Soret and Dufour effects on steady free convective MHD viscoelastic fluid flow confined between a long vertical wavy wall and parallel flat wall

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Abstract

The present paper analyzes the Soret and Dufour effects along with viscoelastic effects on a two dimensional steady free convective MHD flow of a slow and slowly varying viscoelastic incompressible fluid between a long vertical wavy wall and a parallel flat wall. A uniform magnetic field is assumed to be applied perpendicular to the flat wall. The governing equations of the fluid and the heat transfer are solved subject to relevant boundary conditions. It is assumed that the fluid consists of two parts: a mean part and a perturbed part. To obtain the perturbed part of the solution, we perform a long wave approximation. The perturbed part of the solution is the contribution from the waviness of the wall. Expressions for the zeroth-order and first-order velocity, temperature, skin friction and heat transfer at the wall are obtained. The profiles of the velocity components are presented graphically for different combinations of parameters involved in the problem to observe the effects of the viscoelastic parameter on the governing flow taking into considerations of Soret and Dufour effects.

Keywords: MHD; Soret and Dufour effects; viscoelastic parameter; skin-friction.

1. Introduction

The problem of viscous fluid flow over a wavy wall is of interest because of its application in different areas such as cooling of re-entry vehicles and rocket boosters, cross-hatching on ablative surfaces and film vaporization in combustion chambers. Benjamin [1] was probably the first to consider the problem of the flow over a wavy wall. His analysis was based on the assumption of parallel flow in absence of waviness. The steady streaming generated by an oscillatory viscous flow over a wavy wall under the assumption that the amplitude of the wave is smaller than the Stokes boundary layer thickness was studied by Lyne [2]. Lekoudis, Nayfeh and Saric [3] made a linear analysis of compressible boundary layer flows over a wavy wall. Sankar and Sinha [4] studied in detail the Rayleigh problem for a wavy wall. They found that at low Reynolds number, the waviness of the wall quickly ceases to be of importance as the liquid is dragged along by the wall, while at large Reynolds number the effects of viscosity are confined to a thin layer close to the wall. The effect of small amplitude wall waviness upon the stability of the boundary layer was studied by Lessen and Gangwani [5]. An analysis of the free convective heat transfer in a viscous incompressible fluid between a long vertical

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wavy wall and a parallel flat wall was made by Vajravelu and Sastri [6]. Das and Ahmed [7] have extended this problem by including the effects of transverse magnetic field. Choudhury and Das [8] studied the problem for non-Newtonian fluid. Aziz *et al.* [9] analyzed the effects of suction and injection on the free convective steady flow and heat transfer in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall of equal transpiration. The free convection of a viscous incompressible fluid in porous medium between two long vertical wavy walls was investigated by Patidar and Purohit [10].

Energy flux can be generated by a temperature gradient and a concentration gradient when heat and mass transfer occur simultaneously. The Soret effect (thermal diffusion) is an occurrence of a diffusion flux due to a temperature gradient while the Dufour effect (diffusion-thermo) is an occurrence of a heat flux due to a concentration gradient. These effects are very significant when temperature and concentration gradients are very large. In many studies, the thermal diffusion (Dufour) and the diffusion-thermo (Soret) effects are of a smaller order of' magnitude than the effects described by Fourier's or Fick's laws and are often neglected in heat and mass-transfer processes. The effects of diffusion-thermo and thermal-diffusion on the transport of heat and mass were developed from the kinetic theory of gases by Chapman and Cowling [11] in 1952. Eckert and Drake [12] found that the diffusion-thermo effect cannot be neglected in concerning isotope separation and in mixtures between gases with very light molecular weight (H_2, He) and for medium molecular weight (N_2, air).

Alam*et al.* [13] investigated the Dufour and Soret effects on steady combined free-forced convective and mass transfer flow past a semi-infinite vertical flat plate in the presence of a uniform transverse magnetic field.

The problem of thermal diffusion and diffusion-thermo in non-Newtonian fluid have great importance in engineering applications like the thermal design of industrial equipment dealing with molten plastics, polymeric liquids, foodstuffs, or slurries. Several authors [14]-[17] have studied this problem.

The viscoelastic parameter can affect the flow and consequently the properties and quality of the final product. This fact motivates the present study to provide an investigation of the effects of viscoelastic parameter taking into account the Soret and Dufour effects. In the present study, free convective MHD flow, heat and mass transfer of a slow and slowly varying viscous incompressible non-Newtonian second-order fluid through a channel bounded by a long vertical wavy wall and a flat wall is considered.

2. Basic equations and Formulations

Consider the problem of steady two-dimensional laminar free convective MHD flow of a viscoelastic fluid along a vertical channel with a chemical reaction, taking into account the Soret and Dufour effects. The $\bar{x}$ -axis is taken vertically upwards and parallel to the flat wall while the $\bar{y}$ -axis is taken perpendicularly to it. The wavy and the flat walls are represented by $\bar{y} = \varepsilon^* \cos k\bar{x}$ and $\bar{y} = d$, respectively. The wavy and flat walls are maintained at constant temperatures of T_w and T_1 respectively.

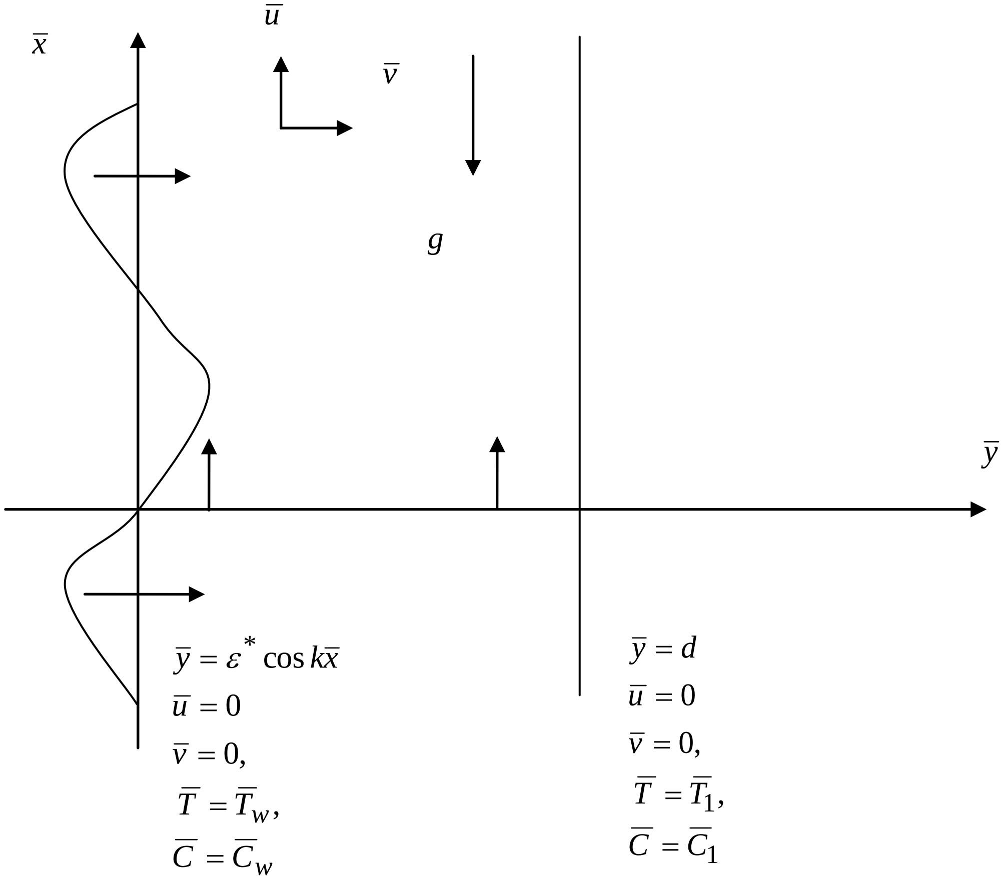

Fig.(i). Flow configuration of the physical problem.

The constitutive equation for the incompressible second-order fluid is

$$S = -pI + \mu_1 A_1 + \mu_2 A_2 + \mu_3 (A_1)^2 \qquad (1)$$

where S is the stress tensor, p is the hydrostatic pressure, $A_n, n = 1,2$ are the kinematic Rivlin-Ericksen tensors, and μ_1, μ_2, μ_3 are the material co-efficients describing the viscosity, visco-elasticity and cross-viscosity respectively, where μ_1 and μ_3 are positive and μ_2 is negative (Coleman and Markovitz [18]). Equation (2) was derived by Coleman and Noll [19] from that of simple fluids by assuming that the stress is more sensitive to the recent deformation than to the deformation that occurred in some distant past. The expressions for A_1 and A_2 are given by

$$A_{(1)ij} = v_{i,j} + v_{j,i}$$

$$A_{(2)ij} = a_{i,j} + a_{j,i} + 2v^m{}_{,i} v_{m,j}$$

where v_i and a_i are the i th component of the velocity and acceleration vectors respectively and a comma denotes co-variant differentiation with respect to the symbol following it.

The following assumptions are made:

(i) All the fluid properties except the density in the buoyancy force are constant.

(ii) The viscous and magnetic dissipative effects can be neglected in the energy equation.

(iii) The volumetric heat source /sink term in the energy equation is constant.

(iv) The magnetic Reynolds number is small so that the induced magnetic field can be neglected.

(v) The wavelength of the wavy wall is large such that k is small.

Under the above assumptions, the equations governing the steady flow and heat transfer problem are the momentum equations

$$\begin{aligned}
\rho\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right) &= -\frac{\partial p^*}{\partial \bar{x}} + \mu_1\left(\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right) + \mu_2\left(\bar{u}\frac{\partial^3 \bar{u}}{\partial \bar{x}^3} + \bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3} + \bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{x}^2 \partial \bar{y}}\right. \\
&+ \bar{u}\frac{\partial^3 \bar{u}}{\partial \bar{x}\partial \bar{y}^2} \\
&+ 3\frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}} + \frac{\partial \bar{u}}{\partial \bar{x}}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + 3\frac{\partial^2 \bar{v}}{\partial \bar{x}^2}\frac{\partial \bar{u}}{\partial \bar{y}} + 13\frac{\partial \bar{u}}{\partial \bar{x}}\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} \\
&\left. + 2\frac{\partial \bar{v}}{\partial \bar{x}}\frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}} + 4\frac{\partial^2 \bar{v}}{\partial \bar{x}^2}\frac{\partial \bar{v}}{\partial \bar{x}}\right) + 2\mu_3\left(4\frac{\partial \bar{u}}{\partial \bar{x}}\frac{\partial^2 \bar{u}}{\partial \bar{x}^2}\right. \\
&\left. + \frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}} + \frac{\partial \bar{v}}{\partial \bar{x}}\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{x}\partial \bar{y}}\frac{\partial \bar{v}}{\partial \bar{x}} + \frac{\partial^2 \bar{v}}{\partial \bar{x}^2}\frac{\partial \bar{u}}{\partial \bar{y}}\right) \\
&- \rho g - \sigma B^2 \bar{u} \qquad (2)
\end{aligned}$$

$$\begin{aligned}
\rho\left(\bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}}\right) &= -\frac{\partial p^*}{\partial \bar{y}} + \mu_1\left(\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{y}^2}\right) + \mu_2\left(\bar{u}\frac{\partial^3 \bar{v}}{\partial \bar{x}^3} + \bar{v}\frac{\partial^3 \bar{v}}{\partial \bar{y}^3} + \bar{v}\frac{\partial^3 \bar{v}}{\partial \bar{x}^2 \partial \bar{y}}\right. \\
&+ \bar{u}\frac{\partial^3 \bar{v}}{\partial \bar{x}\partial \bar{y}^2} \\
&+ 3\frac{\partial \bar{v}}{\partial \bar{x}}\frac{\partial^2 \bar{v}}{\partial \bar{x}\partial \bar{y}} + \frac{\partial \bar{v}}{\partial \bar{y}}\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + 3\frac{\partial \bar{v}}{\partial \bar{x}}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + 13\frac{\partial \bar{v}}{\partial \bar{y}}\frac{\partial^2 \bar{v}}{\partial \bar{y}^2} \\
&\left. + 2\frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{v}}{\partial \bar{x}\partial \bar{y}} + 4\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\frac{\partial \bar{u}}{\partial \bar{y}}\right) \\
&+ 2\mu_3\left(4\frac{\partial \bar{v}}{\partial \bar{y}}\frac{\partial^2 \bar{v}}{\partial \bar{y}^2} + \frac{\partial \bar{v}}{\partial \bar{x}}\frac{\partial^2 \bar{v}}{\partial \bar{x}\partial \bar{y}} + \frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right. \\
&\left. + \frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial^2 \bar{v}}{\partial \bar{x}\partial \bar{y}} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\frac{\partial \bar{v}}{\partial \bar{x}}\right), \qquad (3)
\end{aligned}$$

the continuity equation

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \quad , \qquad (4)$$

the energy equation

$$\rho C_p\left(\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{T}}{\partial \bar{y}}\right)=k\left(\frac{\partial^2\bar{T}}{\partial \bar{x}^2}+\frac{\partial^2\bar{T}}{\partial \bar{y}^2}\right)+\frac{\rho DK}{C_s}\left(\frac{\partial^2\bar{C}}{\partial \bar{x}^2}+\frac{\partial^2\bar{C}}{\partial \bar{y}^2}\right)+Q, \quad (5)$$

and the species continuity equation

$$\bar{u}\frac{\partial \bar{C}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{C}}{\partial \bar{y}}=D\left(\frac{\partial^2\bar{C}}{\partial \bar{x}^2}+\frac{\partial^2\bar{C}}{\partial \bar{y}^2}\right)+\frac{DK}{T_m}\left(\frac{\partial^2\bar{T}}{\partial \bar{x}^2}+\frac{\partial^2\bar{T}}{\partial \bar{y}^2}\right). \quad (6)$$

The boundary conditions relevant to the problem are:

$\bar{u}=\bar{v}=0,\ \bar{T}=\bar{T}_w, \bar{C}=\bar{C}_w$

on $\bar{y}=\varepsilon^*\cos k\bar{x}$

$\bar{u}=\bar{v}=0,\ \bar{T}=\bar{T}_1, \bar{C}=\bar{C}_1$ on $\bar{y}=d$ (7)

where $\bar{u}$, $\bar{v}$ are the velocity components in the directions of x and y respectively, p^* the fluid pressure, ρ the fluid density, g the acceleration due to gravity, σ the electrical conductivity, B strength of the applied magnetic field, C_p the specific heat at constant pressure, k the thermal conductivity, Q the constant heat addition/absorption, D the coefficient of mass diffusion, K the thermal diffusion ratio, $\bar{T}$ the fluid temperature, $\bar{T}_1$ the temperature of the flat wall, T_m the mean fluid temperature, $\bar{T}_w$ the temperature of the wavy wall, $\bar{C}$ the species concentration, $\bar{C}_w$ the species concentration at the wavy wall, $\bar{C}_1$ the species concentration at the flat wall, and ε^* a small amplitude parameter.

We introduce the following non dimensional quantities:

$$x=\frac{\bar{x}}{d},\ y=\frac{\bar{y}}{d},\ u=\frac{\bar{u}d}{\upsilon_1},\ v=\frac{\bar{v}d}{\upsilon_1},\ \theta=\frac{\bar{T}-\bar{T}_s}{\bar{T}_w-\bar{T}_s},$$

$$\bar{p}=\frac{p^*d^2}{\rho\upsilon_1^2},\ \lambda=kd,\ p_s=\frac{p_s^*d^2}{\rho\upsilon_1^2},$$

$$\alpha=\frac{Qd^2}{k(\bar{T}_w-\bar{T}_s)},\ m=\frac{\bar{T}_1-\bar{T}_s}{\bar{T}_w-\bar{T}_s},$$

$$n=\frac{\bar{C}_1-\bar{C}_s}{\bar{C}_w-\bar{C}_s},\ C=\frac{\bar{C}-\bar{C}_s}{\bar{C}_w-\bar{C}_s},\ \varepsilon=\frac{\varepsilon^*}{d},$$

$$P_r=\frac{\mu_1 C_p}{k},\ D_u=\frac{DK(\bar{C}_w-\bar{C}_s)}{\upsilon_1 C_s C_p(\bar{T}_w-\bar{T}_s)},$$

$$S_c=\frac{\upsilon_1}{D},\ M=\frac{\sigma' B^2 d^2}{\rho\upsilon_1^2},$$

$$S_r=\frac{DK(\bar{T}_w-\bar{T}_s)}{\upsilon_1 T_m(\bar{C}_w-\bar{C}_s)},$$

$$G_r=\frac{d^3 g\beta(\bar{T}_w-\bar{T}_s)}{\upsilon_1^2},$$

$$G_m=\frac{d^3 g\beta_c(\bar{C}_w-\bar{C}_s)}{\upsilon_1^2}.$$

where P_r the Prandtl number, D_u the Dufor number, S_c the Schmidt number, S_r the Soret number, G_r the Grashof number for heat transfer, G_m Grashof number for mass transfer, M the Hartmann number and the subscript s denotes quantities in the static fluid condition.

With the help of non-dimensional quantities introduced, we rewrite the equations (2)-(6) and the boundary conditions (7) as

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial \bar{p}}{\partial x}+\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\alpha_1\left(u\frac{\partial^3 u}{\partial x^3}+v\frac{\partial^3 u}{\partial y^3}+v\frac{\partial^3 u}{\partial x^2\partial y}+u\frac{\partial^3 u}{\partial x\partial y^2}\right.$$

$$+3\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}+3\frac{\partial^2 v}{\partial x^2}\frac{\partial u}{\partial y}+13\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2}$$

$$+2\frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial x\partial y}+4\frac{\partial^2 v}{\partial x^2}\frac{\partial v}{\partial x}\bigg)+\beta_1\bigg(4\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2}+\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial v}{\partial x}\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 u}{\partial x\partial y}\frac{\partial v}{\partial x}+\frac{\partial^2 v}{\partial x^2}\frac{\partial u}{\partial y}\bigg)+G_r\theta+G_mC-Mu, \quad (8)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial \bar{p}}{\partial y}+\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\alpha_1\bigg(u\frac{\partial^3 v}{\partial x^3}+v\frac{\partial^3 v}{\partial y^3}+v\frac{\partial^3 v}{\partial x^2\partial y}+u\frac{\partial^3 v}{\partial x\partial y^2}$$

$$+3\frac{\partial v}{\partial x}\frac{\partial^2 v}{\partial x\partial y}+\frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial x^2}+3\frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial y^2}+13\frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial y^2}+2\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial x\partial y}+4\frac{\partial^2 u}{\partial y^2}\frac{\partial u}{\partial y}\bigg)+\beta_1\bigg(4\frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial y^2}+\frac{\partial v}{\partial x}\frac{\partial^2 v}{\partial x\partial y}+\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 v}{\partial x\partial y}\frac{\partial u}{\partial y}+\frac{\partial^2 u}{\partial y^2}\frac{\partial v}{\partial x}\bigg), \quad (9)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \quad (10)$$

$$\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}+P_rD_u\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)=P_r\left(u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}\right)-\alpha, \quad (11)$$

$$\frac{1}{S_r}\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)+S_r\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)=u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}. \quad (12)$$

With boundary conditions

$$u=0,\ v=0,\ \theta=1, C=1 \text{ on } y=\varepsilon\cos\lambda x$$

$$u=0,\ v=0,\ \theta=m, C=n \text{ on } y=1 \quad (13)$$

where

$$\alpha_1=\frac{\mu_2}{\rho d^2},\quad \beta_1=\frac{2\mu_3}{\rho d^2}$$

Then, we use the well knownBoussinesq approximation $\rho=\rho_s\left[1-\rho\left(\bar{T}-\bar{T}_s\right)\right]$ in Equation (8) in the static fluid condition and adopt the perturbation scheme

$$u(x,y)=u_0(y)+\varepsilon u_1(x,y), v(x,y)=\varepsilon v_1(x,y),$$

$$\bar{p}=p_0(x)+\varepsilon p_1(x,y), \theta(x,y)=\theta_0(y)+\varepsilon\theta_1(x,y),$$

$$C(x,y)=C_0(y)+\varepsilon C_1(x,y). \quad (14)$$

The zeroth-order equations are

$$\frac{d^2u_0}{dy^2}-Mu_0=-G_r\theta_0-G_mC_0, \quad (15)$$

$$\frac{d^2\theta_0}{dy^2}+P_rD_u\frac{d^2C_0}{dy^2}=-\alpha, \quad (16)$$

$$\frac{d^2C_0}{dy^2}+S_rS_c\frac{d^2\theta_0}{dy^2}=-\alpha, \text{ and} \quad (17)$$

The first-order equations are

$$u_0\frac{\partial u_1}{\partial x}+v_1\frac{\partial u_0}{\partial y}=-\frac{\partial p_1}{\partial x}+\frac{\partial^2 u_1}{\partial x^2}+\frac{\partial^2 u_1}{\partial y^2}+\alpha_1\bigg(u_0\frac{\partial^3 u_1}{\partial x^3}+v_1\frac{\partial^3 u_0}{\partial y^3}+u_0\frac{\partial^3 u_1}{\partial x\partial y^2}+3\frac{\partial u_0}{\partial y}\frac{\partial^2 u_1}{\partial x\partial y}+\frac{\partial u_1}{\partial x}\frac{\partial^2 u_0}{\partial y^2}+3\frac{\partial^2 v_1}{\partial x^2}\frac{\partial u_0}{\partial y}\bigg)+\beta_1\bigg(\frac{\partial u_0}{\partial y}\frac{\partial^2 u_1}{\partial x\partial y}+\frac{\partial^2 v_1}{\partial x^2}\frac{\partial u_0}{\partial y}\bigg)$$

$$+G_r\theta_1+G_mC_1-Mu_1, \quad (18)$$

$$u_0\frac{\partial v_1}{\partial x}=-\frac{\partial p_1}{\partial y}+\frac{\partial^2 v_1}{\partial x^2}+\frac{\partial^2 v_1}{\partial y^2}+\alpha_1\left(u_0\frac{\partial^3 v_1}{\partial x^3}+u_0\frac{\partial^3 v_1}{\partial x\partial y^2}+3\frac{\partial v_1}{\partial x}\frac{\partial^2 u_0}{\partial y^2}\right.$$
$$\left.+2\frac{\partial u_0}{\partial y}\frac{\partial^2 v_1}{\partial x\partial y}+4\frac{\partial u_0}{\partial y}\frac{\partial^2 u_1}{\partial y^2}+4\frac{\partial u_1}{\partial y}\frac{\partial^2 u_0}{\partial y^2}\right)$$
$$+\beta_1\left(\frac{\partial^2 u_0}{\partial y^2}\frac{\partial u_1}{\partial y}+\frac{\partial^2 u_1}{\partial y^2}\frac{\partial u_0}{\partial y}+\frac{\partial v_1}{\partial x}\frac{\partial^2 u_0}{\partial y^2}\right.$$
$$\left.+\frac{\partial^2 v_1}{\partial x\partial y}\frac{\partial u_0}{\partial y}\right), \tag{19}$$

$$\frac{\partial u_1}{\partial x}+\frac{\partial v_1}{\partial y}=0, \tag{20}$$

$$\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}+P_rD_u\left(\frac{\partial^2 C_1}{\partial x^2}+\frac{\partial^2 C_1}{\partial y^2}\right)=P_r\left(u_0\frac{\partial\theta_1}{\partial x}+v\frac{\partial\theta_0}{\partial y}\right), \tag{21}$$

and

$$\frac{1}{S_r}\left(\frac{\partial^2 C_1}{\partial x^2}+\frac{\partial^2 C_1}{\partial y^2}\right)+S_r\left(\frac{\partial^2\theta_1}{\partial x^2}+\frac{\partial^2\theta_1}{\partial y^2}\right)=u_0\frac{\partial C_1}{\partial x}+v\frac{\partial C_0}{\partial y}. \tag{22}$$

The boundary conditions (13) can be split into the following two parts:

$$u_0=0,\ \ \theta_0=1, C_0=1 \text{ on } y=0$$
$$u_0=0,\ \ \theta_0=m, C_0=n \text{ on } y=1, \tag{23}$$

and

$$u_1=-\mathrm{Re}\left(u_0'e^{i\lambda x}\right),\ v_1=0,\ \theta_1=-\mathrm{Re}\left(\theta_0'e^{i\lambda x}\right),$$
$$C_1=-\mathrm{Re}\left(C_0'e^{i\lambda x}\right) \qquad \text{on } y=0$$

$$u_1=0,\ v_1=0,\ \theta_1=0,\ C_1=0 \text{ on } y=1 \tag{24}$$

where the prime denotes differentiation with respect to y.

3. Solution of the problem

The solutions for the zeroth-order velocity (u_0), the zeroth-order temperature (θ_0), and the zeroth-order concentration (C_0) in Equations (15), (16), and (17) subject to the boundary conditions (23) are given by

$$\theta_0=1+\left(m-1-\frac{A_1}{2}\right)y+\frac{A_1}{2}y^2, \tag{25}$$

$$C_0=1+S_rS_c+\{n+(m-1)S_rS_c-1\}y-S_rS_cT_0, \tag{26}$$

$$u_0=A_7e^{\sqrt{M}y}+A_8e^{-\sqrt{M}y}-\frac{A_3}{M}y-\frac{A_4}{M}y^2-A_4. \tag{27}$$

Where

$$A_1=\frac{\alpha}{P_rD_uS_rS_c-1},\ A_2=-G_r-G_m,$$
$$A_3=-G_r\left(m-\frac{A_1}{2}-1\right)+G_m(n-1)+\frac{G_mS_rS_cA_1}{2},$$
$$A_4=\frac{A_1}{2}\left(G_mS_rS_c-G_r\right),$$
$$A_5=\frac{1}{M}\left(A_1+\frac{2A_4}{M}\right),$$
$$A_6=\frac{1}{M}\left(A_2+A_3+A_4+\frac{2A_4}{M}\right),$$
$$A_7=\frac{A_6-A_5e^{-\sqrt{M}}}{e^{\sqrt{M}}-e^{-\sqrt{M}}},$$
$$A_8=\frac{A_5e^{-\sqrt{M}}-A_5}{e^{\sqrt{M}}-e^{-\sqrt{M}}}.$$

In order to solve Equations (18) to (22) for the first-order quantities, it is convenient to introduce the stream function $\overline{\psi}_1$, defined by

$$u_1=-\frac{\partial\overline{\psi}_1}{\partial y},\ v_1=\frac{\partial\overline{\psi}_1}{\partial x}.$$

Further, eliminating p_1 from (18) and (19) and assuming

$\overline{\psi}_1(x,y)=e^{i\lambda x}\psi(y)$, $\theta_1(x,y)=e^{i\lambda x}t(y)$ and
$C_1(x,y)=e^{i\lambda x}\phi(y)$, (28)

We get the ordinary differential equations:

$$\psi^{IV}-\psi''\left(M+2\lambda^2+i\lambda u_0-u_0\right)+\psi\left(u_0\lambda^2+\lambda^4+i\lambda u_0''+iu_0\lambda^3\right)+i\alpha_1\left(u_0\lambda\psi^{IV}-2u_0\lambda^3\psi''-u_0^{IV}\lambda\psi+u_0\lambda^V\psi\right)=G_r t'+G_m\phi', \quad (29)$$

$$t''-t\left(\lambda^2+Pi\lambda u_0\right)+P_rD_u\left(\phi''-\lambda^2\phi\right)=P_r i\lambda\psi\theta_0', \quad (30)$$

$$\phi''-\lambda^2\phi+S_cS_r\left(\theta''+\lambda^2\theta\right)=i\lambda S_c\left(u_0+\psi C_0'\right). \quad (31)$$

If we consider only small values of λ, (or $k<<1$) then substituting

$$\psi(\lambda,y)=\sum_{j=0}^{2}\lambda_j\psi_j\ ,\ \ t(\lambda,y)=\sum_{j=0}^{2}\lambda^j t_j\ ,$$
$$\phi(\lambda,y)=\sum_{j=0}^{2}\lambda_j\phi_j\ .$$

Equations (29), (30) and (31) give, to order of λ^2, the following sets of differential equations

$$\psi_0^{IV}-M\psi_0''=G_r t_0'+G_m\phi_0', \quad (32)$$

$$t_0''+P_rD_u\phi_0''=0, \quad (33)$$

$$\phi_0''+S_cS_r t_0''=0. \quad (34)$$

$$\psi_1^{IV}-M\psi_1''=iu_0\psi_0''-iu_0''\psi_0+G_r t_1'+G_m\phi_1'+i\alpha_1\left(u_0^{IV}\psi_0-u_0\psi_0^{IV}\right), \quad (35)$$

$$t_1''=P_r\left(iu_0t_0+it_0'\psi_0-D_u\phi_1''\right), \quad (36)$$

$$\phi_1''=S_c\left(iu_0\phi_0+i\psi_0C_0'-S_rt_1''\right), \quad (37)$$

$$\psi_2^{IV}-M\psi_2''=2\psi_0''+iu_0\psi_1''-iu_0''\psi_1+G_rt_2'+G_m\phi_2'+i\alpha_1\left(u_0^{IV}\psi_1-u_0\psi_1^{IV}\right), \quad (38)$$

$$t_2''=iP_r\left(u_0t_1+t_0'\psi_1\right)+t_0+P_rD_u\left(\phi_0''-\phi_2''\right), \quad (39)$$

$$\phi_2''=S_rS_c\left(t_0''-t_2''\right)+iS_c\left(u_0\phi_1+\psi_1C_0'\right)+\phi_0. \quad (40)$$

The corresponding boundary conditions are

$$\begin{aligned}
&\psi_0'=u_0',\ \psi_0=0,\ t_0=-\theta_0',\ \phi_0=-C_0' && \text{on } y=0\\
&\psi_0'=0,\ \psi_0=0,\ t_0=0,\ \phi_0=0 && \text{on } y=1\\
&\psi_1'=0,\ \psi_1=0,\ t_1=0,\ \phi_1=0 && \text{on } y=0\\
&\psi_1'=0,\ \psi_1=0,\ t_1=0,\ \phi_1=0 && \text{on } y=1\\
&\psi_2'=0,\ \psi_2=0,\ t_2=0,\ \phi_2=0 && \text{on } y=0\\
&\psi_2'=0,\ \psi_2=0,\ t_2=0,\ \phi_2=0 && \text{on } y=1
\end{aligned} \quad (41)$$

The equations (32) to (40) are solved subject to the boundary conditions (41), but are not presented here for the sake of brevity.

4. Skin friction at the walls

The shear stress τ_{xy} at any point in non-dimensional form is given by $\tau_{xy}=\dfrac{d^2\bar{\tau}_{xy}}{\rho v_1^2}$

$$=\frac{\partial u_0}{\partial y}+\varepsilon e^{i\lambda x}\bar{u}_1'(y)+i\lambda\varepsilon e^{i\lambda x}\bar{v}_1(y)+\alpha_1\varepsilon\left[i\lambda u_0(y)e^{i\lambda x}\bar{u}_1'(y)+e^{i\lambda x}\bar{v}_1(y)u_0''(y)\right.$$

$$-u_0\lambda^2 e^{i\lambda x}\bar{v}_1(y)+2i\lambda e^{i\lambda x}\bar{u}_1(y)\bar{u}_0(y)u_0'(y)]. \quad (42)$$

At the wavy wall $y=\varepsilon\cos\lambda x$ and at the flat wall $y=1$, τ_{xy} becomes, respectively,

$$\tau_w=\tau_0^0+\varepsilon\,\mathrm{Re}\left[e^{i\lambda x}u_0''(0)+e^{i\lambda x}\bar{u}_1'(0)\right]+\alpha_1\varepsilon\left[\lambda u_0(0)\cos\lambda x\psi_i''(0)\right.$$
$$+\lambda u_0(0)\psi_r''(0)\sin\lambda x+2\lambda u_0'(0)\psi_i'(0)\cos\lambda x$$
$$\left.+2\lambda u_0'(0)\psi_r'(0)\sin\lambda x\right] \quad (43)$$

and

$$\tau_1=\tau_1^0+\mathrm{Re}[\varepsilon e^{i\lambda x}\bar{u}_1'(1)]$$
$$+\alpha_1\varepsilon\left[\lambda u_0(1)\psi_i''(1)\cos\lambda x+\lambda u_0(1)\psi_r''(1)\sin\lambda x\right.$$
$$\left.+2\lambda\psi_i'(1)u_0'(1)\cos\lambda x+2\lambda u_0'(1)\psi_r'(1)\sin\lambda x\right] \quad (44)$$

where $\tau_0^0=u_0'(0),\tau_1^0=u_0'(1)$ are the zeroth-order skin frictions at the walls, and $\bar{u}_1(y)$ and $\bar{v}_1(y)$ are given by

$$u_1(x,y)=\varepsilon e^{i\lambda x}\bar{u}_1(y),\ v_1(x,y)=\varepsilon e^{i\lambda x}\bar{v}_1(y).$$

5. Heat transfer and mass transfer coefficient

The non-dimensional heat transfer coefficient in terms of Nusselt number N_u is given by

$$N_u=\frac{\partial\theta}{\partial y}=\theta_0'(y)+\varepsilon e^{i\lambda x}t'(y). \quad (45)$$

At the wavy wall $y=\varepsilon\cos\lambda x$ and the flat wall $y=1$, N_u takes the form

$$Nu_w=Nu_0^0+\varepsilon\left(\cos\lambda x\theta_0''(0)+e^{i\lambda x}t'(0)\right) \quad (46)$$

and

$$Nu_1=Nu_1^0+\varepsilon e^{i\lambda x}\theta'(1) \quad (47)$$

respectively, where

$$Nu_0^0=\theta_0'(0),\ Nu_1^0=\theta_0'(1)$$

The mass transfer coefficient in terms of Sherwood number Sh is given by

$$Sh=\frac{\partial C}{\partial y}=C_0(y)+\varepsilon\frac{\partial C_1}{\partial y}=C_0'(y)+\varepsilon e^{i\lambda x}\phi'(y).$$

At the wavy wall $y=\varepsilon\cos\lambda x$ and the flat wall $y=1$, Sh takes the form

$$Sh_w=Sh_0^0+\varepsilon\left(\cos\lambda xC_0''(0)+e^{i\lambda x}\phi'(0)\right) \quad (48)$$

and

$$Sh_1=Sh_1^0+\varepsilon e^{i\lambda x}\phi'(1) \quad (49)$$

respectively, where

$$Sh_0^0=C_0',\ Sh_1^0=C_0'(1).$$

6. Results and Discussions

The purpose of this study is to bring out the Soret and Dufour effects on the governing flow with the combination of the viscoelastic parameter. Here the real parts of the results are considered throughout for numerical validation.

To examine the nature of variation of various physical quantities associated with the problem under consideration, a particular case characterized by the following values of parameters involved in the analysis is presented. $P=3,\alpha=1,G_r=2,m=4,n=1,\lambda=0.001,\varepsilon=0.01.$ The non-dimensional velocity u against y is plotted in Fig 1-4. It is evident from the figures that velocity profile is parabolic in nature and attains a distinctive maximum in the vicinity of the middle of the channel. This phenomenon is noticed in both Newtonian $(\alpha_1=0)$ and viscoelastic fluid flows $(\alpha_1=-0.05,-0.1)$. The velocity increases with the increasing values of S_r, D_u, G_m, whereas the velocity decreases with the increasing strength of the magnetic field for both Newtonian $(\alpha_1 = 0)$ and viscoelastic fluid flows $(\alpha_1 = -0.05, -0.1)$. Also, velocity decreases with the increasing values of the viscoelastic parameter (α_1) in comparison to the Newtonian fluid for all the cases.

Figures 5 and 6 exhibit the nature of skin friction at both the wavy wall and the flat wall. From the figures, it is observed that the magnitude of shear stress decreases with the increasing values of the viscoelastic parameter.

The Nusselt number and Sherwood number are not affected significantly during the changes made in viscoelasticity of the fluid flow.

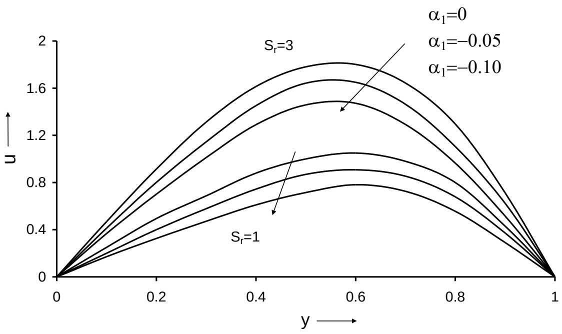

Fig.1. Variation of velocity against y for $D_u = 0.2,\ G_m = 2,\ M = 0.5.$

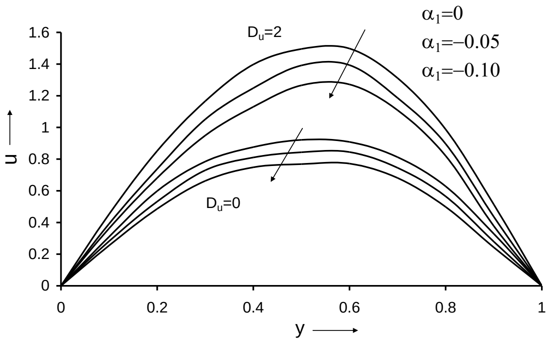

Fig.2. Variation of velocity against y for $S_r = 1,\ G_m = 2,\ M = 0.5.$

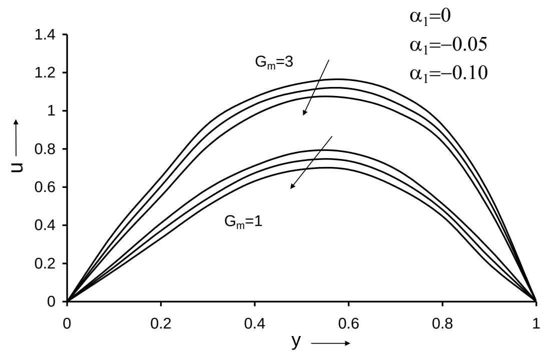

Fig.3. Variation of velocity against y for $S_r = 1,\ D_u = 0.2,\ M = 0.5.$

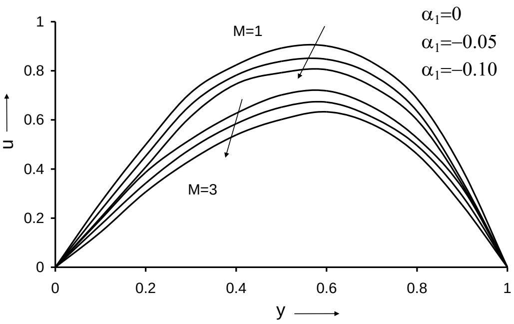

Fig.4. Variation of velocity against y for $S_r = 1,\ D_u = 0.2,\ G_m = 2.$

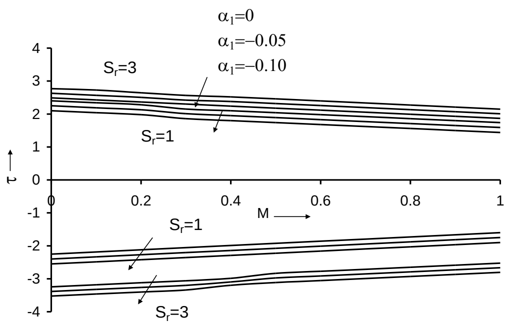

Fig.5. Variation of skin friction τ against M for $D_u = 0.3, \lambda x = \dfrac{\pi}{2}.$

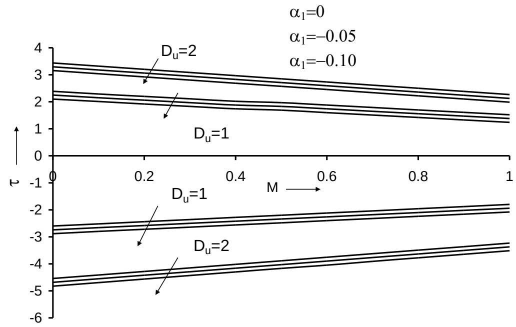

Fig.6. Variation of skin friction τ against M for $S_r = 1$, $\lambda x = \dfrac{\pi}{2}$.

7. Conclusions

In this paper, the Soret and Dufour effects along with the viscoelastic effects on a two dimensional steady free convective MHD flow of viscoelastic incompressible fluid between a long vertical wavy wall and a parallel flat wall are studied. The second order fluid model for a viscoelastic fluid flow is assumed. The following observations are noted through the graphs

(i) The fluid motion retarded with the increasing values of the viscoelastic parameter.

(ii) The fluid motion is accelerated due to thermal diffusion and diffusion thermo effects and retarded under the application of the transverse magnetic field for both Newtonian and viscoelastic fluid flow.

(iii) Magnitudes of the viscous drag at the wavy wall and at the flat wall increase due to thermal diffusion and diffusion thermo effects for both Newtonian and viscoelastic fluid flow.

Nomenclature

g Acceleration due to gravity

$\bar{x}, \bar{y}$ Cartesian coordinates

D Coefficient of mass diffusion

Q Constant heat addition/absorption

d Distance between two walls

D_u Dufour number

p^* Fluid pressure

$\bar{T}$ Fluid temperature

$\bar{T}_s$ Fluid temperature in static condition

G_r Grashof number for heat transfer

G_m Grashof number for mass transfer

M Hartmann number

T_m Mean fluid temperature

P_r Prandtl number

p_s^* Pressure of the fluid in static condition

A_1, A_2 RivlinEricksen tensors

$\bar{C}$ Species concentration

$\bar{C}_w$ Species concentration at the wavy wall

$\bar{C}_1$ Species concentration at the flat wall

C_p Specific heat at constant pressure

$\bar{B}$ Strength of the applied magnetic field

S_r Soret number

S_c Schmidt number

$\bar{T}_1$ Temperature of the flat wall

$\bar{T}_w$ Temperature of the wavy wall

k Thermal conductivity

K Thermal diffusion ratio

$\bar{u}, \bar{v}$ Velocity components in the directions of x and y respectively

n Wall concentration ratio

m Wall temperature ratio

Greek symbols

ε^* Amplitude parameter

β_c Coefficient of expansion with concentration

β Coefficient of volume expansion for heat transfer
μ Co-efficient of viscosity
ρ_S Density of the fluid in static condition
σ' Electrical conductivity
ρ Fluid density
λ Frequency parameter
α Heat source/sink parameter
υ_1 Kinematic viscosity
μ_1, μ_2, μ_3 Material co-efficients describing the viscosity, visco-elasticity and cross-viscosity respectively

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