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Charnes and Cooper's Transformations for Solving Uncertain Multi-Item Fixed Charge Solid Transportation Problem

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ABSTRACT

This paper focusses on the multi-item fixed charge solid transportation problem (MIFCSTP) with parameters like transportation cost, fixed charge, conveyance capacities, and supply and demand values taken as uncertain variables. The model representing MIFCSTP with uncertain variables is known as an uncertain model which cannot be solved directly by standard techniques. So, to deal with uncertain MIFCSTP, the expected value model (EVM) and dependent chance-constrained model (DCCM) are formulated. Later, using the theory of uncertainty, these two models are converted into their corresponding deterministic forms. This crisp formulation of the EVM is solved directly by LINGO 18.0 software whereas the Charnes and Cooper's transformation is applied on the deterministic form of the DCCM for converting it into linear form. Then, the optimal solution of the transformed linear form of the DCCM is obtained using LINGO 18.0 software. At the end, the application of the formulated models is demonstrated with the help of a numerical illustration.

Keywords: Charnes and Cooper's transformations; Dependent chance constrained model; Expected value model; Uncertain multi-item fixed charge solid transportation problem

1. Introduction

A transportation model basically deals with transporting a homogeneous commodity from a number of available origins to various destinations with an objective that the cost of transportation is minimized. The transportation problem which was initially developed by Hitchcock [1] in 1941 involved only demand and source constraints. But, in real world scenarios, different factors (like modes of transportation, different types of items, etc.) are also taken

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into consideration which plays a crucial role in evaluating the total transportation cost. So, if mode of transportation is added as an additional constraint in the basic transportation problem along with its existing source and demand constraints, then this extension of the basic transportation model is called the solid transportation problem (STP). TP is essentially the specialization of the STP because, if we consider only one mode of transportation through which transportation can be done, then it is simply the basic TP. If multiple numbers of items are transported in a STP instead of a single item, then such an STP is known as multi-item STP (MISTP). Haley [2] was the first one to introduce STP in 1962 and later Bhatia et al. [3] worked on STP with an objective of minimizing the transportation time. STP with discounts in the transportation cost, fixed charges along with the cost of vehicles was analyzed by Ojha et al. [4] in 2010. Li et al. [5] studied multi-criteria STP and solved it using a neural network approach. Zimmerman [6] introduced fuzzy programming technique for solving a multi-objective STP. Bit et al. [7] presented multi-objective TP (MOTP) with the fuzzy programming model.

Another variation of TP, called fixed charge TP (FCTP) is introduced in which some fixed charge is also involved in the transportation plan. Hirsch and Dantzig [8] presented the model for FCTP with the objective that the overall cost is minimized. The overall cost consists of the shipping cost and some fixed charge incurred due to toll taxes, permit fees, etc. Unlike the shipping cost, the fixed charge involved here is independent of the quantity of goods getting transported and is applicable only when some transportation movement occurs between sources and destinations. A MISTP studied along with the fixed charge is said to be the MIFCSTP. FCTP has been focused on by researchers like Steinberg [9], Sun et al. [10], Kennington and Unger [11], Balinski [12], Adlakha and Kowalski [13] and diverse solution approaches to deal with FCTP have been proposed.

However, due to the complexity of the real-world scenarios, the relevant parameters involved in the problem cannot be specifically defined and therefore cannot be treated as constants. These parameters can be imprecise and may vary due to the lack of information, road conditions or environmental conditions. This fact motivated the researchers to involve indeterminant parameters in the transportation model. To deal with imprecise data, Zadeh [14] has introduced fuzzy sets and Moore [15] brought in the concept of interval theory. Indeterminacy can be modelled with probability theory as well, if the historical data is available to us. But if we are not provided with the previous information, then using the concepts of probability theory will not lead us to the appropriate results. Thus, according to Liu [16], uncertainty theory is applicable when we have lack of previously available historical data and we estimate the degree of belief for some phenomenon. So, to measure the belief degree, the basic fundamentals of uncertainty theory have been introduced by Liu [16] in 2007 and later modified by him in 2010 [17].

In the literature, Jimenez and Verdegay [18] studied STP with parameters taken in the interval and fuzzy environment. Gen et al. [19] focused on bicriteria STP with fuzzy numbers whereas Kundu et al. [20] focused on MOSTP with budget constraints in the uncertain environment. Paper [21] is based on an evolutionary algorithm with parametric approach studied by Jimenez and Verdegay to solve fuzzy STP. Williams [22] proposed a stochastic transportation model with demand variables assumed as random variables. After that, Szwarc [23] inspected a transportation model in the stochastic environment, where along with the minimization of total transportation cost, expected penalty costs were also taken into consideration. A bicriteria STP along with fixed charge was analyzed by Yang and Feng [24] under the stochastic environment. Different variants of TP under the stochastic environment have also been studied by researchers like Liu and Yao [25], Maity G et al. [26], Dalman H. [27], Mou [28], Sheng and Yao [29,30], Liu L. et al [31], Zhang et al. [32], Majumder et al. [33] and Chen et al. [34], utilizing different approaches to obtain the solutions.

In this paper, we have studied MIFC-STP in the uncertain environment and formulated two models, i.e the EVM and the DCCM model. These two uncertain models are further simplified into deterministic forms with the help of uncertainty theory. The solution of the deterministic form of EVM can be easily achieved in LINGO 18.0 software but the deterministic form of DCCM is obtained in the fractional form. So to solve this model, variable transformations introduced by Charnes and Coopers [35] are used to convert it into linear form. The linear form of DCCM is then solved in LINGO 18.0 software. Lastly, we demonstrate the application of the formulated models with the help of a numerical example.

2. Preliminaries on Uncertainty Theory

This section aims to introduce the basic fundamentals and concepts of uncertainty theory.

Definition 2.1 (Liu [16]). For any real number x, an uncertain variable ξ has an uncertainty distribution $\Phi : \mathcal{R} \to [0, 1]$ defined by

$$\Phi(x) = \mathcal{M}\xi \le x.$$

Definition 2.2 (Liu [17]). An uncertain variable ξ with uncertainty distribution function Φ given by

$$\Phi(x) = \begin{cases} 0, & \text{if } x \le a \\ \frac{x-a}{2(b-a)} & \text{if } a \le x \le b \\ \frac{x+c-2b}{2(c-b)}, & \text{if } b \le x \le c \\ 1, & \text{if } x \ge c \end{cases}$$

is called a zigzag uncertain variable and it is denoted by $\mathcal{Z}(a, b, c)$ with $a, b, c \in \mathcal{R}$ and $a < b < c$.

Definition 2.3 (Liu [17]). The inverse uncertainty distribution function denoted by Φ^{-1} of $\mathcal{Z}(a, b, c)$, with uncertainty distribution is given by

$$\Phi^{-1}(\alpha) = \begin{cases} (1-2\alpha)a + 2\alpha b, & \text{if } \alpha < 0.5 \\ (2-2\alpha)b + (2\alpha-1)c, & \text{if } \alpha \ge 0.5. \end{cases}$$

Theorem 2.4 (Liu [17]). *Consider independent uncertain variables $\xi_1, \xi_2, \cdots, \xi_n$ with uncertainty distribution functions $\Phi_1, \Phi_2, \ldots, \Phi_n$, respectively. Then $\xi = f(\xi_1, \xi_2, \cdots, \xi_n)$ is an uncertain variable with inverse uncertainty distribution*

$$\Psi^{-1}(\alpha) = f(\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \cdots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1-\alpha), \Phi_{m+2}^{-1}(1-\alpha), \cdots, \Phi_n^{-1}(1-\alpha),$$

where $f(x_1, x_2, \cdots, x_n)$ is strictly increasing for $x_1, x_2, \cdots, x_m$ and strictly decreasing for $x_{m+1}, x_{m+2}, \cdots, x_n$.

Theorem 2.5 (Liu [17]). *If the expected value of an uncertain variable ξ with uncertainty distribution Φ exists, then it is given by*

$$E[\xi] = \int_0^1 \Phi^{-1}(\alpha)d\alpha.$$

The zigzag uncertain variable $\mathcal{Z}(a, b, c)$ has the expected value $E[\xi] = \dfrac{a + 2b + c}{4}$.

3. Mathematical Formulation of Multi Item FCSTP

This section introduces the MIFCSTP having m sources, n destinations and r items getting transported using K conveyances. The mathematical formulation for MIFCSTP is:

***Model 1*:**

$$\min Z = \sum_{p=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} c_{ijk}^{p} x_{ijk}^{p} + f_{ijk} y_{ijk}$$

subject to the constraints:

$$\begin{aligned}
\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk}^{p} &\leq a_i^p, \forall i, p,\\
\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk}^{p} &\geq b_j^p, \forall j, p,\\
\sum_{p=1}^{r} x_{ijk}^{p} &\leq e_{ijk}, \forall i, j, k,
\end{aligned}$$

$$x_{ijk}^{p} \geq 0,\ y_{ijk} = \begin{cases} 1, & \text{if} x_{ijk}^{p} > 0,\\ 0, & \text{if} x_{ijk}^{p} = 0.\end{cases}$$

Here c_{ijk}^p, a_i^p, b_j^p and e_{ijk} represent the shipping cost, supplying capacity of the source i for the p item, requirement at destination j for the pth item and capacity of the conveyance k for transporting goods from j^{textth} source to destination respectively. f_{ijk} represents the fixed charge, x_{ijk}^p represents the number of pth items transported from the source i to destination j using the conveyance k. y_{ijk} represents a decision variable in binary form which pertains value 1 only when some transportation activity occurs between the source and destination pair (i, j) by conveyance k otherwise it is equal to zero. y_{ijk} is independent of the type of item getting transported. If in case $x_{ijk}^p > 0$ for more than one item type with same source-destination pair and same conveyance, then $y_{ijk} = 1$ is assumed for only one type of item and for the rest of the items it is assumed to be zero.

The above model (3) is formulated by considering all the parameters a_i^p, b_j^p, e_{ijk}, c_{ijk}^p as crisp numbers. So, to involve indeterminacy of the parameters, the variables $a_i^p, b_j^p, e_{ijk}, c_{ijk}^p$ and f_{ijk} in the MIFCSTP are replaced by $\widetilde{a}_i^p, \widetilde{b}_j^p, \widetilde{e}_{ijk}, \xi_{ijk}^p$ and η_{ijk}, respectively, which are called uncertain variables. Then the MIFCSTP becomes uncertain MIFCSTP, denoted by UMIFCSTP.

4. Uncertain Model for MIFCSTP

The following uncertain programming model for MIFCSTP is obtained by introducing the uncertain variables $\widetilde{a}_i^p, \widetilde{b}_j^p, \widetilde{e}_{ijk}, \xi_{ijk}^p$ and η_{ijk} in Model 1.

***Model 2*:**

$$\min Z(\xi, x)$$

$$= \sum_{p=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} \xi_{ijk}^{p} x_{ijk}^{p} + \eta_{ijk} y_{ijk}$$

subject to the constraints:

$$\begin{aligned}
\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk}^{p} &\leq \widetilde{a}_i^p, \forall i, p,\\
\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk}^{p} &\geq \widetilde{b}_j^p, \forall j, p,\\
\sum_{p=1}^{r} x_{ijk}^{p} &\leq \widetilde{e}_{ijk}, \forall i, j, k,
\end{aligned}$$

and the constraint of Model 1.

The basic problem in the uncertain MIFCSTP model is ranking of the uncertain variables. An expected value criterion is adopted to rank the uncertain variables and it is termed EVM. Dependent chance-constrained programming is another method to deal with optimal problems in the uncertain environment.

4.1 Expected value model

The basic idea of EVM is to optimize the expected value of the objective function subject to the constraints with expected values. The formulation of the EVM model is given by the following model:

***Model 3*:**

$$\min E[Z(\xi, x)]$$

$$= E\left[\sum_{p=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} \xi_{ijk}^{p} x_{ijk}^{p} + \eta_{ijk} y_{ijk}\right]$$

subject to the constraints:

$$E\left[\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk}^{p} \leq \widetilde{a}_{i}^{p}\right], \qquad \forall i, p,$$

$$E\left[\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk}^{p} \geq \widetilde{b}_{j}^{p}\right], \qquad \forall j, p,$$

$$E\left[\sum_{p=1}^{r} x_{ijk}^{p} \leq \widetilde{e}_{ijk}\right], \qquad \forall i, j, k,$$

and the constraint of Model 1.

4.2 Dependent chance-constrained model

Suppose that f is the given predetermined maximal cost. The decision maker wishes to maximize the uncertain measure of the cost less than or equal to f as much as possible of MIFCSTP. So, subject to the chance constraints, the mathematical formulation of the DCCM is given as:

***Model 4*:**

$$\max M$$

$$\left\{\sum_{p=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} (\xi_{ijk}^{p} x_{ijk}^{p} + \eta_{ijk} y_{ijk}) \leq \overline{f}\right\}$$

subject to the constraints:

$$M\left\{\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk}^{p} \leq \widetilde{a}_{i}^{p}\right\} \geq \alpha_{i}^{p}, \forall i, p,$$

$$M\left\{\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk}^{p} \geq \widetilde{b}_{j}^{p}\right\} \geq \beta_{j}^{p}, \forall j, p$$

$$M\left\{\sum_{p=1}^{r} x_{ijk}^{p} \leq \widetilde{e}_{ijk}\right\} \geq \gamma_{ijk}, \forall i, j, k,$$

where $\alpha_{i}^{p}, \beta_{j}^{p}$ and γ_{ijk} are predetermined confidence levels.

5. The Deterministic Formulations

Since the uncertain models cannot be solved directly, it is important for us to convert these models into simpler equivalent models using the concepts of uncertainty defined in Section 2. Suppose $\xi_{ijk}^{p}, \eta_{ijk}, \widetilde{a}_{i}^{p}, \widetilde{b}_{j}^{p}, \widetilde{e}_{ijk}$ represent the independent uncertain variables with uncertainty distributions $\Phi_{\xi_{ijk}^{p}}, \Phi_{\eta_{ijk}}, \Phi_{\widetilde{a}_{i}^{p}}, \Phi_{\widetilde{b}_{j}^{p}}, \Phi_{\widetilde{e}_{ijk}}$, respectively, then the deterministic transformations for the above uncertain models can be given as:

5.1 EVM model

***Model 5*:**

$$\min E[Z(\xi, x)]$$

$$= \sum_{p=1}^{r}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk}^{p} E[\xi_{ijk}^{p}] + y_{ijk} E[\eta_{ijk}]$$

subject to the constraints:

$$\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk}^{p} \leq E[\widetilde{a}_{i}^{p}], \ \forall i, p,$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk}^{p} \geq E[\widetilde{b}_{j}^{p}], \ \forall j, p,$$

$$\sum_{p=1}^{r} x_{ijk}^{p} \leq E[\widetilde{e}_{ijk}], \ \forall i, j, k,$$

and the constraint of Model 1.

5.2 DCCM model

Theorem 5.1. *Let* $\xi_1, \xi_2, \cdots, \xi_n$ *given by* $Z(a_1, b_1, c_1), Z(a_2, b_2, c_2), \cdots, Z(a_n, b_n, c_n)$

be independent zigzag uncertain variables respectively and let $x_1, x_2, \cdots, x_n$ be non-negative decision variables. If

$$\overline{f} \in \left[\sum_{i=1}^{n} a_i x_i, \sum_{i=1}^{n} b_i x_i, \right] \qquad (2)$$

then

$$M\left\{ \sum_{i=1}^{n} \xi_i x_i \le \overline{f} \right\} = \frac{\overline{f} - \sum_{i=1}^{n} a_i x_i}{2 \sum_{i=1}^{n} (b_i - a_i) x_i}.$$

If

$$\overline{f} \in \left[\sum_{i=1}^{n} b_i x_i, \sum_{i=1}^{n} c_i x_i, \right] \qquad (3)$$

then

$$M\left\{ \sum_{i=1}^{n} \xi_i x_i \le \overline{f} \right\} = \frac{\overline{f} + \sum_{i=1}^{n} (c_i - 2b_i) x_i}{2 \sum_{i=1}^{n} (c_i - b_i) x_i}.$$

Otherwise, the measure will be 0 if $\overline{f}$ lies on the left of the interval (2) *or 1 if $\overline{f}$ is on the right of the interval* (3).

Using Theorem 5.1, the objective function in Model 4 is converted into the fractional objective function depending on the value of $\overline{f}$ and we obtain the given deterministic model for DCCM as follows:

***Model 6*:**

$\max M$

$$\left\{ \sum_{p=1}^{r} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} (\xi_{ijk}^{p} x_{ijk}^{p} + \eta_{ijk} y_{ijk}) \le \overline{f} \right\}$$

subject to the constraints:

$$\sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk}^{p} - \Phi_{\widetilde{a}_i^p}^{-1}(1 - \alpha_i^p) \le 0, \forall i, p,$$

$$\Phi_{\widetilde{b}_j^p}^{-1}(\beta_j^p) - \sum_{i=1}^{m} \sum_{k=1}^{K} x_{ijk}^{p} \le 0, \forall j, p,$$

$$\sum_{p=1}^{r} x_{ijk}^{p} - \Phi_{\widetilde{e}_{ijk}}^{-1}(1 - \gamma_{ijk}) \le 0, \forall i, j, k,$$

and the constraint of Model 1, where α_i^p, β_j^p and γ_{ijk} are predetermined confidence levels.

6. Charnes and Cooper's Transformation for DCCM Model

This section describes Charnes and Cooper's transformations [5] used for converting the linear fractional problem (LFP) shown in Model 7 to the linear programming problem shown in Model 8.

***Model 7*:**

$\min = \dfrac{p'x + \alpha}{q'x + \beta}$

subject to the constraints:

$$Ax \le b, x \ge 0.$$

Assume that the set

$$\mathcal{S} = \{x : Ax \le b, x \ge 0\}$$

is compact and $q'x + \beta > 0$ for each $x \in \mathcal{S}$. Letting $t = \dfrac{1}{q'x + \beta}$ and $y = tx$, the LFP in Model 7 converts to the following model:

***Model 8*:**

$\min = p'y + \alpha t$

subject to the constraints:

$$Ay - bt \le 0, \; q'y + \beta t = 1, \; y \ge 0, \; t \ge 0.$$

Note that, if (y, t) i; a feasible solution to Model 8, then $t > 0$ and if $(\overline{y}, \overline{t})$ is an optimal solution to the above linear program, then the fractional program has an optimal solution $\overline{x} = \dfrac{\overline{y}}{\overline{t}}$. If $q'x + \beta < 0$ for all $x \in \mathcal{S}$, then letting $-t = \dfrac{1}{q'x + \beta}$ and $y = tx$ gives the following LPP:

$\min(-p'y - \alpha t)$
subject to the constraints:

$$Ay - bt \leq 0,\ -q'y - \beta t = 1,\ y \geq 0,\ t > 0.$$

The form of the LPP used depends on whether $q'x + \beta > 0$ for all $x \in \mathcal{S}$ or $q'x + \beta < 0$ for all $x \in \mathcal{S}$.

7. Numerical Examples

To demonstrate the applications of the models, we consider a MIFCSTP with two sources, three destinations and two conveyances. The objective is to develop a transportation scheme so that the total cost of transportation is minimum. The notations $\widetilde{a}_i^p, \widetilde{b}_j^p$ and $\widetilde{e}_{ijk}$ are used to denote the capacity of suppliers, demands at destinations, and capacity of conveyances, respectively. All the uncertain variables used here are assumed to be independent zigzag uncertain variables as listed below:

$$\widetilde{a}_1^1 = (36, 45, 52),\ \widetilde{a}_1^2 = (50, 52, 55),$$
$$\widetilde{a}_2^1 = (52, 58, 66),\ \widetilde{a}_2^2 = (33, 35, 36),$$
$$\widetilde{b}_1^1 = (16, 20, 22),\ \widetilde{b}_1^2 = (14, 18, 20),$$
$$\widetilde{b}_2^1 = (25, 28, 31),\ \widetilde{b}_2^2 = (20, 23, 25),$$
$$\widetilde{b}_3^1 = (33, 35, 38),\ \widetilde{b}_3^2 = (18, 20, 24).$$

Here, values of the confidence levels are assumed to be $\alpha_i^p = 0.9, \beta_j^p = 0.9, \gamma_{ijk} = 0.9$.

Firstly, we evaluate the derived costs suggested by Balinski [12] for the shipping costs by both the conveyances. Let us represent the supply, demand, shipping cost and fixed charge uncertain variables, respectively, in the form

$$\widetilde{a}_i^p = (a_i^{1p}, a_i^{2p}, a_i^{3p}),$$
$$\widetilde{b}_j^p = (b_j^{1p}, b_j^{2p}, b_j^{3p}),$$
$$\xi_{ijk}^p = (\xi_{ijk}^{1p}, \xi_{ijk}^{2p}, \xi_{ijk}^{3p}) \text{ and}$$
$$\eta_{ijk} = (\eta_{ijk}^1, \eta_{ijk}^2, \eta_{ijk}^3).$$

Then the derived costs for both of the conveyances are obtained as:

$$\xi_{ijk}^{1p} + \frac{\eta_{ijk}^1}{min(a_i^{1p}, b_j^{1p})},$$
$$\xi_{ijk}^{2p} + \frac{\eta_{ijk}^2}{min(a_i^{2p}, b_j^{2p})},$$
$$\xi_{ijk}^{3p} + \frac{\eta_{ijk}^3}{min(a_i^{3p}, b_j^{3p})},$$

$i = 1, 2;\ j = 1, 2, 3;\ k = 1, 2;\ p = 1, 2.$
Here, for evaluation purpose of the derived costs, we assume that the fixed charge for p items is same. Therefore, the derived costs obtained for this numerical are given in Table 5 to 8.

Table 1. The capacity of the conveyances 1 and 2 from i to j (e_{ijk}).

e_{ij1}	1	2	3
1	(20,22,23)	(23,25,27)	(20,22,25)
2	(18,20,26)	(17,23,25)	(21,25,28)
e_{ij2}	1	2	3
1	(22,26,28)	(23,25,28)	(20,24,26)
2	(23,27,30)	(22,26,34)	(18,22,24)

Table 2. The cost of transportation for items 1 and 2 by conveyance 1.

ξ_{ij1}^1	1	2	3
1	(10,12,15)	(8,10,13)	(6,8,12)
2	(7,9,10)	(6,7,10)	(8,10,14)
ξ_{ij1}^2	1	2	3
1	(4,6,10)	(5,8,12)	(6,8,12)
2	(6,7,10)	(8,10,13)	(8,10,13)

Expected value model:
The deterministic form of the EVM mentioned in Section 5.1 is obtained for the

Table 3. The cost of transportation for items 1 and 2 by conveyance 2.

ξ^1_{ij2}	1	2	3
1	(8,10,12)	(10,13,17)	(6,10,12)
2	(6,8,12)	(6,9,10)	(10,12,13)
ξ^2_{ij2}	1	2	3
1	(6,8,9)	(8,9,11)	(8,9,12)
2	(5,8,10)	(6,8,9)	(10,12,13)

Table 4. The fixed charge by conveyances 1 and 2.

η_{ij1}	1	2	3
1	(6,8,12)	(8,10,13)	(6,8,9)
2	(7,9,10)	(7,9,12)	(7,9,12)
η_{ij2}	1	2	3
1	(7,9,12)	(6,8,9)	(7,10,12)
2	(5,8,10)	(7,9,10)	(8,9,12)

problem by substituting the values of the parameters evaluated in Tables 5 to 8.
Model 9:

min =

$$
\begin{aligned}
&12.68x^1_{111} + 10.6132x^1_{121} + 8.719x^1_{131}\\
&+9.1983x^1_{211} + 7.8272x^1_{221} + 10.7605x^1_{231}\\
&+6.9792x^2_{111} + 8.6975x^2_{121} + 8.877x^2_{131}\\
&+8x^2_{211} + 10.653x^2_{221} + 10.6972x^2_{231}\\
&+10.4707x^1_{112} + 13.5255x^1_{122} + 9.775x^1_{132}\\
&+8.892x^1_{212} + 8.8112x^1_{222} + 12.018x^1_{232}\\
&+8.275x^2_{112} + 9.589x^2_{122} + 9.9722x^2_{132}\\
&+8.1862x^2_{212} + 8.133x^2_{222} + 12.211x^2_{232};
\end{aligned}
$$

subject to the constraints:

$$
\begin{aligned}
&x^1_{111} + x^1_{112} + x^1_{121} + x^1_{122} + x^1_{131} + x^1_{132} - 44.5 \le 0;\\
&x^1_{211} + x^1_{212} + x^1_{221} + x^1_{222} + x^1_{231} + x^1_{232} - 58.5 \le 0;\\
&x^2_{111} + x^2_{112} + x^2_{121} + x^2_{122} + x^2_{131} + x^2_{132} - 52.25 \le 0;\\
&x^2_{211} + x^2_{212} + x^2_{221} + x^2_{222} + x^2_{231} + x^2_{232} - 34.75 \le 0;
\end{aligned}
$$

$$
\begin{aligned}
&19.5 - \left(x^1_{111} + x^1_{112} + x^1_{211} + x^1_{212}\right) \le 0;\\
&28 - \left(x^1_{121} + x^1_{122} + x^1_{221} + x^1_{222}\right) \le 0;\\
&35.25 - \left(x^1_{131} + x^1_{132} + x^1_{231} + x^1_{232}\right) \le 0;\\
&17.5 - \left(x^2_{111} + x^2_{112} + x^2_{211} + x^2_{212}\right) \le 0;\\
&22.75 - \left(x^2_{121} + x^2_{122} + x^2_{221} + x^2_{222}\right) \le 0;\\
&20.5 - \left(x^2_{131} + x^2_{132} + x^2_{231} + x^2_{232}\right) \le 0;
\end{aligned}
$$

$$
\begin{aligned}
&x^1_{111} + x^2_{111} - 21.75 \le 0;\\
&x^1_{121} + x^2_{121} - 25 \le 0;\\
&x^1_{131} + x^2_{131} - 22.25 \le 0;\\
&x^1_{211} + x^2_{211} - 21 \le 0;\\
&x^1_{221} + x^2_{221} - 22 \le 0;\\
&x^1_{231} + x^2_{231} - 24.75 \le 0;\\
&x^1_{112} + x^2_{112} - 25.5 \le 0;\\
&x^1_{122} + x^2_{122} - 25.25 \le 0;\\
&x^1_{132} + x^2_{132} - 23.5 \le 0;\\
&x^1_{212} + x^2_{212} - 26.75 \le 0;\\
&x^1_{222} + x^2_{222} - 27 \le 0;\\
&x^1_{232} + x^2_{232} - 21.5 \le 0.
\end{aligned}
$$

After the crisp formulation of the expected value model, LINGO 18.0 software is usedto obtain the solution. The solution obtained for the EVM is as follows:

Table 5. The derived transportation cost for item 1 by conveyance 1.

ξ'^{1}_{ij1}	1	2	3
1	(10.375,12.4,15.545)	(8.32,10.357,13.419)	(6.182,8.229,12.236)
2	(7.438,9.45,10.455)	(6.28,7.321,10.387)	(8.212,10.257,14.316)

Table 6. The derived transportation cost for item 1 by conveyance 1.

ξ'^{2}_{ij1}	1	2	3
1	(4.429,6.444,10.6)	(5.4,8.435,12.52)	(6.333,8.4,12.375)
2	(6.5,7.5,10.5)	(8.35,10.391,13.48)	(8.389,10.45,13.5)

Table 7. The derived transportation cost for item 1 by conveyance 1.

ξ'^{1}_{ij2}	1	2	3
1	(8.438,10.45,12.545)	(10.24,13.286,17.290)	(6.212,10.286,12.316)
2	(6.313,8.4,12.455)	(6.28,9.321,10.323)	(10.242,12.257,13.316)

Table 8. The derived transportation cost for item 1 by conveyance 1.

ξ'^{2}_{ij2}	1	2	3
1	(6.5,8.5,9.5)	(8.3,9.348,11.36)	(8.389,9.5,12.5)
2	(5.357,8.444,10.5)	(6.35,8.391,9.4)	(10.444,12.45,13.5)

$x^1_{131} = 11.75$, $x^1_{221} = 22$, $x^2_{111} = 17.5$,
$x^2_{121} = 1.75$, $x^2_{131} = 10.5$, $x^2_{231} = 10$,
$x^1_{132} = 23.5$, $x^1_{212} = 19.5$, $x^1_{222} = 6$,
$x^2_{222} = 21$.

Substituting this solution set in the original objective function of the problem formed by taking the expected values of the initially mentioned uncertain variables in Table 1 to 4 as given in Model 10. We are back substituting this solution set in the objective function of Model 10 so that we can obtain the actual expected value of the problem, since the optimal value of the objective function in Model 9 will give us the derived expected value.

Model 10:

min =

$$
\begin{aligned}
&12.25x^1_{111}+10.25x^1_{121}+8.5x^1_{131}\\
&+8.75x^1_{211}+7.5x^1_{221}+10.5x^1_{231}\\
&+6.5x^2_{111}+8.25x^2_{121}+8.5x^2_{131}\\
&+7.5x^2_{211}+10.25x^2_{221}+10.25x^2_{231}\\
&+10.x^1_{112}+13.25x^1_{122}+9.5x^1_{132}\\
&+8.5x^1_{212}+8.5x^1_{222}+11.75x^1_{232}\\
&+7.75x^2_{112}+9.25x^2_{122}+9.5x^2_{132}\\
&+7.75x^2_{212}+7.75x^2_{222}+11.75x^2_{232}
\end{aligned}
$$

subject to the given constraints of Model 9.

The objective function value is obtained as $Z^* = 1187.562$. Now add the expected values of the fixed charges (given in Table 4) corresponding to the decision variables in the solution set because fixed charge is added only for those variables for which transportation movement has occurred. Note here that, for the decision variables x^1_{131}, x^2_{131} and x^1_{222}, x^2_{222}, we are adding the fixed charges corresponding to these variables only once because the fixed charge incurred in the transportation cost is independent of the type of item being transported. So, after adding the fixed charges, we obtain the objective function value as $Z^*_{FC} = 1258.812$.

Dependent Chance Constrained model: ***Model 11*:**

max =

$$\frac{\begin{pmatrix} 1600-(9.255x^1_{111}+7.295x^1_{121}+ \\ 4.222x^1_{131}+8.445x^1_{211}+4.255x^1_{221} \\ +6.198x^1_{231}+2.288x^2_{111}+4.35x^2_{121} \\ +4.425x^2_{131}+4.5x^2_{211}+7.302x^2_{221} \\ +7.4x^2_{231}+8.355x^1_{112}+9.282x^1_{122} \\ +8.256x^1_{132}+4.345x^1_{212}+8.319x^1_{222} \\ +11.198x^1_{232}+7.4x^2_{112}+7.336x^2_{122} \\ +6.5x^2_{132}+6.388x^2_{212}+7.382x^2_{222} \\ +11.4x^2_{232}) \end{pmatrix}}{\begin{pmatrix} 2(3.145x^1_{111}+3.062x^1_{121}+4.007x^1_{131} \\ +1.005x^1_{211}+3.066x^1_{221}+4.059x^1_{231} \\ +4.156x^2_{111}+4.085x^2_{121}+3.975x^2_{131} \\ +3x^2_{211}+3.089x^2_{221}+3.05x^2_{231} \\ +2.095x^1_{112}+4.004x^1_{122}+2.03x^1_{132} \\ +4.055x^1_{212}+1.002x^1_{222}+1.059x^1_{232} \\ +1.1x^2_{112}+2.012x^2_{122}+3x^2_{132} \\ +2.056x^2_{212}+1.009x^2_{222}+1.05x^2_{232}) \end{pmatrix}}$$

subject to the constraints:

$$x^1_{111}+x^1_{112}+x^1_{121}+x^1_{122}+x^1_{131}+x^1_{132} -(0.8*36+0.2*45) \le 0;$$
$$x^1_{211}+x^1_{212}+x^1_{221}+x^1_{222}+x^1_{231}+x^1_{232} -(0.8*52+0.2*58) \le 0;$$
$$x^2_{111}+x^2_{112}+x^2_{121}+x^2_{122}+x^2_{131}+x^2_{132} -(0.8*50+0.2*52) \le 0;$$
$$x^2_{211}+x^2_{212}+x^2_{221}+x^2_{222}+x^2_{231}+x^2_{232} -(0.8*33+0.2*35) \le 0;$$
$$(0.2*20+0.8*22) -(x^1_{111}+x^1_{112}+x^1_{211}+x^1_{212}) \le 0;$$
$$(0.2*28+0.8*31) -(x^1_{121}+x^1_{122}+x^1_{221}+x^1_{222}) \le 0;$$
$$(0.2*35+0.8*38) -(x^1_{131}+x^1_{132}+x^1_{231}+x^1_{232}) \le 0;$$
$$(0.2*18+0.8*20) -(x^2_{111}+x^2_{112}+x^2_{211}+x^2_{212}) \le 0;$$
$$(0.2*23+0.8*25) -(x^2_{121}+x^2_{122}+x^2_{221}+x^2_{222}) \le 0;$$
$$(0.2*20+0.8*24) -(x^2_{131}+x^2_{132}+x^2_{231}+x^2_{232}) \le 0;$$

$$x^1_{111}+x^2_{111}-((0.8*20)+(0.2*22)) \le 0;$$
$$x^1_{121}+x^2_{121}-((0.8*23)+(0.2*25)) \le 0;$$
$$x^1_{131}+x^2_{131}-((0.8*20)+(0.2*22)) \le 0;$$
$$x^1_{211}+x^2_{211}-((0.8*18)+(0.2*20)) \le 0;$$
$$x^1_{221}+x^2_{221}-((0.8*17)+(0.2*23)) \le 0;$$
$$x^1_{231}+x^2_{231}-((0.8*21)+(0.2*25)) \le 0;$$

$$x^1_{112}+x^2_{112}-((0.8*22)+(0.2*26)) \le 0;$$
$$x^1_{122}+x^2_{122}-((0.8*23)+(0.2*25)) \le 0;$$
$$x^1_{132}+x^2_{132}-((0.8*20)+(0.2*24)) \le 0;$$
$$x^1_{212}+x^2_{212}-((0.8*23)+(0.2*27)) \le 0;$$
$$x^1_{222}+x^2_{222}-((0.8*22)+(0.2*26)) \le 0;$$

$x_{232}^1 + x_{232}^2 - ((0.8 * 18) + (0.2 * 22)) \leq 0.$

After applying the Charnes and Cooper's transformations [35] in Model 11, we obtain the following model:

***Model 12*:**

$$\max = 1600t - (9.255y_{111}^1 + 7.295y_{121}^1 + 4.222y_{131}^1 + 8.445y_{211}^1 + 4.255y_{221}^1 + 6.198y_{231}^1 + 2.288y_{111}^2 + 4.35y_{121}^2 + 4.425y_{131}^2 + 4.5y_{211}^2 + 7.302y_{221}^2 + 7.4y_{231}^2 + 8.355y_{112}^1 + 9.282y_{122}^1 + 8.256y_{132}^1 + 4.345y_{212}^1 + 8.319y_{222}^1 + 11.198y_{232}^1 + 7.4y_{112}^2 + 7.336y_{122}^2 + 6.5y_{132}^2 + 6.388y_{212}^2 + 7.382y_{222}^2 + 11.4y_{232}^2)$$

subject to the constraints:

$$2(3.145y_{111}^1 + 3.062y_{121}^1 + 4.007y_{131}^1 + 1.005y_{211}^1 + 3.066y_{221}^1 + 4.059y_{231}^1 + 4.156y_{111}^2 + 4.085y_{121}^2 + 3.975y_{131}^2 + 3y_{211}^2 + 3.089y_{221}^2 + 3.05y_{231}^2 + 2.095y_{112}^1 + 4.004y_{122}^1 + 2.03y_{132}^1 + 4.055y_{212}^1 + 1.002y_{222}^1 + 1.059y_{232}^1 + 1.1y_{112}^2 + 2.012y_{122}^2 + 3y_{132}^2 + 2.056y_{212}^2 + 1.009y_{222}^2 + 1.05y_{232}^2) \leq 1;$$

$$y_{111}^1 + y_{112}^1 + y_{121}^1 + y_{122}^1 + y_{131}^1 + y_{132}^1 - (0.8 * 36 + 0.2 * 45)t \leq 0;$$

$$y_{211}^1 + y_{212}^1 + y_{221}^1 + y_{222}^1 + y_{231}^1 + y_{232}^1 - (0.8 * 52 + 0.2 * 58)t \leq 0;$$

$$y_{111}^2 + y_{112}^2 + y_{121}^2 + y_{122}^2 + y_{131}^2 + y_{132}^2 - (0.8 * 50 + 0.2 * 52)t \leq 0;$$

$$y_{211}^2 + y_{212}^2 + y_{221}^2 + y_{222}^2 + y_{231}^2 + y_{232}^2 - (0.8 * 33 + 0.2 * 35)t \leq 0;$$

$$(0.2 * 20 + 0.8 * 22)t - (y_{111}^1 + y_{112}^1 + y_{211}^1 + y_{212}^1) \leq 0;$$

$$(0.2 * 28 + 0.8 * 31)t - (y_{121}^1 + y_{122}^1 + y_{221}^1 + y_{222}^1) \leq 0;$$

$$(0.2 * 35 + 0.8 * 38)t - (y_{131}^1 + y_{132}^1 + y_{231}^1 + y_{232}^1) \leq 0;$$

$$(0.2 * 18 + 0.8 * 20)t - (y_{111}^2 + y_{112}^2 + y_{211}^2 + y_{212}^2) \leq 0;$$

$$(0.2 * 23 + 0.8 * 25)t - (y_{121}^2 + y_{122}^2 + y_{221}^2 + y_{222}^2) \leq 0;$$

$$(0.2 * 20 + 0.8 * 24)t - (y_{131}^2 + y_{132}^2 + y_{231}^2 + y_{232}^2) \leq 0;$$

$$y_{111}^1 + y_{111}^2 - (0.8 * 20 + 0.2 * 22)t \leq 0;$$
$$y_{121}^1 + y_{121}^2 - (0.8 * 23 + 0.2 * 25)t \leq 0;$$
$$y_{131}^1 + y_{131}^2 - (0.8 * 20 + 0.2 * 22)t \leq 0;$$
$$y_{211}^1 + y_{211}^2 - (0.8 * 18 + 0.2 * 20)t \leq 0;$$
$$y_{221}^1 + y_{221}^2 - (0.8 * 17 + 0.2 * 23)t \leq 0;$$
$$y_{231}^1 + y_{231}^2 - (0.8 * 21 + 0.2 * 25)t \leq 0;$$
$$y_{112}^1 + y_{112}^2 - (0.8 * 22 + 0.2 * 26)t \leq 0;$$
$$y_{122}^1 + y_{122}^2 - (0.8 * 23 + 0.2 * 25)t \leq 0;$$
$$y_{132}^1 + y_{132}^2 - (0.8 * 20 + 0.2 * 24)t \leq 0;$$
$$y_{212}^1 + y_{212}^2 - (0.8 * 23 + 0.2 * 27)t \leq 0;$$
$$y_{222}^1 + y_{222}^2 - (0.8 * 22 + 0.2 * 26)t \leq 0;$$
$$y_{232}^1 + y_{232}^2 - (0.8 * 18 + 0.2 * 22)t \leq 0;$$

The optimal solution obtained for the transformed Model 12 using LINGO 18.0 software is:

t= 0.001229597, y^1_{131}= 0.02508377,
y^1_{211}= 0.02262458, y^1_{221}= 0.02237866,
y^2_{111}= 0.02410010, y^2_{231}= 0.02385418,
y^1_{132}= 0.02090314, y^1_{212}= 0.003934709,
y^1_{222}= 0.01500108, y^2_{122}= 0.01721435,
y^2_{132}= 0.004672467, y^2_{222}= 0.01303372

Using the transformation $y = xt$ of Charnes and Cooper [35], we evaluate the values of x as shown below:

$x^1_{131} = 20.4$, $x^1_{211} = 18.4$, $x^1_{221} = 18.2$,
$x^2_{111} = 19.6$, $x^2_{231} = 19.4$, $x^1_{132} = 17$,
$x^1_{212} = 3.2$, $x^1_{222} = 12.2$, $x^2_{122} = 14$,
$x^2_{132} = 3.8$, $x^2_{222} = 10.6$.

with a measure value of the objective function as 0.7762.

Since, this model gives the optimal value in terms of uncertain measure, we obtain the uncertain optimal value by substituting the obtained solution set in the original uncertain objective function i.e.

$$\begin{aligned} Z_1 = \; & (10,12,15)x^1_{111} && +(8,10,13)x^1_{121} \\ & +(6,8,12)x^1_{131} && +(7,9,10)x^1_{211} \\ & +(6,7,10)x^1_{221} && +(8,10,14)x^1_{231} \\ & +(4,6,10)x^2_{111} && +(5,8,12)x^2_{121} \\ & +(6,8,12)x^2_{131} && +(6,7,10)x^2_{211} \\ & +(8,10,13)x^2_{221} && +(8,10,13)x^2_{231} \\ & +(8,10,12)x^1_{112} && +(10,13,17)x^1_{122} \\ & +(6,10,12)x^1_{132} && +(6,8,12)x^1_{212} \\ & +(6,9,10)x^1_{222} && +(10,12,13)x^1_{232} \\ & +(6,8,9)x^2_{112} && +(8,9,11)x^2_{122} \\ & +(8,9,12)x^2_{132} && +(5,8,10)x^2_{212} \\ & +(6,8,9)x^2_{222} && +(10,12,13)x^2_{232}; \end{aligned}$$

The uncertain optimal solution obtained after substituting the solution set in Z_1 is (994.4, 1318.2, 1718.4). Now, adding the fixed charges (given in Table 4) corresponding to the decision variables in the solution set, we get the uncertain optimal value for the uncertain multi-item fixed charge solid transportation problem as (1052.4, 1396.2, 1814.4). As stated earlier in the expected value model, the fixed charges corresponding to the variables x^1_{132}, x^2_{132} and x^1_{222}, x^2_{222} are added only once in the uncertain optimal solution of the uncertain multi-item fixed charge solid transportation problem.

8. Results and Comparison

The optimal solution obtained for the uncertain multi-item fixed charge solid transportation problem using the expected value model is:

$x^1_{131} = 11.75$, $x^1_{221} = 22$, $x^2_{111} = 17.5$,
$x^2_{121} = 1.75$, $x^2_{131} = 10.5$, $x^2_{231} = 10$,
$x^1_{132} = 23.5$, $x^1_{212} = 19.5$, $x^1_{222} = 6$,
$x^2_{222} = 21$.

and the optimal value corresponding to this solution set is $Z^*_{FC} = 1258.812$. On the other hand, the solution obtained for the UMIFCSTP using the dependent chance constrained model is:

$x^1_{131} = 20.4$, $x^1_{211} = 18.4$, $x^1_{221} = 18.2$,
$x^2_{111} = 19.6$, $x^2_{231} = 19.4$, $x^1_{132} = 17$,
$x^1_{212} = 3.2$, $x^1_{222} = 12.2$, $x^2_{122} = 14$,
$x^2_{132} = 3.8$, $x^2_{222} = 10.6$.

and the optimal value in this model is obtained in terms of the uncertain measure which is 0.7762. The uncertain solution is also obtained for the DCCM model by substituting the solution set in the uncertain objective function and it is obtained as (1052.4, 1396.2, 1814.4).

For obtaining the solution, the EVM model assumes the expected values of the uncertain variables in the objective function and the constraints, which leads us to a single optimal value only, whereas the DCCM model is self-flexible and tries to obtain that solution on which the measure value is maximum which gives a better solution compared to the other method. Also, the DCCM gives the optimal value of the problem in the uncertain range whereas in the case of the expected value model it is a fixed optimal value. Since all the parameters are uncertain numbers, the decision maker might be interested in the uncertain solution instead of a fixed one. So, in that case the solution obtained using the DCCM model will preferable and better.

9. Conclusion

This paper presented MIFCSTP in the uncertain environment. With the help of uncertainty theory, two uncertain models, EVM and DCCM, were developed and their equivalent deterministic formulations were obtained. Charnes and Cooper's transformations were further applied on the deterministic form of the DCCM to convert it into linear form. Applying the transformations suggested by Charnes and Cooper on the fractional form of the DCCM converts it into simpler form which can be easily solved to obtain the solution. Both of these models were then solved in the LINGO 18.0 software to obtain the solutions. A numerical example with the illustration of the application of the models is also provided at the end of the paper and the results obtained by both the models were compared with each other.

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