The flow then diverges into two stream paths passing north and south of Ko Yo (**Figure 6a**). The tidal wave takes about half an hour propagating to Pak Ro.

Three hours later, slack tide is present in the Thale Sap Songkhla while the flood current starts appearing in the narrow channel at Pak Ro (**Figure 6b**). A maximum current speed of around 0.5 m/s is predicted in the channel.

Figure 6c presents the flow turning to ebb current after 6 h. A similar current pattern can be expected as previously shown for the maximum flood. The dominate current prevails in the deep channel and at Ko Yo. The flow continues ebbing after 9 h (**Figure 6d**). As seen, a slack tide occurred throughout the lake while the strong ebb current appears in the Pak Ro channel.



(a) 0 h (b) 3 h



(c) 6 h (d) 9 h

Figure 6 Computed circulation pattern in Thale Sap Songkhla at 3-h interval.

The hydrodynamic complexity of Thale Sap Songkhla can be explained by the relationship between the water level and the current velocity in the lake (**Figure** 7). There is a dramatic shift in the relative phase with surface elevation and velocity from Ko Yo to Pak Ro. At Ko Yo, the sea surface elevations are nearly 90° out of phase with the currents (**Figure 7a**) and can be considered as a standing wave pattern. In contrast, at Pak Ro, the currents tend to follow the water levels and display the characteristics of a partially progressive wave (**Figure 7b**).

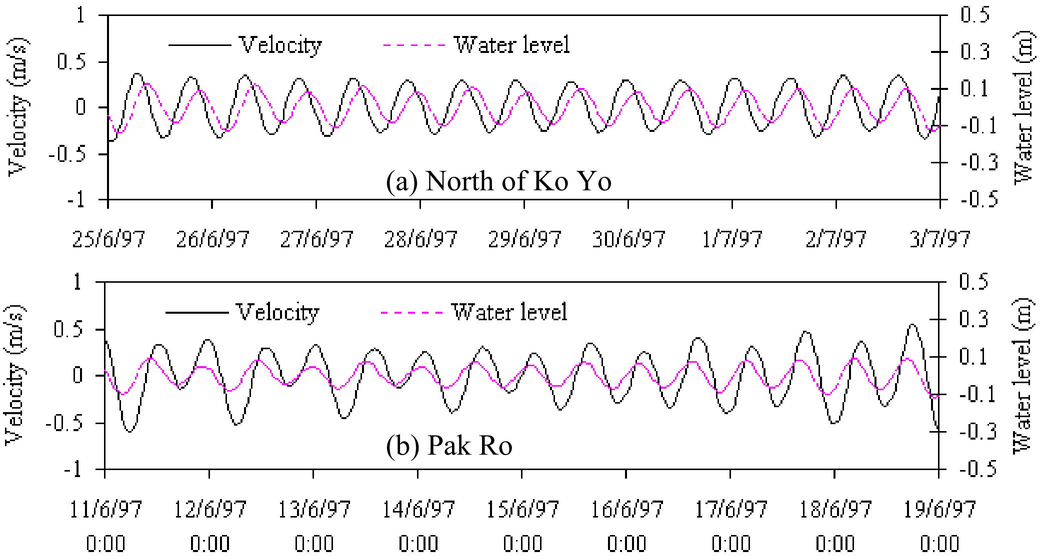
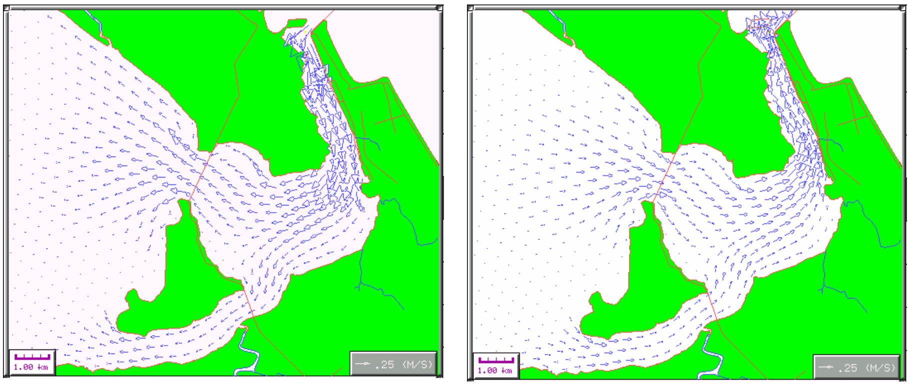


Figure 7 Relationship between water level and current velocity in Thale Sap Songkhla.

To gain insight into the circulation around Ko Yo Island, current vectors were prepared at maximum flood and ebb stages. **Figure 8a** shows the maximum flood current enters the lake entrance and then diverges toward Ko Yo. As the flow turns at the north tip of the island, it induces a significant current along the north shoreline where intensive mariculture is practiced.

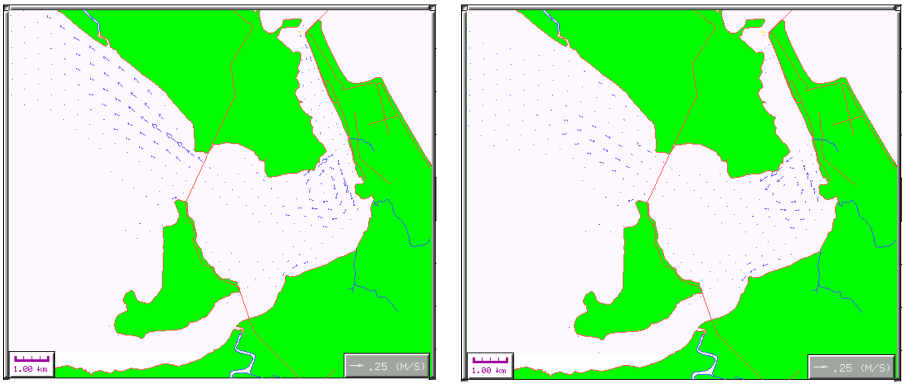
Figure 8b depicts the complexity of the maximum ebb current. Similar current patterns to the flood current can be expected. The flow prevails along the island generating strong currents at the north tip. This unique characteristic is useful for describing the mechanism of water quality changes in the area. The simulations also reveal that the flows are stronger in the deep channel north of the island than in the south channel. Depth-averaged magnitudes around 0.55 m/s and 0.40 m/s, respectively are predicted. It also reveals two stagnation zones on the east and the west of the island, where high sediment deposition can be expected. In the shallow areas, the currents are very small or diminish.



(a) Maximum flood current (b) Maximum ebb current

Figure 8 Computed maximum flood and ebb current around Ko Yo.

It is interesting to note that there exist gyres near the lake entrance as the flow changes its direction whilst returning to the ebb and the flood stages (**Figure 9**). It indicates that the area is relatively sensitive to human activities, such as harboring, dredging, etc. The gyres persist for less than 30 min.



(a) Starting ebb stage (clockwise gyre) (b) Starting flood stage (counterclockwise gyre)

Figure 9 Gyres near the lake entrance whilst returning to ebb and flood stages.

CONCLUSIONS

A 2-D vertically averaged boundary-fitted coordinate model of hydrodynamics was applied to simulate tidal circulation in Thale Sap Songkhla. The model used simplified governing equations invoking the shallow water assumption and the Boussinesq approximation.

The model was calibrated against current velocity data collected between June 14 and July 2, 1997 in general agreement with the observations. The dominate flows prevailed at the lake entrance, at Pak Ro and north of Ko Yo with an estimated current speed of about 0.75, 0.55 and 0.40 m/s, respectively. Circulation in Thale Sap Songkhla was strongly influenced by the lake geometry. The strong flow, coming from the lake entrance, diverged to the north and the south of Ko Yo. Subsequently, there existed stagnation zones on the east and the west of the island, where high sediment deposition was expected. Passing around the northern tip of the island, the turning current induced a significant current along the west shoreline of the island where intensive mariculture is practiced.

The hydrodynamic complexity of the lake can be explained from tidal responses in Thale Sap Songkhla. Simulations showed that the tide at Ko Yo exhibited a standing wave pattern with maximum flood and ebb currents preceding high and low tide by about 3 h. Whereas at Pak Ro, partially progressive wave characteristics were displayed. The model also predicted a gyre near the lake entrance, which persisted for a relatively short time when the current changed its direction from the ebb to the flood and vice versa.

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บทคัดย่อ

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การประยุกต์แบบจำลอง 2 มิติค่าเฉลี่ยตามแนวดิ่งและพิกัดสอดคล้องขอบเขตกับการไหลเวียนของน้ำขึ้นน้ำลงในทะเลสาบสงขลา ประเทศไทย

แบบจำลอง 2 มิติทางอุทกพลศาสตร์สำหรับค่าเฉลี่ยตามแนวดิ่งและมีพิกัดสอดคล้องกับขอบเขต ถูกนำมาใช้เพื่อจำลองการไหลเวียนของน้ำในทะเลสาบสงขลาอันเนื่องมาจากน้ำขึ้นน้ำลงในอ่าวไทย แบบจำลองได้รับการปรับเทียบกับค่าความเร็วกระแสน้ำที่ได้จากการสำรวจระหว่างเดือนมิถุนายนและกรกฎาคม 2540 ที่ให้ผลสอดคล้องอย่างดีกับข้อมูลที่วัดได้ที่เกาะยอและปากรอ เพื่อความเข้าใจพฤติกรรมอุทกพลศาสตร์ในทะเลสาบสงขลา ขนาดและทิศทางของกระแสน้ำได้รับการนำเสนอสำหรับการไหลในช่วงน้ำขึ้นและน้ำลง ผลการวิเคราะห์อย่างละเอียดแสดงให้เห็นว่า มีการไหลอ้อมปลายทิศเหนือของเกาะยอซึ่งเหนี่ยวนำให้เกิดกระแสน้ำลัดเลาะไปตามชายฝั่งทิศเหนือของเกาะ การคำนวณยังแสดงว่ากระแสน้ำในร่องน้ำทิศเหนือของเกาะยอมีความเร็วมากกว่าทางทิศใต้ของเกาะ นอกจากนี้ยังปรากฏกระแสน้ำวนที่ใกล้กับร่องน้ำทางเข้าทะเลสาบสงขลา ซึ่งคงอยู่ชั่วขณะหนึ่งในช่วงที่กระแสน้ำมีการเปลี่ยนทิศทางของกระแสน้ำขึ้นและน้ำลง

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