

# Effect of Thermophoresis on MHD Free Convective Heat and Mass Transfer Flow along an Inclined Stretching Sheet under the Influence of Dufour-Soret Effects with Variable Wall Temperature

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## Abstract

In this paper, the effect of thermophoresis on MHD free convective heat and mass transfer flow along an inclined permeable stretching sheet under the influence of Dufour and Soret effects with variable wall temperature and concentration is presented. The governing non-linear partial differential equations are transformed into ordinary ones by using similarity transformation. The resulting similarity equations are solved numerically by applying sixth-order Runge-Kutta method with Nachtsheim-Swigert shooting iteration technique. The numerical results have been analyzed for the effect of different physical parameters such as magnetic field parameter, suction parameter, angle of inclination, wall temperature parameter and thermophoresis parameter to investigate the flow, heat, and mass transfer characteristics. The results show that higher order temperature and concentration indices have more decreasing effect on the hydrodynamic, thermal and concentration boundary layers compared to the zero order (constant plate temperature and concentration) indices. From the numerical computations, the rate of heat transfer is also calculated and presented in tabular form.

**Keywords:** MHD; Inclined stretching sheet; Dufour-Soret effects; Thermophoresis

## 1. Introduction

The study of boundary layer flow on continuous moving surfaces has many practical applications in industrial and technological processes. Aerodynamic extrusion of plastic sheets; cooling of an infinite metallic plate in a cooling path, which may be an electrolyte; crystal growing; the boundary layer along a liquid film in condensation processes; and heat treated material traveling between a feed roll and a wind-up roll are some examples of continuous moving surfaces. Sakiadis [1]

initiated the study of boundary layer flow over a continuous solid surface moving with constant speed. Erickson et al. [2] extended the work of Sakiadis to include blowing or suction at the moving surface and investigated its effects on the heat and mass transfer in the boundary layer. Gupta and Gupta [3] studied the heat and mass transfer characteristics over an isothermal stretching sheet with suction or blowing with the help of similarity solutions. Chen and Char [4] studied the heat transfer of a continuous stretching surface with suction or blowing.

Anderson *et al.* [5] studied the diffusion of a chemically reactive species from a linearly stretching sheet. Heat and mass transfer over an accelerating surface with heat source in the presence of suction and blowing is studied by Acharya *et al.* [6]. Recently, Abo-Eldahad and El -Aziz [7] studied the blowing/suction effect on hydromagnetic heat transfer by mixed convection from an inclined continuously stretching surface with internal heat generation/absorption.

In the previous papers, the diffusion-thermo (Dufour) and thermal-diffusion (Soret) terms have been neglected from the energy and concentration equations respectively. But when heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of more intricate nature. It has been found that an energy flux can be generated not only by temperature gradients but by composition gradients as well. The energy flux caused by a composition gradient is called the Dufour or diffusion-thermo effect.

On the other hand, mass fluxes can also be created by temperature gradients and this is the Soret or thermal-diffusion effect. In general, the thermal-diffusion and diffusion-thermo effects are of a smaller order of magnitude than the effects described by Fourier's or Fick's law and are often neglected in heat and mass transfer processes. However, exceptions are observed therein. The thermal-diffusion (Soret) effect, for instance, has been utilized for isotope separation, and in mixture between gases with very light molecular weight ( $H_2$ , He) and of medium molecular weight ( $N_2$ , air) the diffusion-thermo (Dufour) effect was found to be of a considerable magnitude such that it cannot be ignored (Eckert and Drake [8]). In view of the importance of these above mentioned effects, Dursunkaya and Worek [9] studied diffusion-thermo and thermal-diffusion effects in transient and steady natural convection from a vertical surface whereas Kafoussias and Williams [10]

studied the same effects on mixed free-forced convective and mass transfer boundary layer flow with temperature dependent viscosity. Anghel *et al.* [11] investigated the Dufour and Soret effects on a free convection boundary layer over a vertical surface embedded in a porous medium. Eldabe *et al.* [12] investigated the thermal-diffusion and diffusion-thermo effects on mixed free-forced convection and mass transfer boundary layer flow for non-Newtonian fluid with temperature dependent viscosity. Salem [13] analyzed thermal-diffusion and diffusion-thermo effects on convective heat and mass transfer in a visco-elastic fluid flow through a porous medium over a stretching sheet. Alam *et al.* [14] investigated the Dufour and Soret effects on unsteady MHD free convection and mass transfer flow past a vertical porous plate in a porous medium. Postelnicu [15] studied the influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects.

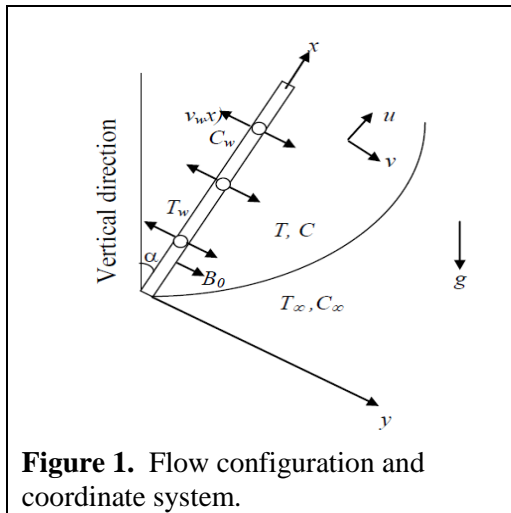
However, studies in small particle (such as dust or aerosol etc.) deposition due to thermophoresis, in the presence of large temperature gradients, have gained importance in many engineering applications over the last few decades. Thermophoresis has many engineering applications in removing small particles from gas streams, in determining exhaust gas particle trajectories from combustion devices, and in studying the particulate material deposition on turbine blades. Thermophoresis is also important in thermal precipitators, which are sometimes more effective than electrostatic precipitators in removing submicron-sized particles from gas streams. Since industrial air pollution is of great concern in the world, this phenomenon can be utilized to control air pollution by removing small particles from gas streams and other flue gases. This phenomenon commonly contributes significantly to the atmospheric and environmental sciences, aerosol science and

technology. Thermophoresis can also be used for the production of fine ceramic powders like aluminum nitride in the high temperature aerosol flow reactors. In aerosol flow reactors, the thermophoretic depositions are important since it is desired to decrease the deposition during the process in order to increase product yield. Thermophoretic deposition of radioactive particles is one of the major factors causing accidents in nuclear reactors. Thermophoresis is considered to be the dominant mass transfer mechanism in the modified chemical vapor deposition (MCVD) processes as currently used in the manufacturing of graded index optical fiber preforms (i. e. the production of optical fiber preforms by using MCVD). In optical fiber process, high deposition levels are desired since the goal is to coat the interior of the tube with particles. The fabrication of high yield processors is highly dependent on thermophoresis because of the repulsion and/or deposition of impurities on the wafer as it heats up during fabrication. In light of various applications of thermophoresis, Chiou [16] studied the particle deposition from natural convection boundary layer flow onto an isothermal vertical cylinder. Chamkha and Pop [17] investigated the effect of thermophoresis particle deposition in free convection boundary layer flow from a vertical flat plate embedded in a porous medium. Thermophoretic deposition of aerosol particles in laminar tube flow with mixed convection is studied by Walsh *et al.* [18]. El-Kabeir *et al.* [19] studied the combined heat and mass transfer on non-Darcy natural convection in a fluid saturated porous medium with thermophoresis. Alam *et al.* [20] studied the effects of variable suction and thermophoresis on steady MHD free-forced convective heat and mass transfer flow over a semi-infinite permeable inclined flat plate in the presence of thermal radiation. As per author's knowledge, the literature review revealed that hydromagnetic natural convective heat and mass transfer flow in an inclined stretching sheet with

thermophoresis in the presence of variable wall temperature and concentration considering Soret-Dufour effects has not been studied yet. Therefore, the purpose of the present paper is to investigate the effect of thermophoresis on MHD free convective flow with heat and mass transfer over an inclined permeable stretching sheet under the influence of Dufour and Soret effects with variable wall temperature and concentration.

## 2. Mathematical Modeling

We consider a steady two-dimensional laminar MHD free convective heat and mass transfer flow of a viscous and incompressible fluid along a linearly stretching semi-infinite sheet that is inclined from the vertical with an acute angle  $\alpha$ . The surface is assumed to be permeable and moving with velocity,  $u_w(x) = bx$  (where  $b$  is a constant called stretching rate). Fluid suction/injection is imposed at the stretching surface. The  $x$ -axis runs along the stretching surface in the direction of motion with the slot as the origin and the  $y$ -axis is measured normally from the sheet to the fluid. A magnetic field of uniform strength  $B_0$  is applied to the sheet in the  $y$ -direction, which produces magnetic effect in the  $x$ -direction. We further assume that (a) due to the boundary layer behavior the temperature gradient in the  $y$ -direction is much larger than that in the  $x$ -direction and hence only the thermophoretic velocity component which is normal to the surface is of importance, (b) the fluid has constant kinematic viscosity and thermal diffusivity, and that the Boussinesq approximation may be adopted for steady laminar flow, and (c) the magnetic Reynolds number is small so that the induced magnetic field can be neglected. The flow configuration and coordinate system are shown in Figure 1.



**Figure 1.** Flow configuration and coordinate system.

Under the above assumptions the governing equations describing the conservation of mass, momentum, energy and concentration respectively are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g \beta (T - T_\infty) \cos \alpha + g \beta^* (C - C_\infty) \cos \alpha - \frac{\sigma B_0^2}{\rho} u \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\lambda_g}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{D_m k_T}{c_s c_p} \frac{\partial^2 C}{\partial y^2} \tag{3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} + \frac{D_m k_T}{T_m} \frac{\partial^2 T}{\partial y^2} \tag{4}$$

$$-\frac{\partial}{\partial y} [V_T (C - C_\infty)]$$

where the the thermophoretic deposition velocity in the  $y$ -direction is given by

$$V_T = -k\nu \frac{\nabla T}{T_{ref}} = -\frac{k\nu}{T_{ref}} \frac{\partial T}{\partial y} \tag{5}$$

where  $k$  is the thermophoretic coefficient and  $T_{ref}$  is some reference temperature.

The boundary conditions for the above model are as follows:

$$u = u_w(x) = bx, v = \pm v_w(x), T_w - T_\infty = A_1 x^n, C_w - C_\infty = A_2 x^n \quad \text{at } y = 0 \tag{6a}$$

$$u = 0, T = T_\infty, C = C_\infty \text{ as } y \rightarrow \infty, \tag{6b}$$

where  $b$  is a constant called stretching rate;  $A_1, A_2$  are proportionality constants, and  $v_w(x)$  represents the permeability of the porous surface where its sign indicates suction ( $< 0$ ) or injection ( $> 0$ ). Here  $n$  is the temperature parameter and for  $n = 0$ , the thermal boundary conditions become isothermal.

### 3. Dimensional analysis

Dimensional analysis is one of the most important mathematical tools in the study of fluid mechanics. To describe several transport mechanisms in fluid dynamics, it is meaningful to express the conservation equations in non-dimensional form. The advantages of non-dimensionalization are as follows: (i) one can analyze any system irrespective of its material properties, (ii) one can easily understand the controlling flow parameters of the system, (iii) one can make a generalization of the size and shape of the geometry, and (iv) before doing experiments one can get insight into the physical problem. These aims can be achieved through the appropriate choice of scales. Therefore, in order to obtain the dimensionless form of the governing equations (1)-(4) together with the boundary conditions (6) we introduce the following non-dimensional variables:

$$\left. \begin{aligned} u &= \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}, \psi = (\nu b)^{1/2} x f(\eta), \\ \eta &= (b/\nu)^{1/2} y, \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \\ \phi(\eta) &= \frac{C - C_\infty}{C_w - C_\infty}. \end{aligned} \right\} \tag{7}$$

Now employing (7) in equations (1)-(4), we obtain the following nonlinear ordinary differential equations:

$$f''' + ff'' - (f')^2 + g_s \theta \cos \alpha + g_c \phi \cos \alpha - Mf' = 0 \tag{8}$$

$$\theta'' - n \text{Pr} f' \theta + \text{Pr} f \theta' + \text{Pr} Df \phi'' = 0 \tag{9}$$

$$\phi'' - n \text{Sc} f' \phi + \text{Sc} (f + \tau \theta') \phi' + \text{Sc} S_0 \theta'' + \text{Sc} \tau \phi \theta'' = 0 \tag{10}$$

The boundary conditions (6) then turn into  $f = f_w, f' = 1, \theta = 1, \varphi = 1$  at  $\eta = 0$  (11a)

$$f' = 0, \theta = 0, \varphi = 0 \quad \text{as } \eta \rightarrow \infty \quad (11b)$$

where  $f_w = -v_w / (b\nu)^{1/2}$  is the dimensionless wall mass transfer coefficient such that  $f_w > 0$  indicates wall suction and  $f_w < 0$  indicates wall injection.

The dimensionless parameters introduced in the above equations are defined as follows:

$M = \frac{\sigma B_0^2 x}{\rho u_w(x)}$  is the local magnetic field

parameter,  $Gr = \frac{g\beta(T_w - T_\infty)x^3}{\nu^2}$  is the local

Grashof number,  $Gm = \frac{g\beta^*(C_w - C_\infty)x^3}{\nu^2}$  is

the local modified Grashof number,

$Re_x = \frac{u_w(x)x}{\nu}$  is the local Reynolds number,

$g_s = \frac{Gr}{Re_x^2}$  is the temperature buoyancy

parameter,  $g_c = \frac{Gm}{Re_x^2}$  is the mass buoyancy

parameter,  $Pr = \frac{\nu\rho c_p}{\lambda_g}$  is the Prandtl number,

$Df = \frac{D_m k_T (C_w - C_\infty)}{\nu c_s c_p (T_w - T_\infty)}$  is the Dufour number,

$S_0 = \frac{D_m (T_w - T_\infty)}{\nu (C_w - C_\infty) T_m}$  is the Soret number,

$Sc = \frac{\nu}{D_m}$  is the Schmidt number and

$\tau = \frac{k(T_w - T_\infty)}{T_{ref}}$  is the thermophoretic

parameter.

The parameter of engineering interest for the present problem is the local Nusselt number

$(Nu)$  which is obtained from the following expression:

$$Nu = -Re_x^{\frac{1}{2}} \theta'(0) \quad (12)$$

### 4. Numerical method validation

The transformed set of non-linear ordinary differential equations (8)-(10) together with boundary conditions (11) have been solved numerically by applying Nachtsheim-Swigert [21] shooting iteration technique along with sixth order Runge-Kutta integration scheme. A step size of  $\Delta\eta = 0.01$  was selected to be satisfactory for a convergence criterion of  $10^{-6}$  in all cases. In order to see the accuracy of the present numerical method, we have compared our results with those with Tsai [22]. Thus, Table 1 presents a comparison of the local Stanton number obtained in the present work and those obtained by Tsai [22]. It is clearly observed that very good agreement between the results exists. This lends confidence in the present numerical method.

**Table 1.** Comparison of the local Stanton number obtained in the present work and those obtained by Tsai [22] for  $Sc=1000$ ,  $Pr=0.70$ ,  $\alpha=90^\circ$ ,  $n=1$  and  $g_s=g_c=S_0=Df=0$ .

$\tau$	$f_w$	Tsai [22]	Present study
0.10	1.0	0.7346	0.7273
0.10	0.5	0.3810	0.3724
0.10	0.0	0.0275	0.0273
1.00	1.0	0.9134	0.8925
1.00	0.5	0.5598	0.5580
1.00	0.0	0.2063	0.2060

### 5. Results and Discussion

The results of the numerical computations are displayed graphically in Figures 2-7 and in Tables 2-7 for prescribed surface temperature. Results are obtained for  $Pr = 0.70$  (air),  $Sc = 0.22$  (hydrogen),  $g_s=10$ ;  $g_c = 4$  (due to free convection problem) and

various values of the magnetic field parameter  $M$ , suction parameter  $f_w$ , angle of inclination  $\alpha$  to vertical, surface temperature parameter  $n$ , Dufour number  $Df$ , Soret number  $So$  and thermophoretic parameter  $\tau$ .

Figures 2(a)-(c) represent, respectively, the dimensionless velocity, temperature and concentration for various values of the magnetic field parameter ( $M$ ). The presence of a magnetic field normal to the flow in an electrically conducting fluid produces a Lorentz force, which acts against the flow. This resistive force tends to slow down the flow and hence the fluid velocity decreases with the increase of the magnetic field parameter as observed in Figure 2(a). From Figure 2(b) we see that the temperature profiles increase with the increase of the magnetic field parameter, which implies that the applied magnetic field tends to heat the fluid, and thus reduces the heat transfer from the wall. In Figure 2(c), the effect of an applied magnetic field is found to increase the concentration profiles, and hence increase the concentration boundary layer.

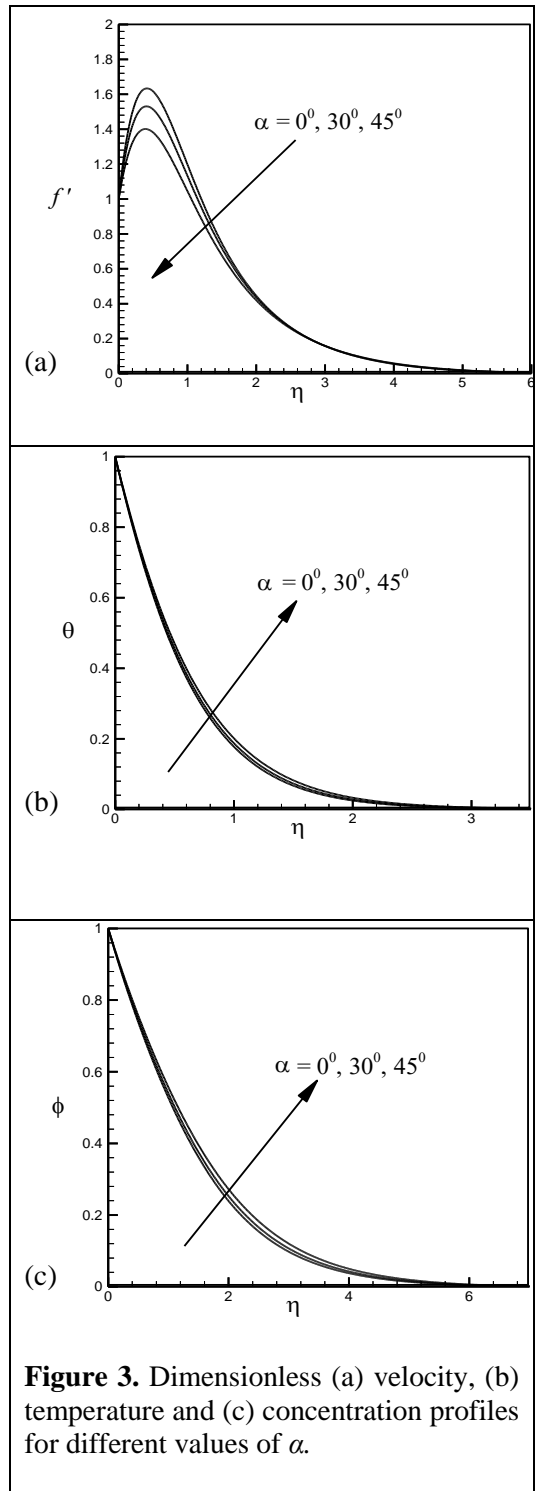
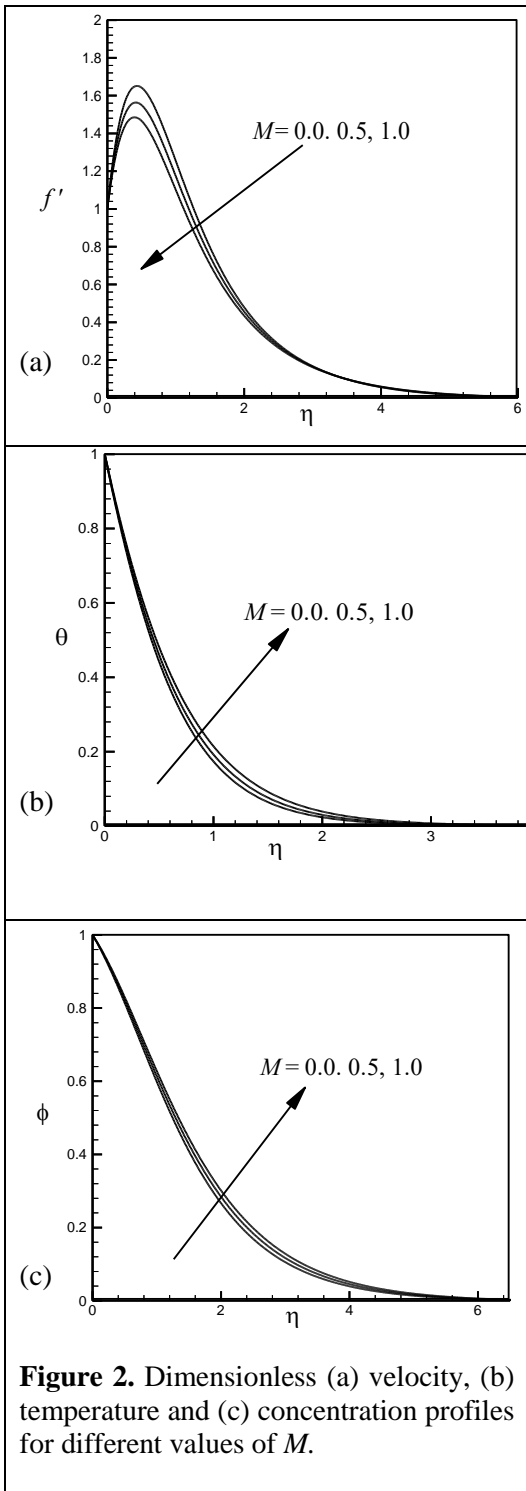
Representative velocity profiles for three typical angles of inclination ( $\alpha = 0^\circ, 30^\circ$  and  $45^\circ$ ) are presented in Figure 3(a). It is revealed from Figure 3(a) that increasing the angle of inclination decreases the velocity. The fact is that, as the angle of inclination increases, the effect of the buoyancy force due to thermal diffusion decreases by a factor of  $\cos\alpha$ . Consequently the driving force to the fluid decreases; as a result velocity profiles decrease. From Figures 3 (b)-(c) we also observe that both the thermal and concentration boundary layer thickness increase as the angle of inclination increases.

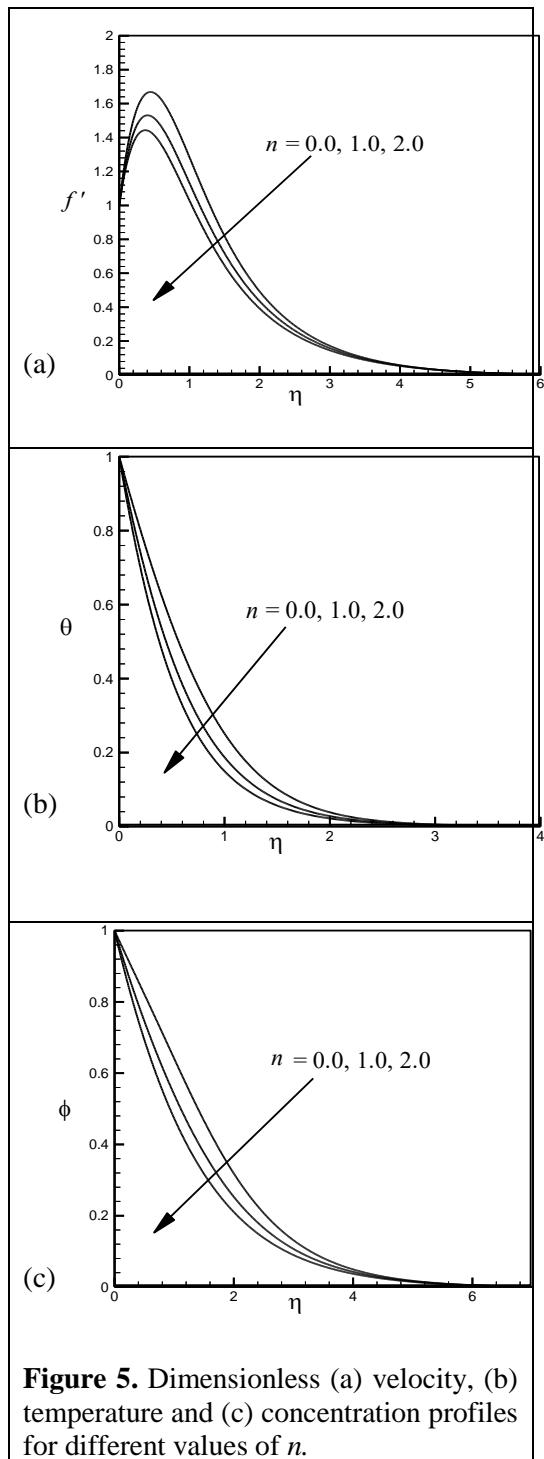
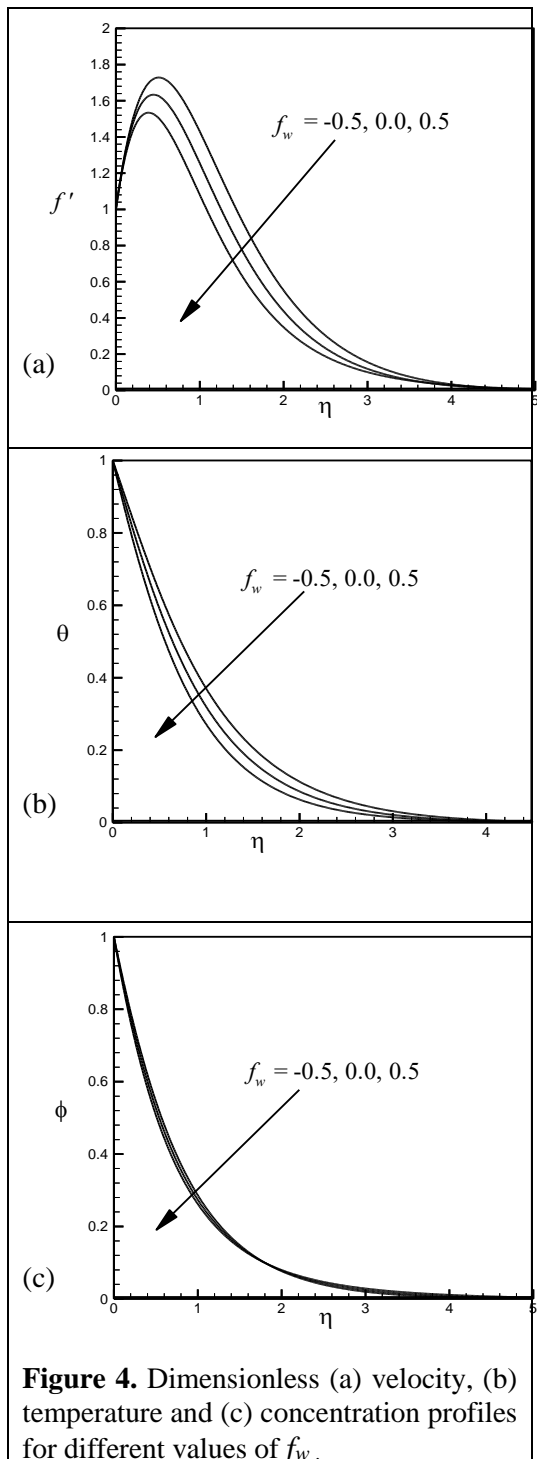
Figures 4(a)-(c) depict the influence of the suction/injection parameter  $f_w$  on the velocity, temperature and concentration profiles in the boundary layer, respectively. It is known that the imposition of wall suction ( $f_w > 0$ ) has the tendency to reduce all the momentum, thermal as well as

concentration boundary layer thickness. This causes reduction in all the velocity, temperature and concentration profiles. The opposite effect is found for the case of injection ( $f_w < 0$ ).

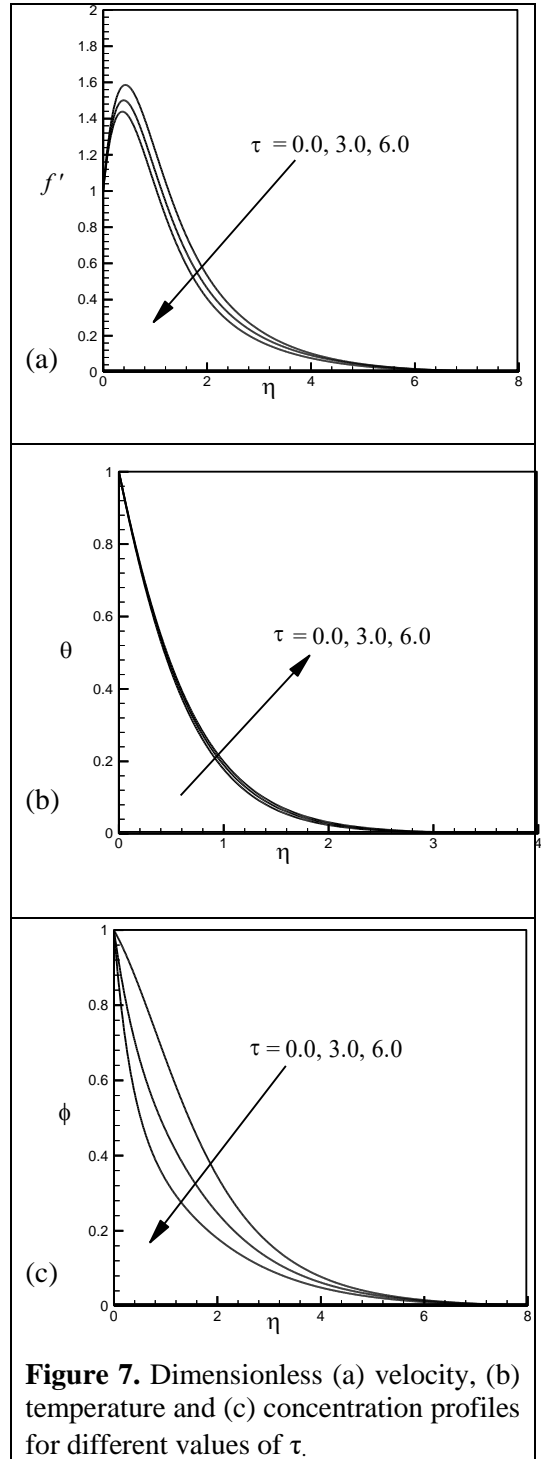
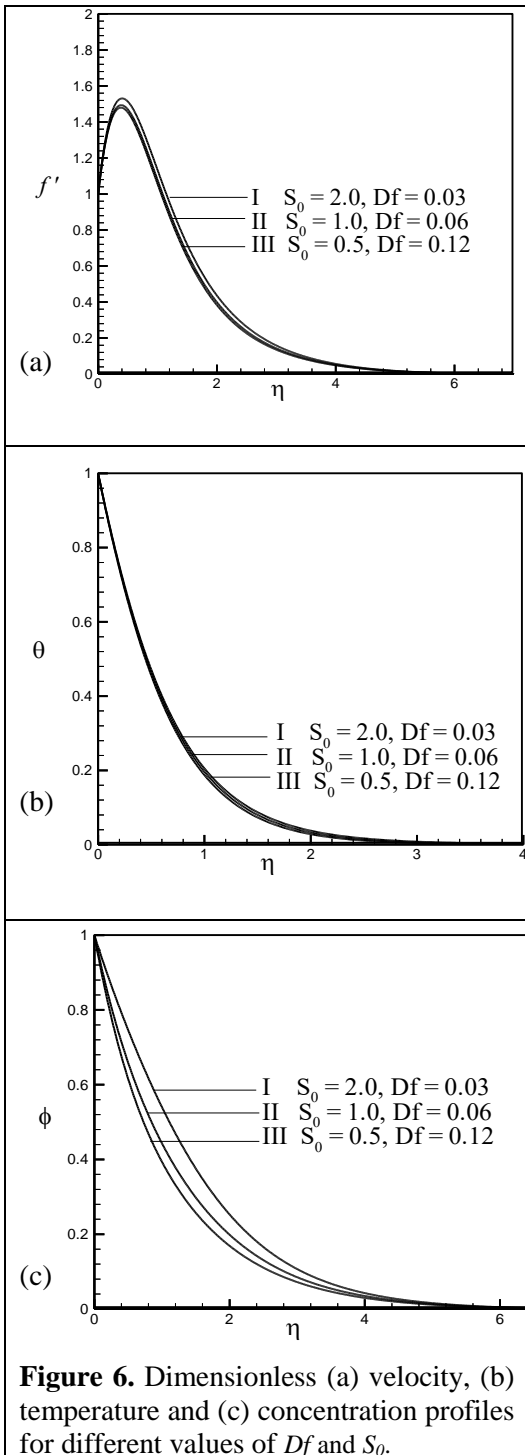
The effects of the surface temperature parameter  $n$  on the dimensionless velocity, temperature and concentration profiles are displayed in Figures 5(a)-(c), respectively. From Figure 5(a) it is seen that, the velocity gradient at the wall increases and hence the momentum boundary layer thickness decreases as  $n$  increases. Furthermore, from Figure 5(b) we can see that as  $n$  increases, the thermal boundary layer thickness decreases and the temperature gradient at the wall increases. This means a higher value of the heat transfer rate is associated with higher values of  $n$ . We also observe from Figure 5(c) that the concentration boundary layer thickness decreases as the exponent  $n$  increases.

The influence of Soret number  $So$  and Dufour number  $Df$  on the velocity field are shown in Figure 6(a). Quantitatively, when  $\eta = 1.0$  and  $So$  decreases from 2.0 to 1.0 (or  $Df$  increases from 0.03 to 0.06), there is 4.09% decrease in the velocity value whereas the corresponding decrease is 2.05% when  $So$  decreases from 1.0 to 0.5 (or  $Df$  increases from 0.06 to 0.12). From Figure 6(b), when  $\eta = 1.0$  and  $So$  decreases from 2.0 to 1.0 (or  $Df$  increases from 0.03 to 0.06), there is 4.97% increase in the temperature, whereas the corresponding increase is 4.47% when  $So$  decreases from 1.0 to 0.5. In Figure 6(c), when  $\eta = 1.0$  and  $So$  decreases from 2.0 to 1.0 (or  $Df$  increases from 0.03 to 0.06), there is 17.95% decrease in the concentration, whereas the corresponding decrease is 11.15% when  $So$  decreases from 1.0 to 0.5.









**Table 2.** Effects of  $n$ ,  $S_o$  and  $Df$  on local Nusselt number ( $Nu$ ) for  $g_s = 10$ ,  $g_c = 4$ ,  $Pr = 0.70$ ,  $Sc = 0.22$ ,  $\tau = 1$ ,  $M = 0.50$ ,  $f_w = 0.50$  and  $\alpha = 30^0$ .

$n$	$S_o$	$Df$	$Nu$
0	2.0	0.03	1.0328385
0	1.0	0.06	1.0184757
0	0.5	0.12	1.0043688
1	2.0	0.03	1.4236964
1	1.0	0.06	1.4047218
1	0.5	0.12	1.3856793
2	2.0	0.03	1.7119614
2	1.0	0.06	1.6902783
2	0.5	0.12	1.6680382

**Table 3.** Effects of  $n$  and  $M$  on local Nusselt number ( $Nu$ ) for  $g_s = 10$ ,  $g_c = 4$ ,  $Pr = 0.70$ ,  $Sc = 0.22$ ,  $\tau = 1$ ,  $S_o = 2.0$ ,  $Df = 0.03$ ,  $f_w = 0.50$  and  $\alpha = 30^0$ .

$n$	$M$	$Nu$
0	0.0	1.0529847
0	0.5	1.0328385
0	1.0	1.0137268
1	0.0	1.1451303
1	0.5	1.4236964
1	1.0	1.3975979
2	0.0	1.7436518
2	0.5	1.7119614
2	1.0	1.6821309

**Table 4.** Effects of  $n$  and  $\alpha$  on local Nusselt number ( $Nu$ ) for  $g_s = 10$ ,  $g_c = 4$ ,  $Pr = 0.70$ ,  $Sc = 0.22$ ,  $\tau = 1$ ,  $S_o = 2.0$ ,  $Df = 0.03$ ,  $f_w = 0.50$  and  $M = 0.50$

$n$	$\alpha$	$Nu$
0	$0^0$	1.0565044
0	$30^0$	1.0328385
0	$60^0$	0.9516741
1	$0^0$	1.4557354
1	$30^0$	1.4236964
1	$60^0$	1.3142996
2	$0^0$	1.7484968
2	$30^0$	1.7119614
2	$60^0$	1.5879576

**Table 5.** Effects of  $n$  and  $f_w$  on local Nusselt number ( $Nu$ ) for  $g_s = 10$ ,  $g_c = 4$ ,  $Pr = 0.70$ ,  $Sc = 0.22$ ,  $\tau = 1$ ,  $S_o = 2.0$ ,  $Df = 0.03$ ,  $M = 0.50$  and  $\alpha = 30^0$ .

$n$	$f_w$	$Nu$
0	0	0.8195905
0	2	1.7934338
0	4	2.9959838
1	0	1.2375598
1	2	2.0849775
1	4	3.1712275
2	0	1.5362973
2	2	2.3229164
2	4	3.3306333

**Table 6.** Effects of  $n$  and  $\tau$  on local Nusselt number ( $Nu$ ) for  $g_s = 10$ ,  $g_c = 4$ ,  $Pr = 0.70$ ,  $Sc = 0.22$ ,  $M = 0.50$ ,  $S_o = 2.0$ ,  $Df = 0.03$ ,  $f_w = 0.50$  and  $\alpha = 30^0$ .

$n$	$\tau$	$Nu$
0	0	1.0419615
0	3	1.0185657
0	6	1.0051294
1	0	1.4354632
1	3	1.4047350
1	6	1.3858538
2	0	1.7251409
2	3	1.6905053
2	6	1.6693653

**Table 7.** Effects of  $\tau$ ,  $S_o$  and  $Df$  on local Nusselt number ( $Nu$ ) for  $g_s = 10$ ,  $g_c = 4$ ,  $Pr = 0.70$ ,  $Sc = 0.22$ ,  $n = 1$ ,  $M = 0.50$ ,  $f_w = 0.50$  and  $\alpha = 30^0$ .

$\tau$	$S_o$	$Df$	$Nu$
0	0.0	0.00	1.4087188
3	0.0	0.00	1.3792736
6	0.0	0.00	1.3560627
0	1.0	0.06	1.4169181
3	1.0	0.06	1.2874784
6	1.0	0.06	1.3754719
0	2.0	0.03	1.4354632
3	2.0	0.03	1.4047350
6	2.0	0.03	1.3858538

The effects of thermophoretic parameter  $\tau$  on the velocity, temperature and concentration distributions are displayed in Figures 7(a)-(c), respectively. It is observed from these figures that an increase in the thermophoretic parameter  $\tau$  leads to decrease in the velocity across the boundary layer. This is accompanied by a decrease in the concentration and a slight increase in the fluid temperature. This means that the effect of increasing  $\tau$  is limited to increasing the wall slope of the convection profile without any significant effect on the concentration boundary layer.

Finally, the effects of surface temperature parameter, Soret number, Dufour number, magnetic field parameter, angle of inclination to vertical, suction parameter, and thermophoretic parameter on the Nusselt number are shown in Tables 2-7. The behavior of these parameters is self-evident from Tables 2-7 and hence they will not be discussed any further due to brevity.

**5. Conclusions**

In this paper, the effect of thermophoresis on hydromagnetic buoyancy-induced natural convection flow of a viscous, incompressible, electrically-conducting fluid along an inclined permeable surface with variable wall temperature and concentration has been investigated numerically. The governing equations are developed and transformed using appropriate similarity transformations. The transformed similarity equations are then solved numerically by applying the shooting method. From the present numerical investigations the following conclusions may be drawn:

1. The fluid velocity inside the boundary layer decreases with the increasing values of the magnetic field parameter, suction parameter, angle of inclination, and the thermophoretic parameter.

2. The temperature distribution increases with the increasing values of the magnetic field parameter, angle of inclination, and the thermophoretic parameter, whereas it decreases with an increasing value of the suction parameter.
3. The concentration profile increases with an increasing value of the magnetic field parameter and angle of inclination, whereas it decreases with the increasing values of the suction parameter and the thermophoretic parameter.
4. Higher order temperature and concentration indices have more decreasing effect on the hydrodynamic, thermal and concentration boundary layers compared to the zero order (constant plate temperature and concentration) index.
5. Dufour and Soret parameters have significant effects on the heat and mass transfer flow of a hydrogen-air mixture fluid.

**6. Nomenclature**

$B_0$	Magnetic induction
$C$	Concentration
$c_p$	Specific heat at constant pressure
$D_m$	Mass diffusivity
$f$	Dimensionless stream function
$f_w$	Dimensionless wall suction/injection
$g$	Acceleration due to gravity
$Gr_x$	Local Grashof number
$Gm_x$	Local modified Grashof number
$g_s$	Temperature buoyancy parameter
$g_c$	Mass buoyancy parameter
$M$	Magnetic field parameter
$Nu$	Local Nusselt number
$Pr$	Prandtl number
$Sc$	Schmidt number
$T$	Temperature
$u, v$	Velocity components in the $x$ - and $y$ -direction respectively
$x, y$	Axis in direction along and normal to the plate

$\eta$	Pseudo-similarity variable
$\alpha$	Angle of inclination to the vertical
$\beta$	Coefficient of thermal expansion
$\beta^*$	Coefficient of concentration expansion
$\sigma$	Electrical conductivity
$\rho$	Density of the fluid
$\nu$	Kinematic viscosity
$\lambda_g$	Thermal conductivity of fluid
$\tau$	Thermophoretic parameter
$\theta$	Dimensionless temperature
$\phi$	Dimensionless concentration
<b>Subscripts</b>	
$w$	Condition at wall
$\infty$	Condition at infinity

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## 8. References

- [1] Sakiadis, B. C, Boundary Layer Behavior on Continuous Solid Surface: I. Boundary-layer Equations for Two-Dimensional and Axisymmetric Flow, *AIChE J.*, vol.7, pp.26-28, 1961.
- [2] Erickson, L. E., Fan L. T. and Fox V. G., Heat and Mass Transfer on a Moving Continuous Flat Plate with Suction or Injection, *Ind. Eng. Chem. Fundam.*, Vol. 5, pp.19-25, 1966.
- [3] Gupta P. S. and Gupta A. S. , Heat and Mass Transfer on a Stretching Sheet with Suction or Blowing, *Can. J. Chem. Engng.*, Vol. 55, pp. 744-746, 1977.
- [4] Chen C. K. and Char M., Heat Transfer of a Continuous Stretching Surface with Suction or Blowing, *J. Math. Anal. Appl.*, Vol. 135, pp.568-580, 1988.
- [5] Anderson H. I., Hansen O. R. and Holmedal B., Diffusion of a Chemically Reactive Species from a Stretching Sheet, *Int. J. Heat and Mass Transfer*, Vol. 37, pp.659-664, 1994.
- [6] Acharya M., Singh L. P. and Dash G. C., Heat and Mass Transfer over an Accelerating Surface with Heat Source in the Presence of Suction and Blowing, *Int. J. Engng. Sci.*, Vol. 37, pp. 189-211, 1999.
- [7] Abo-Eldahad, E. M. and El -Aziz, M. A, Blowing/Suction Effect on Hydromagnetic Heat Transfer by Mixed Convection from an Inclined Continuously Stretching Surface with Internal Heat Generation/Absorption, *Int. J. Thermal Sciences*, Vol. 43, pp.709-719, 2004.
- [8] Eckert, E. R. G. and Drake, R. M., *Analysis of Heat and Mass Transfer*, McGraw-Hill, New York, 1972.
- [9] Dursunkaya, Z. and Worek, W. M., Diffusion-thermo and Thermal-Diffusion Effects in Transient and Steady Natural Convection from Vertical Surface, *Int. J. Heat Mass Transfer*, Vol. 35, pp. 2060-2065, 1992.
- [10] Kafoussias, N. G. and Williams, E. M., Thermal-diffusion and Diffusion-Thermo Effects on Mixed Free-forced Convective and Mass Transfer Boundary Layer Flow with Temperature Dependent Viscosity, *Int. J. Engng. Sci.* Vol. 13, pp.1369-1384, 1995.
- [11] Anghel, M., Takhar, H. S. and I. Pop, Dufour and Soret Effects on Free-Convection Boundary Layer over a Vertical Surface Embedded in a Porous Medium, *Studia Universitatis*

- Babes-Bolyai, *Mathematica*, Vol. XLV, pp.11-21, 2000.
- [12] Eldabe N. T., El-Saka A. G. and Fouad A., Thermal-diffusion and Diffusion-Thermo Effects on Mixed Free-forced Convection and Mass Transfer Boundary Layer Flow for Non-Newtonian Fluid with Temperature Dependent Viscosity, *Appl. Math. and Compt.*, Vol. 152, pp. 867-883, 2004.
- [13] Alam M. S., Rahman M. M. and Samad M. A., Dufour and Soret Effects on Unsteady MHD Free Convection and Mass Transfer Flow Past a Vertical Porous Plate in a Porous Medium, *Non-Linear Analysis: Modeling and Control*, Vol. 11(3), pp.217-226, 2006.
- [14] Salem, A. M., Thermal-diffusion and Diffusion-thermo Effects on Convective Heat and Mass Transfer in a Visco-elastic Fluid Flow through a Porous Medium over a Stretching Sheet, *Commun. in Numer. Meth in Engng.*, Vol. 22, pp.955-966, 2006.
- [15] Postelnicu A., Influence of Chemical Reaction on Heat and Mass Transfer by Natural Convection from Vertical Surfaces in Porous Media Considering Soret and Dufour Effects, *Heat and Mass Transfer*, Vol. 43, pp. 595-602, 2007.
- [16] Chiou M. C, Particle Deposition from Natural Convection Boundary Layer Flow onto an Isothermal Vertical Cylinder, *Acta Mechanica*, Vol. 129, pp.163-178, 1998.
- [17] Chamkha A. J. and Pop I., Effect of Thermophoresis Particle Deposition in Free Convection Boundary Layer Flow from a Vertical Flat Plate Embedded in a Porous Medium, *Int. Cumm. In Heat and mass transfer*, Vol. 31, pp.421-430, 2004.
- [18] Walsh J. K., Weimer A. W. and Hrenya, Thermophoretic Deposition of Aerosol Particles in Laminar Tube Flow with Mixed Convection, *J. Aerosol Sci.*, Vol.37, pp.715-734, 2006.
- [19] El- Kabeir S. M. M., El-Hakim M. A. and Rashad M. A., Combined Heat and Mass Transfer on Non-Darcy Natural Convection in a Fluid Saturated Porous Medium with Thermophoresis, *Int. J. Appl. Mech. Eng.*, Vol.12, pp.9-18, 2007.
- [20] Alam, M. S., Rahman, M. M. and Sattar, M. A., Effects of Variable Suction and Thermophoresis on Steady MHD Free-forced Convective Heat and Mass Transfer Flow over a Semi-infinite Permeable Inclined Flat Plate in the Presence of Thermal Radiation, *Int. J. Thermal Sciences*, Vol. 47(6), pp. 758-765, 2008.
- [21] Nachtsheim, P. R and Swigert, P., Satisfaction of the Asymptotic Boundary Conditions in Numerical Solution of the System of Non-linear Equations of Boundary Layer Type, *NASA TND-3004*, 1965.
- [22] Tsai, R., A Simple Approach for Evaluating the Effect of Wall Suction and Thermophoresis on Aerosol Particle Deposition from a Laminar Flow over a Flat Plate, *Int. Comm. Heat Mass Transfer*, Vol. 26, pp. 249–257, 1999.