P-ISSN 2586-9000
E-ISSN 2586-9027
Homepage : *https://tci-thaijo.org/index.php/SciTechAsia* Science & Technology Asia
Vol.25 No.1 January - March 2020 Page: [157-165]
Original research article

On a Paradox in Multi-Objective Linear and Fractional Transportation Problem

Vishwas Deep Joshi[1,*], Jagdev Singh[2], Rachana Saini[2]

[1] *Department of Mathematics, School of Liberal Arts and Sciences, Mody University of Science and Technology, Sikar-332311, India*
[2] *Department of Mathematics, Faculty of Science, JECRC University, Jaipur-303905, India*

Received 25 August 2019; Received in revised form 9 January 2020
Accepted 14 January 2020; Available online 26 March 2020

ABSTRACT

In this paper, a more-for-less (MFL) paradox situation is discussed for a multi-objective transportation problem with linear and fractional objective function. By using the MFL paradox in multi-objective programming, we can transfer more goods from source to destination with less or equal compromise optimal solution. In this approach, it is not necessary that a paradox is present in every objective. If a paradox is found in one of the objectives, then we can use this approach. We compare the paradoxical solution with compromise solution using ranking procedure [1] and show the superiority of the proposed paradoxical approach. For proper explanation of theory two examples are discussed.

Keywords: Linear programming; Multi-objective transportation problem; Multi-objective fractional transportation problem; MFL paradox

1. Introduction

The transportation problem (TP) of transporting stock from different sources to various destinations is known as a classical TP with a single objective. But in practical situations we want to satisfy multiple objectives in a single problem. The MFL paradox in TP occurs when we want to relax our supply and demand constraints to ship more things for less (or equal) cost. A paradoxical situation in a linear fractional transportation problem (LFTP) was first discussed by Verma V. and Puri M.C. [2]. They used the classical cost minimization TP with fractional objective function, developed a sufficient condition to identify the paradoxical situation, and gave a complete paradoxical range of flow. Gupta and Arora [3] developed a new approach to calculate the lowest cost per unit transport in a capacitive TP by placing a limit on the terms of the rim condition. Joshi and Saini [4] proposed a pro-

***Corresponding author:** vdjoshi.or@gmail.com **doi:** 10.14456/scitechasia.2020.15

cedure for solving the MOLFTP problem.

Deineko et al. [5] gives an exact characterization for linear transportation problem (LTP) cost matrices that are immune against the transportation paradox. MFL analysis is mostly discussed in LTP. In the early 1970s, Charnes and Klingman [6] and Szwarc [7], independent of each other, identified the transportation paradox in the LTP. The MFL paradox has been intensively studied by Adlakha V et al [8–10]. Almost every type of LTP with paradoxical solutions has been discussed by them.

Porchelvi and Anitha [11] developed an algorithm to search the paradoxical solutions of the multi-objective linear transportation problem (MOLTP) by linear constraints. The algorithm tries to achieve its best paradoxical pair and paradoxical range of flow by using the sufficient condition of the existing paradox. But in this approach, paradoxical position must be common in all the objectives. Joshi and Gupta [12] reported using an objective matrix the identifying of an MFL paradox in an LFTP. Joshi and Gupta [13] presented a procedure for handling the MFL paradox in a linear plus linear fractional transportation problem (LPLFTP).

A new approach for solving an MFL paradox in MOLTP and MOLFTP is discussed without common paradoxical situation in each objective in this paper. The paper is organized as follows: Section 2 outlines the basic concepts and definitions related to the MFL paradox; mathematical formulation of the problems is discussed in Section 3; In Section 4 the procedure for handling the MFL paradox is discussed; In Section 5 a numerical example discussed in support of the theory for the problems discussed in Section 3; Conclusions of the article are discussed in last section.

2. Basic Concepts and Definitions

Theorem 2.1 ([5]). *For LTP an $m \times n$ cost matrix $C = (c_{ij})$ is immune against the transportation paradox, if and only if for all q, r, s, t with $1 \le q, s \le m$, $1 \le r, t \le n$, $q \neq s$ and $r \neq t$ the inequality $c_{qr} \le c_{qt} + c_{sr}$ are satisfied.*

Theorem 2.2 ([12]). *If there exists a bad quadruple for the objective matrix in LFTP $P = \left(c_{ij}, d_{ij}\right)$, then P is not immune against the transportation paradox.*

Consider some fixed $m \times n$ cost matrix $Z = \left(c_{ij}, d_{ij}\right)$. Then four integers q, r, s, t with $1 \le q, s \le m$, $1 \le r, t \le n$ (where $q \neq s$ and $r \neq t$) form a bad quadruple if

$$\frac{c_{qt} + c_{sr}}{d_{qt} + d_{sr}} < \frac{c_{qr}}{d_{qr}}.$$

Identify MFL paradox using shadow price matrix: The MFL paradox is presented in the LTP and LFTP if at least one of the shadow prices are negative corresponding to the optimal allocation table [2, 7].

3. Problem Formulation

3.1 Mathematical formulation of MOLTP

Consider the following MOLTP problem (P1)

Min $Z_k = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^{k} x_{ij}$, $k = 1, 2, 3, \ldots, K$

subject to

$$\begin{aligned} \textstyle\sum_{i=1}^{m} x_{ij} &= a_i, \quad & j &= 1, 2, 3, \ldots, n, \\ \textstyle\sum_{j=1}^{n} x_{ij} &= b_j, \quad & i &= 1, 2, 3, \ldots, m, \\ x_{ij} &\ge 0, \quad & i &= 1, 2, 3 \ldots, m; \\ & & j &= 1, 2, 3 \ldots, n, \end{aligned}$$

where

a_i = the i^{th} origin,
b_j = the j^{th} destination,
x_{ij} = from the i^{th} origin to the j^{th} destination the amount transported,
c_{ij}^k = from i^{th} origin to j^{th} destination the cost is transporting per unit.

3.2 Mathematical formulation of MOLFTP

The MOLFTP is formulated as follows:
(P2)

Min $Z_k(x_{ij}) = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^k x_{ij}}$,
$k = 1, 2, 3, \ldots, K$
subject to

$$\begin{aligned} \textstyle\sum_{i=1}^{m} x_{ij} &= a_i, & j &= 1,2,3,\ldots,n,\\ \textstyle\sum_{j=1}^{n} x_{ij} &= b_j, & i &= 1,2,3,\ldots,m,\\ x_{ij} &\geq 0, & i &= 1,2,3,\ldots,m;\\ & & j &= 1,2,3\ldots,n. \end{aligned}$$

Supply points a_i, $i = 1, 2, 3, \ldots, m$ are m supply points at which the goods are transported. The demand points b_j, $j = 1, 2, 3, \ldots, n$, are n demand centers where goods are required. The cost of transporting a unit from i origin to j destination is c_{ij}^k. The d_{ij}^k is the profit of transporting one unit from i origin to j destination. Suppose variable x_{ij} denotes the number of units that will be transported from i^{th} origin to j^{th} destination. In equaled form (supply's sum = demand's sum) must be the problem.

3.3 Ranking the solutions

We compare our MFL solution with a compromise optimal solution obtained using [4, 14]. For this comparison we are using the ranking framework procedure given in [1].

4. Step by Step MFL Algorithm for MOLTP and MOLFTP

Step 1: Solve the problems (P1/P2) using [4, 14] and find the compromise optimal solution. Find individual ideal and anti-ideal optimal solutions for each objective using the modified distribution method.

Step 2: Create the combined (for each objective) shadow price matrix for the MOLTP/MOLFTP.

Step 3: Identify the location of negative shadow prices in the table obtained in Step 2. If no negative entries are found in the shadow price matrix go to Step 6.

Step 4: Choose the most negative entry found in Step 3 for the MFL solution. Relax the demand and supply ($\max(a_i, b_j)$) to find the MFL solution for the MOLTP/MOLFTP.

Step 5: Repeat the procedure for finding the other paradoxical solution.

Step 6: Solve the reduced problem as a regular unbalanced problem.

Remark: In this MFL procedure, it is not necessary that an MFL situation is present in every objective function.

5. Numerical Examples

In this paper, we solve two different examples of MOLTP and MOLFTP. We solve these examples using LINGO 17.0 software on a core i3 processor PC.

5.1 MOLTP example

Let us consider the MOLTP with three objectives as discussed in Table 1.

We obtain the individual optimum solution for each objective for flow 80 as follows:

$$\begin{aligned} X^1 &= (0, 20, 0, 0, 5, 5, 0, 0, 15, 0, 0, 0, 5, 0, 15, 15),\\ Z_1(X^1) &= 810,\\ X^2 &= (10, 10, 0, 0, 0, 0, 10, 0, 0, 15, 0, 0, 15, 0, 5, 15),\\ Z_2(X^2) &= 415,\\ X^3 &= (20, 0, 0, 0, 5, 5, 0, 0, 0, 15, 0, 0, 0, 5, 15, 15),\\ Z_3(X^3) &= 215. \end{aligned}$$

Table 1. Cost matrix for numerical example 1.

		$\mathbf{D}_1$			$\mathbf{D}_2$			$\mathbf{D}_3$			$\mathbf{D}_4$			a_i
Objective		1^{st}	2^{nd}	3^{rd}	1^{st}	2^{nd}	3^{rd}	1^{st}	2^{nd}	3^{rd}	1^{st}	2^{nd}	3^{rd}	
$\mathbf{O}_1$	c_{ij}^k	5	1	1	8	9	6	7	9	6	18	27	20	20
$\mathbf{O}_2$	c_{ij}^k	6	12	5	10	22	2	5	6	13	15	42	15	10
$\mathbf{O}_3$	c_{ij}^k	7	17	6	15	2	1	3	22	10	16	28	11	15
$\mathbf{O}_4$	c_{ij}^k	15	6	6	21	15	2	8	6	4	18	7	5	35
$\mathbf{b}_i$		25			25			15			15			80

Solving the MOLTP for flow 80 and weight $(0.6, 0.2, 0.2)$, we get the compromise optimal solution as follows:

$$\begin{aligned} X &= (20, 0, 0, 0, 5, 5, 0, 0, 0, 15, 0, 0, 0, 5, 15, 15), \\ Z_1(X) &= 900, \\ Z_2(X) &= 490, \\ Z_3(X) &= 215. \end{aligned}$$

Shadow price matrix for the compromise optimal solution is presented in Table 2.

Individual optimum solution for the flow 85 at point $(1, 2)$

We can't find any common cell where shadow prices are negative for each individual objective. So, we choose the most negative shadow price entry among three objectives. We found negative entries in the shadow price matrix in cells (1,2), (1,3), (2,3), (3,1), (3,3), and (3,4). If we increase the demand and supply for the corresponding row and column by 5, the compromising, paradoxical solution and ranking are given in Table 3. Ideal and anti-ideal solutions are also given in Table 3. Comparison of the proposed method with other approach [14] using [1] is discussed in Table 3.

5.2 MOLFTP example

Let us consider the MOLFTP with three objectives as discussed in Table 4.

We obtain the individual optimum solution for each objective for flow 60 as follows:

$$\begin{aligned} X^1 &= (5, 5, 5, 0, 10, 0, 0, 15, 0, 20, 0, 0,), \\ Z_1(X^1) &= 0.604, \\ X^2 &= (0, 15, 0, 0, 10, 10, 5, 0, 5, 0, 0, 15), \\ Z_2(X^2) &= 0.536, \\ X^3 &= (0, 5, 0, 10, 15, 0, 5, 5, 0, 20, 0, 0,), \\ Z_3(X^3) &= 0.641. \end{aligned}$$

Solving the MOLFTP for flow 60 and weight $(0.6, 0.1, 0.3)$, we get the compromise optimal solution as follows:

$$\begin{aligned} X &= (0, 15, 0, 0, 15, 0, 5, 5, 0, 10, 0, 10), \\ Z1(X) &= 0.837, \\ Z2(X) &= 0.575, \\ Z3(X) &= 0.717. \end{aligned}$$

Shadow price matrix for the compromise optimal solution is presented in Table 5.

Individual optimum solution for the flow 85 at point $(1, 1)$

We can't find any common cell where shadow prices are negative for each individual objective. So, we choose the most negative shadow price entry among three objectives. We found negative entries in the shadow price matrix in cells (1,1), (1,2), (1,3), (3,1), (3,3), and (3,4). If we increase the demand and supply for the corre-

Table 2. Shadow price matrix (for compromise optimal solution) for MOLTP example.

	$\mathbf{D}_1$			$\mathbf{D}_2$			$\mathbf{D}_3$			$\mathbf{D}_4$			$\mathbf{U}_i^1\mathbf{U}_i^2\mathbf{U}_i^3$		
$\mathbf{O}_1$	5	1	1	9	11	-2	-4	2	0	6	3	1	-12	-4	-4
$\mathbf{O}_2$	6	12	5	10	22	2	-3	13	4	7	14	5	-11	7	0
$\mathbf{O}_3$	11	-8	4	15	2	1	2	-7	3	12	-6	4	-6	-13	-1
$\mathbf{O}_4$	17	5	5	21	15	2	8	6	4	18	7	5	0	0	0
$\mathbf{V}_j^1\mathbf{V}_j^2\mathbf{V}_j^3$	17	5	5	21	15	2	8	6	4	18	7	5			

Table 3. Comparison of solution between proposed and MFL solution for flow 85.

FLOW 85	Solution of MOLFTP	First objective	Second objective	Third objective	D_i^+	D_i^-	R	Rank
MFL at cell (1, 2)	Ideal Solution	850	460	205	0	494.3177	1	1
	Anti-Ideal solution	980	875	440	494.3177	0	0	4
	More for less solution	945	545	205	127.4755	406.6325	0.76133	2
	Compromise solution	985	510	250	150.831	411.5216	0.731786	3
MFL at cell (1, 3)	Ideal Solution	795	420	215	0	484.9227	1	1
	Anti-Ideal solution	930	825	445	484.9227	0	0	4
	More for less solution	880	500	215	116.7262	401.2792	0.774662	2
	Compromise solution	920	465	260	140.2676	404.8765	0.742696	3
MFL at cell (2, 3)	Ideal Solution	805	445	235	0	525.8564	1	1
	Anti-Ideal solution	965	890	465	525.8564	0	0	4
	More for less solution	885	555	235	136.0147	414.1558	0.752777	2
	Compromise solution	941	506	298	161.8209	419.4294	0.721599	3
MFL at cell (3, 1)	Ideal Solution	845	385	235	0	589.025	1	1
	Anti-Ideal solution	1000	915	440	589.025	0	0	4
	More for less solution	955	450	235	127.769	510.171	0.7997	2
	Compromise solution	970	431	274	138.788	512.554	0.7869	3
MFL at cell (3, 3)	Ideal Solution	810	385	230	0	587.303	1	1
	Anti-Ideal solution	965	915	430	587.303	0	0	4
	More for less solution	910	455	230	122.066	504.604	0.8052	2
	Compromise solution	950	420	275	151.162	518.917	0.7744	3
MFL at cell (3, 4)	Ideal Solution	860	390	235	0	587.303	1	1
	Anti-Ideal solution	1015	920	435	587.303	0	0	4
	More for less solution	960	460	235	122.066	504.604	0.8052	2
	Compromise solution	988	429	288	143.9236	513.2436	0.780994	3

Table 4. Fractional objective matrix for MOLFTP example.

		$\mathbf{D}_1$			$\mathbf{D}_2$			$\mathbf{D}_3$			$\mathbf{D}_4$			$\mathbf{a}_i$
Objective		1^{st}	2^{nd}	3^{rd}	1^{st}	2^{nd}	3^{rd}	1^{st}	2^{nd}	3^{rd}	1^{st}	2^{nd}	3^{rd}	
$\mathbf{O}_1$	c_{ij}^k	10	14	6	14	9	3	8	11	9	12	9	9	15
	d_{ij}^k	15	12	15	12	24	7	16	7	8	8	12	15	
$\mathbf{O}_2$	c_{ij}^k	8	12	2	12	9	9	14	6	2	8	15	10	25
	d_{ij}^k	10	16	8	6	11	8	13	20	7	12	10	9	
$\mathbf{O}_3$	c_{ij}^k	9	6	5	6	9	9	15	12	8	9	10	13	20
	d_{ij}^k	13	13	8	15	15	10	12	14	7	10	16	9	
$\mathbf{b}_i$		15			25			5			15			60

Table 5. Shadow price matrix (for compromise optimal solution) for MOLFTP example.

	$\mathbf{D}_1$			$\mathbf{D}_2$			$\mathbf{D}_3$			$\mathbf{D}_4$			$\mathbf{U}_i^1\mathbf{U}_i^2\mathbf{U}_i^3$		
Objective	1^{st}	2^{nd}	3^{rd}	1^{st}	2^{nd}	3^{rd}	1^{st}	2^{nd}	3^{rd}	1^{st}	2^{nd}	3^{rd}			
$\mathbf{O}_1$	0.67	0.31	0.4	1.17	0.38	-0.42	0.5	-0.14	0.43	0.53	0.75	0.6	0	-0.44	-0.51
$\mathbf{O}_2$	0.8	0.75	0.25	1.3	0.82	1.12	0.64	0.3	0.29	0.67	1.19	0.45	0.13	0	0
$\mathbf{O}_3$	-0.1	0.46	1.22	0.4	0.53	0.9	-0.27	0.01	0.06	-0.23	0.9	0.22	-0.77	-0.29	-0.04
$\mathbf{V}_j^1\mathbf{V}_j^2\mathbf{V}_j^3$	0.67	0.75	0.25	1.17	0.82	0.94	0.5	0.3	0.29	0.53	0.91	1.11	60		

sponding row and column by 5, the compromising, paradoxical solution and ranking are given in Table 6. Ideal and anti-ideal solutions are also given in this table. Comparison of proposed method with the other approach [4] using [1] is discussed in Table 6.

6. Conclusion

An efficient procedure to solve the MFL paradox in MOLTP and MOLFTP is presented in this paper. No symmetric method yet exists in the literature to find the MFL solution for the above-mentioned problem with no common MFL situation. The approach here allows easy identification of such MFL paradox cells in the objective matrix and the calculation of the maximal allowable units and distribution of these excesses in a systematic approach. We compare our MFL solution with the compromise solution with the same flow. We found that our approach gives a better result in ranking [1] in comparison with the compromise optimal solutions obtained by [1,4]. The reader can see its graphs in Figs. 1, 2.

Table 6. Comparison of solution between proposed and MFL solution for flow 65.

FLOW 85	Solution of MOLFTP	First objective	Second objective	Third objective	D_i^+	D_i^-	R	Rank
MFL at cell (1, 1)	Ideal Solution	0.609195	0.521186	0.580153	0	0.55289	1	1
	Anti-Ideal solution	1.014286	0.864407	0.734375	0.55289	0	0	4
	More for less solution	0.758	0.66466	0.60698	0.208964	0.348233	0.624973	2
	Compromise solution	0.86538	0.564444	0.673077	0.275743	0.340473	0.552523	3
MFL at cell (1, 2)	Ideal Solution	0.64327	0.61429	0.62903	0	0.487568	1	1
	Anti-Ideal solution	1.07353	0.78307	0.78431	0.487568	0	0	4
	More for less solution	0.709838	0.69596	0.703333	0.128845	0.382553	0.747979	2
	Compromise solution	0.764187	0.636802	0.643087	0.123639	0.370289	0.749682	3
MFL at cell (1, 3)	Ideal Solution	0.59429	0.4875	0.58462	0	0.623867	1	1
	Anti-Ideal solution	1.03497	0.87209	0.80165	0.623867	0	0	4
	More for less solution	0.759214	0.645418	0.625	0.229297	0.40077	0.636076	2
	Compromise solution	0.875776	0.535049	0.679612	0.301188	0.392046	0.565532	3
MFL at cell (3, 1)	Ideal Solution	0.6092	0.53182	0.61194	0	0.566514	1	1
	Anti-Ideal solution	1.0292	0.86441	0.79612	0.566514	0	0	4
	More for less solution	0.736842	0.646154	0.61194	0.171377	0.408637	0.70453	2
	Compromise solution	0.825641	0.569676	0.722426	0.239933	0.369298	0.606171	3
MFL at cell (3, 3)	Ideal Solution	0.53933	0.49554	0.61654	0	0.697858	1	1
	Anti-Ideal solution	1.05	0.92025	0.83065	0.697858	0	0	4
	More for less solution	0.703518	0.666316	0.667692	0.241642	0.460254	0.65573	2
	Compromise solution	0.83125	0.542934	0.716981	0.312115	0.450771	0.590876	3
MFL at cell (3, 4)	Ideal Solution	0.547486	0.542601	0.666667	0	0.686266	1	1
	Anti-Ideal solution	1.052239	0.962963	0.865385	0.686266	0	0	4
	More for less solution	0.707851	0.705376	0.683582	0.228219	0.467971	0.672189	2
	Compromise solution	0.824675	0.590296	0.777778	0.302742	0.44539	0.595336	3

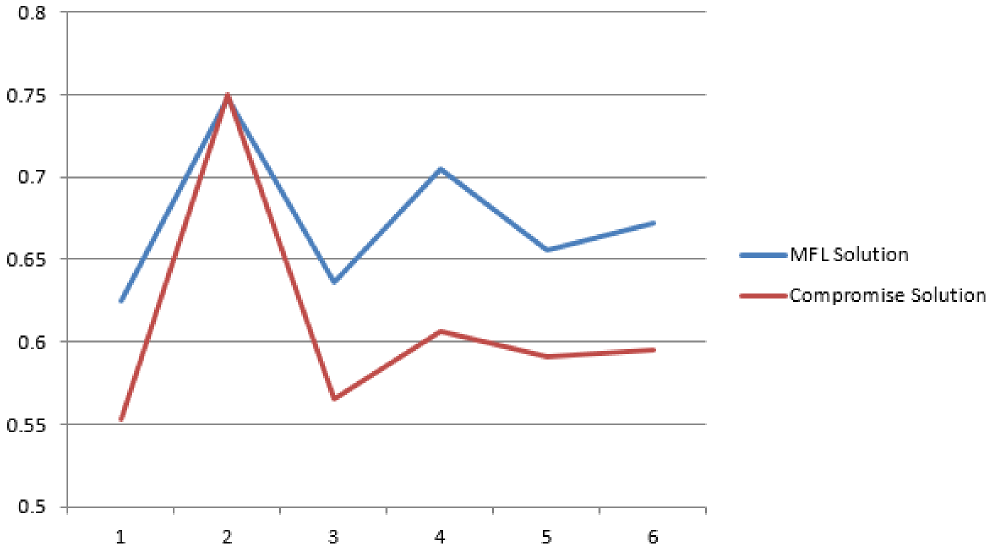

Fig. 1. Ranking comparison for MOLTP between MFL and compromise solution.

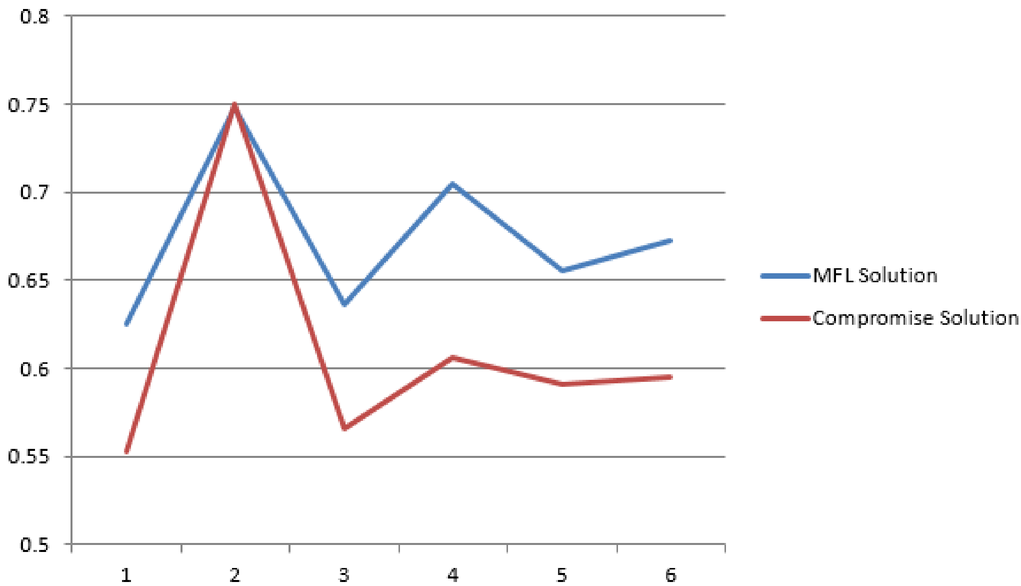

Fig. 2. Ranking comparison for MOLFTP between MFL and compromise solution.

References

[1] Mousa AA, Hamdy MG, Adel YE. Efficient evolutionary algorithm for solving multi-objective transportation problem. J Nat Sci Math 2010;4:77–102.

[2] Verma V, Puri MC. On a Paradox in Linear Fractional Transportation Problem. Recent Developments in Mathematical Programming, published on behalf of Australian Society of Operation Research (ed. Suntosh Kumar), Gordan and Breach Science Publishers 1991; 413-24.

[3] Gupta K, Arora R. More for less method to minimize the unit transportation cost of a capacitated transportation problem with bounds on rim conditions. OPSEARCH 2017;54: 460–74.

[4] Joshi VD, Saini R. Solving Multi-objective Fractional Transportation Problem. In: Singh J, Kumar D, Dutta H, Baleanu D, Purohit SD, editors. Mathematical Modelling, Applied Analysis and Computation; 2018 July 6-8; Jaipur, India.Singapore Pvt: Springer Nature Ltd; 2019. p.221-8.

[5] Deineko VG, Klinz B, Woeginger GJ. Which matrices are immune against the transportation paradox? Discret Appl Math 2003;130:495-501.

[6] Charnes A and Klingman D. The more-for-less paradox in distribution models. Cah Cent étud rech opér 1971;13:11-22.

[7] Szwarc W. The transportation paradox. Nav Res Logist Q 1971;18:185-202.

[8] Adlakha V, Kowalski K. A quick sufficient solution to the more-for-less paradox in the transportation problem. Omega 1997;26:541-7.

[9] Adlakha V, Kowalski K, Lev B. Solving transportation problem with mixed constraints. Int J Manag Sci Eng Manag 2006;1:47-52.

[10] Adlakha V, Kowalski K, Vemuganti RR, Lev B. More-for-less algorithm for fixed charge transportation problems. Omega 2007;35:116-27.

[11] Porchelvi RS, Anitha M. Paradox in a linear multi-objective transportation problem. Pure Appl Math 2017;15:185-92.

[12] Joshi VD, Gupta N. Identifying more-for-less paradox in the linear fractional transportation problem using objective matrix. Mathematika 2012;28:173-80.

[13] Joshi VD, Gupta N. On a paradox in linear plus linear fractional transportation problem. Mathematika 2010;26:167-78.

[14] Nomani MA, Ali I, Ahmed A. A new approach for solving multi-objective transportation problems. Int J Manage Sci Eng Manage 2016;12:1–9.