Application of a Pipelining Technique in Concatenated Tomlinson Harashima Precoder for Downlink Multiuser MIMO Systems

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Abstract

We show that our new concatenated pipelining Tomlinson Harashima precoding (C-PIPTHP) can significantly improve the performance of downlink multiple-input multiple-output (MIMO) systems. We then combine this new scheme with the use of block diagonalization (BD) and Tomlinson Harashima precoding (THP) in a multiuser (MU) setting. In such a setting, BD is used to suppress interuser interference (IUI), and THP is used to pre-eliminate the interstream interference between streams. The new scheme proposed is called BD C-PIPTHP. Its improved processing speed and bit error rate (BER) performance over traditional BD THP MU-MIMO scheme are demonstrated via MATLAB simulations.

Keywords: Tomlinson Harashima precoding (THP), block diagonalization, MU-MIMO, Pipelining, Highspeed, Concatenated pipelining Tomlinson Harashima precoding (C-PIPTHP)

1. Introduction

Over the past several decades, we have seen development of multiple-input and multiple-output (MIMO) in mainstream wireless local area networks (LANs) and mobile networking. MIMO increases efficiency for systems operating in a multipath-rich environment. It was initially considered in a single user scenario with one transmitter (tx) and one receiver (rx), each potentially equipped with several antennas. MIMO has also been an interesting area for next generation wireless communication in a multiuser scenario which is referred to as a multiuser MIMO system (MU-MIMO) [1].

MU-MIMO techniques are currently being introduced in next-generation wireless LAN. With the introduction of multiple antennas in the 802.11n standard, it is possible for each user to communicate multiple streams of data at the same time. In MU-MIMO system, these data streams from each user encounter spatial interferences from other users' data streams and among a user's own data streams. Therefore, eliminating interference is crucial [2-4]. The schemes analyzed in this paper all focus on the downlink direction.

Tomlinson Harashima precoding (TH precoding or simply THP) is a non-linear precoding technique originally used to combat intersymbol interferences (ISI) for single-input single-output (SISO) channels [5-6]. This technique was further extended to be used in MIMO system (MIMO THP) in which the interferences can be eliminated with the help of QR decomposition of the channel matrix [2]. In recent years, research on MIMO THP has been extended to multiple-user systems. [4, 7-9].

The original THP is slow because it uses a modulo device and a feedback filter to implement its pre-equalizer. Consequently, it is challenging to implement TH precoders for high-speed applications. Pipelining Tomlinson Harashima precoding (PIPTHP) has been proposed [10-12] to alleviate such a scenario. In our previous work, we developed pipelining THP technique for MIMO systems. Our proposed system was called concatenated PIPTHP (C-PIPTHP). This precoding scheme can yield better BER performance than MIMO THP [13]. In this paper, we further extend our system to operate in a multiuser scenario where block diagonalization (BD) is utilized to eliminate interference among several users.

Block diagonalization uses transmit precoding matrices designed to ensure no interuser interference among received signals. Every user is restricted to be in the null space of all other users' channels. Therefore, if the channel matrices of all users are perfectly known at the transmitting side, each user is capable of experiencing an interference-free channel. The resulting MU-MIMO effective channel matrix has a block diagonal form, which allows separate consideration for each user. [14-16]. In this study, we explore the idea of adding concatenation of two pre-equalizers into a BD multi-user system. We first analyze our concatenated PIPTHP (C-PIPTHP) [13], which is capable of improving the overall performance by applying two pre-equalizers. To our knowledge, there has been no analysis with a pipelining technique in an MU-MIMO system. Here, we exploit BD so that our pipelining system can support multiple users. Our new scheme, BD C-PIPTHP, is shown to improve system performance, both in terms of reliability and processing time. Table 1 shows the comparison between THP based schemes and our proposed systems.

The remainder of the paper is organized as follows. Section 2 reviews the basic concept of a downlink MU-MIMO system with block diagonalization and Tomlinson Harashima precoding. Section 3 then discusses the proposed scheme, which uses concatenated pipelining TH precoding (C-PIPTHP) in a downlink MU-MIMO system with block diagonalization. In Section 4, performance of the traditional system from Section 2 is compared with our proposed system from Section 3. Finally, Section 5 summarizes our study and further provides potential research directions.

2. Downlink MU-MIMO System with Block Diagonalization and Tomlinson Harashima Precoding

This section reviews the concepts of block diagonalization and Tomlinson Harahsima precoding and also shows how these two particular techniques operate in downlink multiuser scenarios.

Table 1 Comparison among THP based systems. The scheme numbers are used throughout the paper to reduce confusion.

Non-linear Precoding in downlink MIMO				
Schemes	Pipeline	MIMO	Concatenated THP	Multiuser
(1) Traditional THP [5, 6]	×	×	×	×
(2) PIPTHP [10-12]	√	×	×	×
(3) MIMO THP [4]	×	√	×	×
(4) Proposed C-THP	×	√	√	×
(5) Proposed C-PIPTHP	√	√	√	×
(6) BD THP [14]	×	√	×	√
(7) Proposed BD C-PIPTHP	√	√	√	√

Remark: √ = Included
× = Not Included

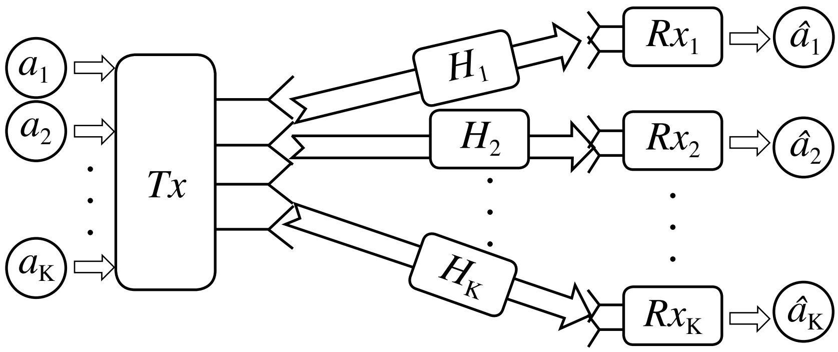
Fig. 1a shows a block diagram for a generic downlink MU-MIMO system. For the system under consideration, we assume that *Nt* transmitting antennas are located at the base station which serves *K* users. Each user has *Nr* receiving antennas. The data vector $\vec{a}_k$ of length *Nr* is to be transmitted to the *k*th user for $k = 1, 2, \ldots, K$. The components of the data vector $\vec{a}_i$ are *M*-QAM symbols randomly chosen from a signal constellation [2, 3-4].

$$\{a_R + ja_I \mid a_R, a_I \in \{\pm 1, \pm 3, \ldots \pm(\sqrt{M}-1)\}\}$$

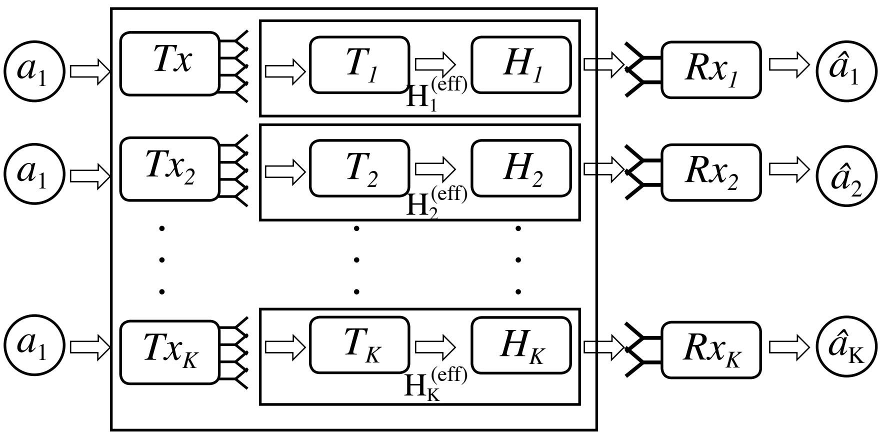
The $N_r \times N_t$ channel matrix of the k^{th} user, representing the channel between k^{th} client to the base station, is denoted by H_k. Flat Rayleigh fading is assumed; that is, the channel coefficients for all pairs of transmitting antennas and receiving antennas are assumed to be independent zero-mean circularly symmetric complex Gaussian random variables with unit variance [1].

2.1 Supressing interuser interference using block diagonalization (BD)

BD provides non-iterative selection of downlink transmit vectors using the null space of other channel matrices [3]. In particular, to suppress inter-user interference (IUI) at each receiver, BD uses a collection of transmit precoding matrices $T_k \in \mathbb{C}^{Nt \times Nr}$ which utilize null space. The precoding matrices for the k^{th} user is constructed to satisfy the following condition [3,14-16]:



(a)



(b)

Fig. 1. System model of Downlink MU-MIMO systuem (a) the overall system, (b) the effective system with block diagonalization.

$$H_i T_j = 0 \qquad \text{for all } i \neq j \text{ and } 1 \leq i, j \leq K \tag{1}$$

from which $\widetilde{H}_k$. isdefined as:

$$\widetilde{H}_k = \left[H_1^T, H_2^T, \dots, H_{k-1}^T, H_{k+1}^T, \dots, H_K^T\right] \tag{2}$$

To do this, we first decompose $\widetilde{H}_k$ via SVD to get :

$$\widetilde{H}_k = \widetilde{U}_k \begin{bmatrix} \tilde{\Delta}_k & 0 \\ 0 & 0 \end{bmatrix} [\tilde{V}_k^1 \quad \tilde{V}_k^0]^H \tag{3}$$

where $\widetilde{U}_k$ is a unitary vector matrix, $\tilde{\Delta}_k$ is a diagonal matrix, and $\tilde{V}_k^0$ is constructed from right singular vectors corresponding to zero singular values. To satisfy condition (1) above, we require that every column of T_k belong to the null space of $\widetilde{H}_k$.

From the decomposition (3), T_k can be directly constructed from $\tilde{V}_k^0$. Combining the precoding matrices with the data vectors, we have the transmitted signal $\vec{x} = \sum_{i=1}^{K} T_i \vec{a}_i$. The received signal at the j^{th} user (after experiencing the channel fading H_j and the additive noise $\vec{n}_j$), is then given by:

$$\vec{y}_j = H_j \sum_{i=1}^{K} T_i \vec{a}_i + \vec{n}_j = H_j T_j \vec{a}_j + \vec{n}_j \tag{4}$$

for $j = 1, \dots, K$. Notice that the data vectors from different users are now completely separated. We only have $\vec{a}_j$ in $\vec{y}_j$. Hence, the MU-MIMO channel is now reduced to K parallel non-interfering SU-MIMO channels whose effective channel matrix $H_k^{(eff)}$ for the kth user is $H_j T_j$. These properties are applied to convert the system in Fig. 1a to the one in Fig. 1b.

2.2. Pre-eliminating interstream interference via Tomlinson Harashima precoding (THP)

To eliminate the interstream (or multi-antenna) interference for each user, we apply Tomlinson Harashima precoding (THP). The procedure for each user is the same and therefore, in this subsection, we shall use H to represent $H_k^{(eff)}$ discussed in the previous subsection. We first apply QR decomposition to H^H which gives:

$$H^H = FR = (SF^H)^H \tag{5}$$

where F, R, and S are unitary feedforward matrix, upper triangular matrix, and lower triangular matrix, respectively [8-9, 16].

Fig. 2 shows the MU-MIMO system in which THP is applied to each user. Here, the strictly lower-left triangular feedback matrix B-I is set to be GS-I where the scaling matrix G is $(\text{diag}(S))^{-1}$ [1, 4, 7-9]. The task of the feedback matrix is to pre-eliminate the interstream interference caused by the S component of H. The feedforward filter F will then cancel the F^H component of H. Because of the triangular nature of the feedback filter, the signal going into the feedforward filter without the modulo device can be represented as:

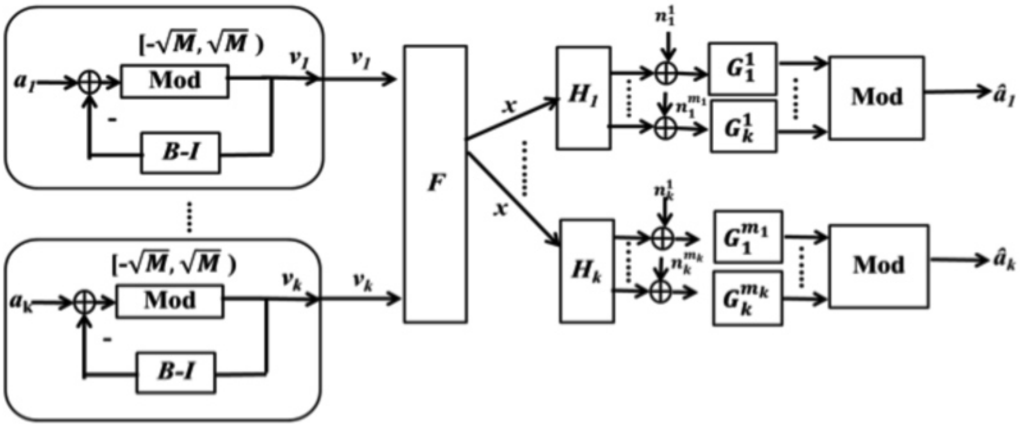


Fig. 2. System model of downlink MU-MIMO THP.

$$v_i = a_i - \sum_{m=1}^{i-1} b_{im} v_m, \quad i = 1,..,K \tag{6}$$

where b_{im} is the (i,m) entry of the matrix $B = GS$.

A modulo device is applied to prevent the increase in the transmitted signal's magnitude resulted from the recursive operation of the feedback filter.

In particular, the modulo device confines the real and imaginary parts of the transmitted signal to be within $[-\sqrt{M},\sqrt{M})$. Equivalently, its output can be expressed as:

$$v_i = a_i + d_i - \sum_{m-1}^{i} b_{\text{im}}\ v_m\ ,\ i = 1,...,K \tag{7}$$

The precoding symbols d_i, which realize the modulo function, are complex numbers whose real and imaginary parts are integer multiples of $2\sqrt{M}$:

$$d_i \epsilon \{2\sqrt{M}(d_R + jd_I) |\ d_R, d_I \epsilon\ \ Z\} \tag{8}$$

Again, d_R and d_i are selected so that the real and imaginary parts of X_i are both within the interva [-√M, √M) in [2, 4, 17]. At the receiver, another modulo device is used to eliminate the precoding signal so that the original signal can be recovered.

When the THP discussed here is used with the effective SU-MIMO channels resulted from the BD reviewed in the previous subsection, we refer to the system as BD THP.

3. Downlink MU-MIMO System with Block Diagonalization and Concatenated Pipelining THP

This section describes the implementation of concatenated pipelining Tomlinson Harashima precoding (C-PIPTHP) and then extends the use of the technique to MU-MIMO systems by the application of BD.

3.1. Concatenated pipelining Tomlinson Harashima precoding (C-PIPTHP)

In this section, we describe the operation of concatenated pipelining Tomlinson Harashima precoding (C-PIPTHP), which was implemented utilizing low complexity PIPTHP as a main design. Necessary transfer functions in Z-transform representation for low complexity PIPTHP are expressed as follows:

$$N(z) = 1 + \sum_{i=1}^{LN} N_i Z^{-i} \quad (9)$$

$$Ne(z) = \sum_{i=1}^{LN} N_i Z^{-i+1} \quad (10)$$

$$N_1(z) = \sum_{i=1}^{L1} N_i Z^{-i-1} \quad (11)$$

$$N_2(z) = \sum_{i=L1+1}^{LN} N_i Z^{-i-L1-1} \quad (12)$$

$$D(z) = 1 + \sum_{i=1}^{LH} d_i Z^{-iK_{pipe}} \quad (13)$$

where LN, LH, and K_{pipe} denote the pipelining parameters defined in [10-11]. Concatenated PIPTHP (C-PIPTHP) combines MIMO THP described in Section 2 and low complexity PIPTHP to support MIMO systems. This nonlinear precoding scheme improves the bit error rate performance because the IIR pipelining filters and the feedback filter are directly connected together.

For the intended input of this scheme, the filters are produced from a digital modulation scheme, and the pipelining filters are capable of operating only one stream per process. Essential components in C-PIPTHP are similar to MIMO THP. The channel matrix of C-PIPTHP can be calculated by help of QR decomposition and after finishing the decomposition technique, the channel matrix is identical with multiplication between lower triangular matrix and unitary matrix [13]. Equations of lower triangular matrix and unitary matrix have been explained in Section 2.2

3.2. Implementation of concatenated Tomlinson Harashima precoding and block diagonalization in MU-MIMO system

Our proposed scheme in this paper is to use the C-PIPTHP reviewed in the previous subsection in the effective SU-MIMO channels, which resulted from the BD reviewed in Section 2.1. We replace the THP discussed in Section 2.2 by concatenated pipelining THP discussed in the previous subsection. Again, the goal is to pre-eliminate interstreams interference between each antenna.

To enable separation of streams for each user in the multiple-user setting, the matrices G, F, and B discussed in Section 2.2 are now constructed for each user where we use subscripts to denote user index. Their key properties are:

$$G_j H_j^{(eff)} F_j B_j^{-1} = I, \; j = 1,\dots,K \quad (14)$$

The Nt streams are operated in C-PIPTHP block, and this block is capable of manipulating one streams per process. Transfer functions of filters in the discrete time domain are represented as follows:

$$N(z): y(n) = \sum_{l=0}^{LN} n_l t(n+l) \quad (15)$$

$$D(z): y(n) = -\sum_{l=1}^{LH} d_l t(n - lK_{pipe}) \quad (16)$$

$$N_1(z): y(n) = \sum_{l=1}^{L1} n_l t(n-l-1) \quad (17)$$

$$N_2(z): y(n) = \sum_{l=L1+1}^{LN} n_l t(n-l-L1-1) \quad (18)$$

where *t(n)* and *y(n)* are denoted as the input and output of the filters and these equations are transformed from Z-domain transfer functions. They are considered as an encoder and a decoder for eliminating interstream interference between each antenna.

Consequential structures of PIPTHP are shown in Fig. 3 and Fig. 4 The feedback matrix *(B-I)*, the feedforward filter *(F)*, the scaling matrix (G) and two modulo devices at transmitter and receiver are the essential components for THP. There are also several significant advantages for our proposed system. The first one is that our proposed system comprises more pre-equalizers at the transmitter; therefore, the MU-MIMO THP scheme, which incorporates only a pre-equalizer part, can yield a worse error rate performance due to more channel distortion and interferences.

Originally, the C-PIPTHP, which was developed by [13], has been successfully implemented in MIMO only. Consequently, the second advantage is the proposed scheme is capable of operating in an MU-MIMO system, which is an important benefit to improve the precoding. Nevertheless, for our proposed design, there are two main disadvantages. The first drawback is there is only a one dimensional filter of pipelining THP. This means that C-PIPTHP is capable of performing one stream per process only.

The second disadvantage is the cost of the components. Despite adding more FIR at the transmitter in the communication schemes and not being concerned with the hardware overhead, the cost of the components will be more expensive than a multiuser MIMO THP scheme.

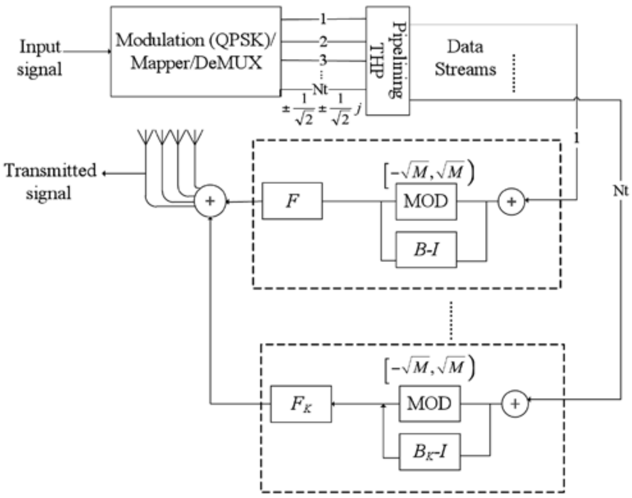


Fig. 3. Concatenated PIPTHP (C-PIPTHP) for Downlink MU-MIMO at Transmitter (Tx).

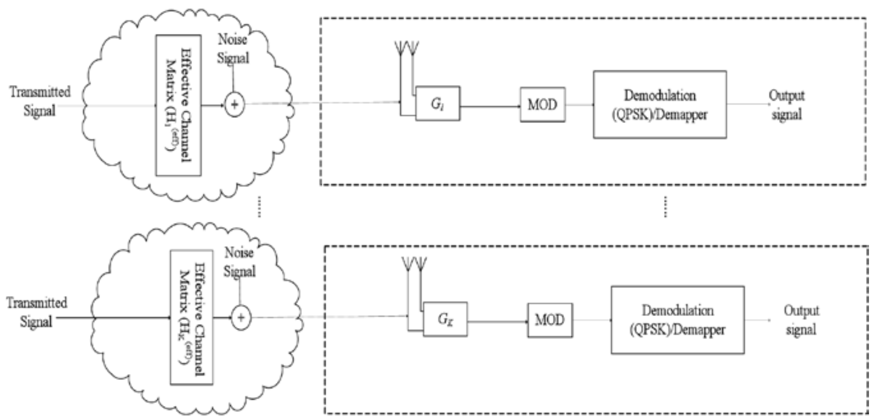


Fig. 4. The effective MU-MIMO systems for concatenated PIPTHP (C-PIPTHP).

4. Simulation Results

There are three new schemes analyzed in this paper, namely C-THP (4), C-PIPTHP (5), and BD C-PIPTHP (7). In Section 4.1, we compare the BER performance of the traditional MIMO scheme with the new C-THP (4) and C-PIPTHP (5). In Section 4.2, we show the performance gained from applying the concept of concatenated pipelining to the multi-user BD THP (6).

We analyze the downlink MIMO communication schemes in MATLAB. In all simulations, QPSK is used. Additionally, we assume that the channel state information is known perfectly at both the transmitter and receivers. The signal to noise ratio (Eb/No) is varied by adjusting the standard variation of the noise.

4.1. MIMO C-PIPTHP

For SU-MIMO, one high-rate data stream is generated and then split into multiple data streams. The BER is found by considering the error occurred at the receiver after the received streams are combined back to the high-rate stream.

Fig. 5 compares BER performance of the new C-THP (4) and C-PIPTHP (5) systems against a system with traditional MIMO THP (3). At BER = 0.001, C-THP (4) and C-PIPTHP (5) outperform MIMO THP (3) by more than 10 dB. Therefore, the new concatenated systems improve the BER performance significantly. Note that their BER plots are almost the same for low SNR with small differences at high SNR.

Table 2 compares the processing time of the THP-based precoding scheme for downlink MIMO analyzed earlier. The new C- PIPTHP (5) is much faster than C-THP (4); the two concatenated pre-equalizers take a long time to operate without pipelining. Although it takes a bit more time than the traditional MIMO THP (3), the performance is much better as shown in Fig. 5.

Table 2 Processing time comparison for downlink MIMO THP-based schemes (Nt = 4 and Nr = 4). Pipelining in C-PIPTHP (5) reduces the processing time of C-THP (4) significantly.

C-PIPTHP (5)	2.53 seconds
C-THP (4)	506.19 seconds
MIMO THP (3)	0.25 seconds

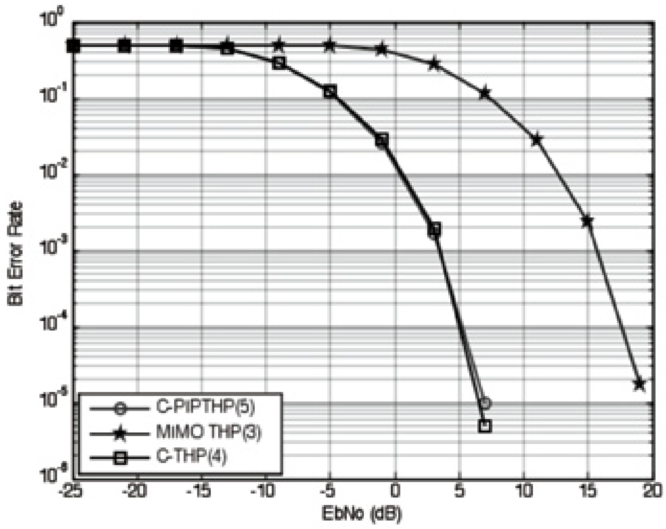


Fig. 5. BER performance for downlink MIMO THP-based schemes. The new concatenated systems, C-THP (4) and C-PIPTHP (5), give better BER than the traditional MIMO THP (3). In C-PIPTHP (5), although its performance is comparable to C-THP (4), pipelining also improves the processing time as shown in Table 2.

4.2. BD C-PIPTHP and BD THP

For the multiple-user setting, we compare the new BD C-PIPTHP (7) against the traditional BD THP (6). We consider the situation where there are two users, each equipped with two antennas. The number of transmitting antennas at the base station is four.

From Figs. 6 and 7, our system, which is BD C-PIPTHP (7), is capable of achieving better BER and SER performance than the traditional BD THP (6). For example, at the 0.001 level of BER or SER, BD C-PIPTHP (7) provides an SNR gain of more than 10 dB over BD THP (6). As in Section 4.1, this improvement comes from incorporating concatenated PIP-THP (with two pre-equalizers).

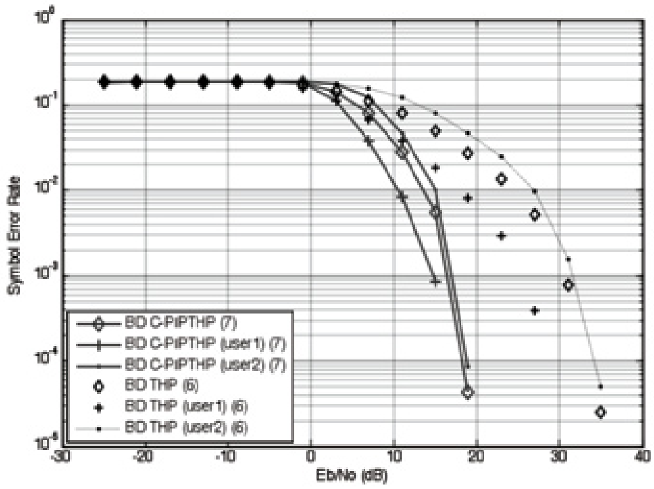


Fig. 6. BER of BD C-PIPTHP (7) and BD- THP (6). BD C-PIPTHP (7) gives better BER performance than BD THP (6).

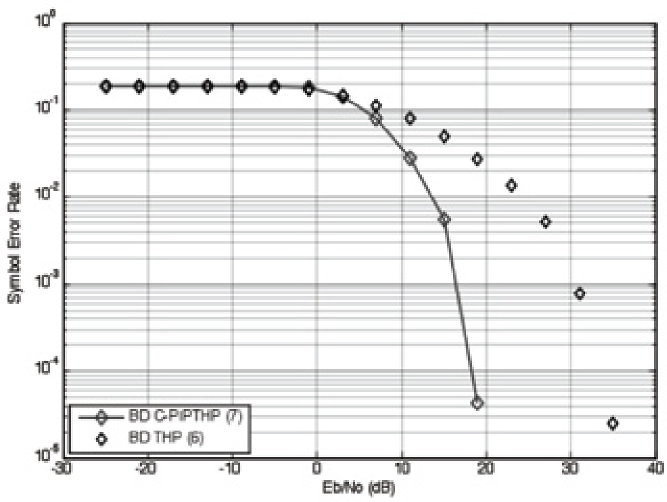


Fig. 7. SER of BD C-PIPTHP (7) and BD- THP(6). BD C-PIPTHP (7) gives better SER performance than BD THP (6).

In Figs. 8 and 9, the BER and SER plots for each user are added on top of the average BER and SER plots in Figs. 6 and Fig. 7, respectively. We see similar improvement as in Figs. 6 and 7. In particular, at the 0.001 level of BER or SER, BD C-PIPTHP (7) has above 10 dB gain in SNR than BD THP (6) for any user.

Table 3 shows the processing time of precoding schemes for multiuser MIMO systems. The new BD C-PIPTHP (7) is much faster than BD THP (6) due to the effect of pipelining system. The non-pipelining MU-MIMO system takes longer time to operate.

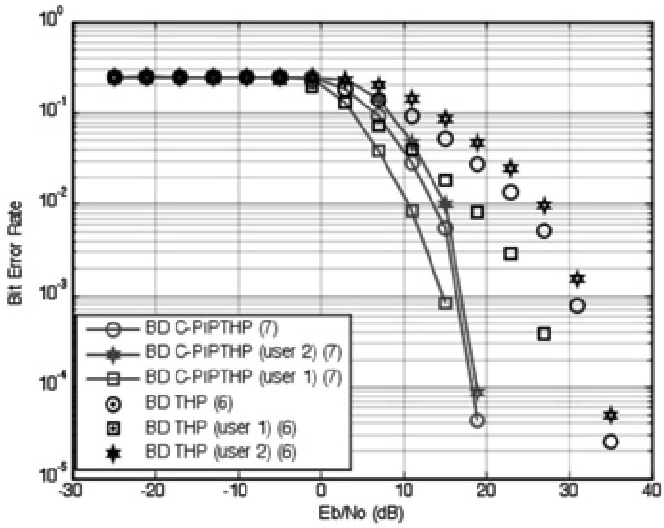


Fig. 8. BD C-PIPTHP (7) gives better BER performance than BD THP (6) for each user.

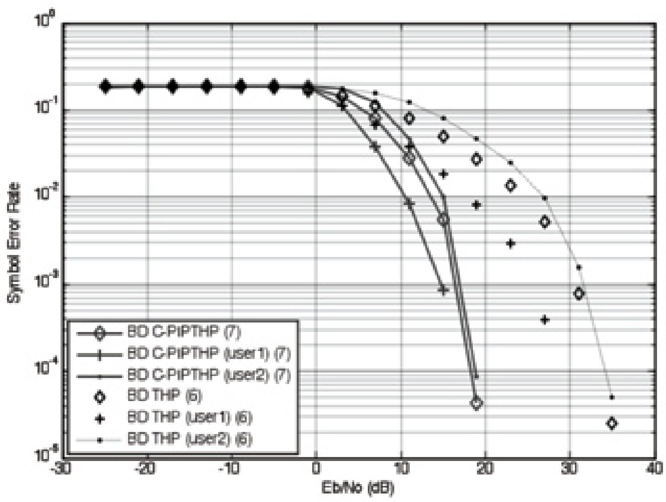


Fig. 9. Performance comparison for SER of BD C-PIPTHP and BD-THP for one user. BD C-PIPTHP (7) for user 1 gives the best SER performance of all downlink MU-MIMO THP-based schemes.

Table 3 Processing time for MU-MIMO downlink precoding schemes 4×[2×2].

BD THP (6)	4.48 seconds
BD C-PIPTHP (7)	1.30 seconds

5. Conclusion

An MU-MIMO system has played an important role in improving spectrum efficiency in mobile communication systems. We start by analyzing the improved performance of concatenated PIPTHP (C-PIPTHP) scheme in MIMO systems. Then, C-PIPTHP is extended to multiuser systems. This is achieved by the application of block diagonalization (BD). We found that our enhanced technique can operate efficiently in a downlink MU-MIMO system. The proposed system is not only capable of accomplishing high speed processing, but also improving the overall reliability. These advantages come from the use of pipelining and two pre-equalizers, respectively.

For our future work, we plan to test the use of generalized triangular decomposition (GTD) due to its capability of providing flexibility for quality of service constraints. We also plan to analytically derive the BER performance for our proposed system. Finally, we will attempt to combine IIR filters of low complexity PIPTHP and the feedback filter as one PIP system in order to reduce the cost of components.

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