Indirect Boundary Element Method for Calculation of Oseen's Flow Past a Circular Cylinder in the Case of Constant Variation

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Abstract

In the present paper, the indirect boundary element method (IDBEM) has been applied to calculate Oseen 's flow past a circular cylinder in the case of constant variation. The boundary of the circular cylinder is discretized into constant boundary elements over which the velocity distribution is calculated. The calculated results are also compared with the exact results.

Keywords: Indirect boundary element method, Oseen's flow, Circular cylinder, Constant variation.

1. Introduction

The boundary element method (BEM) is a numerical technique used to solve the different problems in science and technology. Computational methods such as the finite difference method (FDM) and finite element method (FEM) are very costly and time- consuming because in these methods the whole domain is discretized into a number of elements, whereas in boundary element methods the process of discretization takes place on the surface of a body. This reduces the size of system of the equations with a considerable reduction in data, which is needed to run a computer program efficiently. Boundary element methods are superior in several aspects to other computational methods because of their surface modeling approach. Thus Complicated structures can be more easily modeled by these methods and are therefore preferred by engineers. The results of boundary element methods are more accurate and reliable than those of classical methods. This establishes the fact that these methods (BEMs) are time-saving, accurate, efficient, and economical as compared to other numerical techniques (Mushtaq, M et al. 2008 and 2009).

These salient features of BEMs make them popular in the communities of engineering and science. These methods are essentially the methods for solving the partial differential equations arising in a wide range of fields, e.g., fluid mechanics, solid and fracture mechanics, heat transfer and electromagnetic theory, potential theory, elasticity, elatostatics and elastodynamics, etc. as detailed in (Brebbia and Walker, 1980). Furthermore, the area of their applications is increasing day by day. The indirect method has been used for many years in the past for flow field calculations due to its simplicity. The first work on flow field calculations around three-dimensional bodies was probably done by (Hess and Smith, 1962 & 1967). The direct boundary element method (DBEM) for potential flow calculations around objects was first applied in the past by (Morino et al. 1975). In recent past, boundary element methods have been applied by the author for flow field calculations around two- and three-dimensional bodies (Muhammad; G et al. 2008 and 2010).

2. Calculation of Oseen's Flow Past a Circular Cylinder

Boundary element methods are applied for both problems of exterior and interior flows in two dimensional space.

In this case, an indirect boundary element method is used to calculate the Oseen's flow around a circular cylinder. A circular cylinder of radius 'a' is held fixed in a uniform stream of incompressible viscous fluid flowing steadily around it. Let the centre of a cylinder be taken as the origin, and U_s be the velocity of uniform stream in the positive x – direction as given in Figure 1 (Shah, 2008).

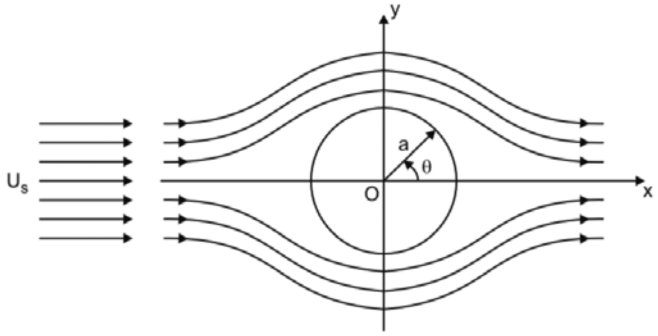

Fig. 1. Oseen's flow around a circular cylinder.

The hydrodynamical equations (Mine-Thomson,1968 and Lamb, 1932) are.

$$\left.\begin{aligned} U_s \frac{\partial u}{\partial x} &= -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \nabla^2 u \\ U_s \frac{\partial v}{\partial x} &= -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu \nabla^2 v \end{aligned}\right\} \tag{1}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$\text{Also} \quad \nabla^2 p = 0 \tag{3}$$

$$\text{and} \qquad p = \rho U_s \frac{\partial \phi}{\partial x} \tag{4}$$

The relations for the velocity components (Milne Thomson, 1968 and Lamb, 1932) are

$$\left.\begin{aligned} u &= \frac{\partial \phi}{\partial x} + \frac{1}{2k}\frac{\partial \chi}{\partial x} - \chi \\ v &= \frac{\partial \phi}{\partial y} + \frac{1}{2k}\frac{\partial \chi}{\partial y} \end{aligned}\right] \tag{5}$$

where k is an inertia coefficient. Also, we know that

$$\nabla^2 \phi = 0 \tag{6}$$

$$\text{Now} \quad \frac{\partial u}{\partial x} = \frac{\partial^2 \phi}{\partial x^2} + \frac{1}{2k}\frac{\partial^2 \chi}{\partial x^2} - \frac{\partial \chi}{\partial x}$$

$$\frac{\partial u}{\partial y} = \frac{\partial^2 \phi}{\partial y^2} + \frac{1}{2k}\frac{\partial^2 \chi}{\partial y^2}$$

Using these relations in equation (2), we get

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{1}{2k}\frac{\partial^2 \chi}{\partial x^2} - \frac{\partial \chi}{\partial x} - \frac{\partial^2 \phi}{\partial y^2} + \frac{1}{2k}\frac{\partial^2 \chi}{\partial y^2} = 0$$

$$\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right) + \frac{1}{2k}\left(\frac{\partial^2 \chi}{\partial x^2} + \frac{\partial^2 \chi}{\partial y^2}\right) - \frac{\partial \chi}{\partial x} = 0$$

$$\nabla^2 \phi + \frac{1}{2k}\left(\nabla^2 \chi - \frac{\partial \chi}{\partial x}\right) = 0$$

Since $\nabla^2 \phi = 0$

$$\frac{1}{2k}\left(\nabla^2 \chi - 2k\frac{\partial \chi}{\partial x}\right) = 0$$

$$\nabla^2 \chi - 2k\frac{\partial \chi}{\partial x} = 0$$

or $$\left(\nabla^2 - 2k\frac{\partial}{\partial x}\right)\chi = 0 \tag{7}$$

Let the appropriate solution of equation (7) for small values of kr be

$$\chi = -C(1 + kx)\left(\gamma + \ln\frac{1}{2}kr\right)$$

Let us assume :

$$\chi = -C(1 + kx)\left(\gamma + \ln\frac{1}{2}kr\right)$$

so that the boundary conditions $u = 0$ and $v = 0$ are satisfied on the boundary of a circular cylinder.

$$\frac{\partial \chi}{\partial x} = -Ck\left(\gamma + \ln\frac{1}{2}kr\right) - C(1 + kx)\left(\frac{x}{r^2}\right)$$

$$\frac{1}{2k}\frac{\partial \chi}{\partial x} = -\frac{C}{2k}\left\{k\left(\gamma + \ln\frac{1}{2}kr\right) + \frac{x}{r^2} + \frac{kx^2}{r^2}\right\}$$

$$\frac{1}{2k}\frac{\partial \chi}{\partial x} - \chi = -\frac{C}{2k}\left\{k\left(\gamma + \ln\frac{1}{2}kr\right) + \frac{x}{r^2} + \frac{kx^2}{r^2}\right\} - C(1 + kx)\left(\gamma + \ln\frac{1}{2}kr\right) \tag{8}$$

Also

$$\frac{\partial \phi}{\partial x} = -U_s + A_0\frac{\partial}{\partial x}(\ln r) + A_1\frac{\partial^2}{\partial x^2}(\ln r) + \ldots \tag{9}$$

Similarly,

$$\frac{\partial \chi}{\partial y} = -C(1+kx)\left(\frac{y}{r^2}\right)$$

$$\frac{1}{2k}\frac{\partial \chi}{\partial y} = -\frac{C}{2k}\left\{\frac{y}{r^2} + k\frac{xy}{r^2}\right\}$$

$$= -\frac{C}{2k}\left\{\frac{\partial}{\partial y}(\ln r) - \frac{1}{2}kr^2\frac{\partial^2}{\partial x\,\partial y}(\ln r) + ..\right\} \tag{10}$$

and

$$\frac{\partial \phi}{\partial y} = A_0\frac{\partial}{\partial y}(\ln r) + A_1\frac{\partial^2}{\partial x\,\partial y}(\ln r) + \ldots \tag{11}$$

Using the equations (8) - (11) in equation (5), we have

$$u = -U_s - C\left(\frac{1}{2} - \gamma - \ln\frac{1}{2}kr\right) + \left(A_0 - \frac{C}{2k}\right)\frac{\partial}{\partial x}(\ln r) + \left(A_1 + \frac{C}{4}r^2\right)\frac{\partial^2}{\partial x^2}(\ln r) + \ldots\ldots. \tag{12}$$

$$v = A_0\frac{\partial}{\partial y}\ln r + A_1\frac{\partial^2}{\partial x\,\partial y}\ln r + \ldots\ldots. - \frac{C}{2k}\frac{\partial}{\partial y}\ln r + \frac{C}{4}r^2\frac{\partial^2}{\partial x\,\partial y}\ln r + \ldots\ldots.$$

$$v = \left(A_0 - \frac{C}{2k}\right)\frac{\partial}{\partial y}\ln r + \left(A_1 + \frac{C}{4}r^2\right)\frac{\partial^2}{\partial x\,\partial y}\ln r + \ldots\ldots\ldots. \tag{13}$$

Boundary conditions are

$u = 0$ and $v = 0$ for $r = a$

Using the above boundary conditions in the equations (12) and (13), we obtain

$$0 = -U_s - C\left(\frac{1}{2} - \gamma - \ln\frac{1}{2}ka\right) + \left(A_0 - \frac{C}{2k}\right)\frac{\partial}{\partial x}(\ln a) + \left(A_1 + \frac{C}{4}a^2\right)\frac{\partial^2}{\partial x^2}(\ln r) + \ldots\ldots.$$

and

$$0 = \left(A_0 - \frac{C}{2k}\right)\frac{\partial}{\partial y}(\ln a) + \left(A_1 + \frac{C}{4}a^2\right)\frac{\partial^2}{\partial x\,\partial y}(\ln a) + \ldots.$$

which give on comparison

$$-U_s - C\left(\frac{1}{2} - \gamma - \ln\frac{1}{2}ka\right) = 0,$$

$$A_0 - \frac{C}{2k} = 0$$

and

$$A_1 + \frac{C}{4}a^2 = 0$$

$$\text{or} \qquad C = -\frac{2\,U_s}{\frac{1}{2} - \gamma - \ln \frac{1}{2} k a}$$

$$A_0 = \frac{C}{2k}$$

and

$$A_1 = -\frac{C}{4} a^2$$

Using these constants in equations (12) and (13), we obtain

$$u = -U_s - C\left(\frac{1}{2} - \gamma - \ln\frac{1}{2} k r\right) + \left(-\frac{C}{2k} + \frac{C}{2k}\right)\frac{\partial}{\partial x}(\ln r) + \left(\frac{C}{4} a^2 - \frac{C}{4} r^2\right)\frac{\partial^2}{\partial x^2}(\ln r)$$

$$= -U_s \left[\frac{1}{\gamma + \ln\frac{1}{2} k a - \frac{1}{2}}\left\{\left(\frac{1}{2} - \gamma - \ln\frac{1}{2} k r\right) - \frac{1}{2}(r^2 - a^2)\frac{\partial^2}{\partial x^2}(\ln r)\right\} + 1\right] \tag{14}$$

$$v = \frac{C}{4}(r^2 - a^2)\frac{\partial^2}{\partial x\,\partial y}(\ln r)$$

$$= \frac{U_s}{2(\gamma + \ln\frac{1}{2} k a - \frac{1}{2})}(r^2 - a^2)\frac{\partial^2}{\partial x\,\partial y}(\ln r) \tag{15}$$

The magnitude of velocity is given by the relation

$$V = \sqrt{u^2 + v^2} \tag{16}$$

Now to approximate the surface of a circular cylinder, the coordinates of extreme points on the boundary elements are generated in a computer program as under (Muhammad, G et al. 2008 and 2011).

The surface of the circular cylinder is discretized into m elements in a clockwise direction using the following formula.

$$\theta_k = [(m+3) - 2k]\,\pi / m, \quad k = 1, 2, \ldots\ldots, m \tag{17}$$

Then the coordinates of the extreme points of these m elements are calculated from

$$\left.\begin{aligned} x_k &= a \cos\theta_k \\ y_k &= a \sin\theta_k \end{aligned}\right\}, \quad k = 1, 2, \ldots, m \tag{18}$$

3. Constant Variation

Let us consider the constant element case in which nodal points are to be taken at the middle of each element. Also ϕ and $\frac{\partial \phi}{\partial n}$ are constant in this case over such elements and equal to the value at the mid-node of the element.

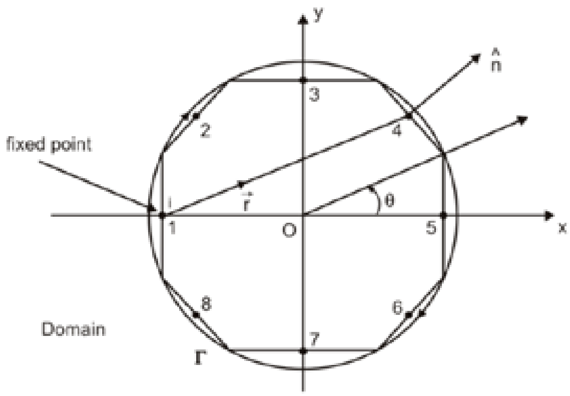

Fig. 2 The discretization of the surface of a circular cylinder into constant boundary elements

The mid-node coordinates over every element are defined by the formula

$$\left.\begin{aligned} x_m &= \frac{x_k + x_{k+1}}{2} \\ y_m &= \frac{y_k + y_{k+1}}{2} \end{aligned}\right\} \quad k, m = 1, 2, \ldots, 8 \tag{19}$$

The equation for the indirect method in the case of a doublet distribution for the problems of two-dimensional exterior flow is given by

$$-\frac{1}{2}\Phi_i + \frac{1}{2\pi}\int_{\Gamma - i} \Phi \frac{\partial}{\partial n}\left(\ln\frac{1}{r}\right) d\Gamma + \phi_\infty = -(\phi_{u.s.})_i \tag{20}$$

Since $(\phi_{u.s.})_i = -x_i$, equation(20) becomes

$$-\frac{1}{2}\Phi_i + \frac{1}{2\pi}\int_{\Gamma - i} \Phi \frac{\partial}{\partial n}\left(\ln\frac{1}{r}\right) d\Gamma + \phi_\infty = x_i \tag{21}$$

Matrix Form Equation (21) can be written as

$$-\frac{1}{2}\Phi_i + \sum_{j=1}^{m} \hat{H}_{ij}\,\Phi_i + \phi_\infty = x_i \tag{22}$$

or

$$\sum_{j=1}^{m} H_{ij}\,\Phi j + \phi_\infty = x_i \tag{23}$$

when all the nodal points are taken into consideration, then equation (23) can be put into the form

$$[H]\ \{\underline{U}\} = \{\underline{R}\} \tag{24}$$

where [H] is a matrix of influence coefficients, {U} is a vector of unknown total potentials and {R} on the R.H.S. is a known vector whose elements are the negative values of the velocity potential of the uniform stream at the nodal points on the boundary of the circular cylinder. Since $\frac{\partial \phi}{\partial n}$ is specified at each node of the element, the values of the perturbation velocity potential ϕ can be found at each node on the boundary. The total potential Φ is then found, which will then be used to calculate the velocity on the circular cylinder.

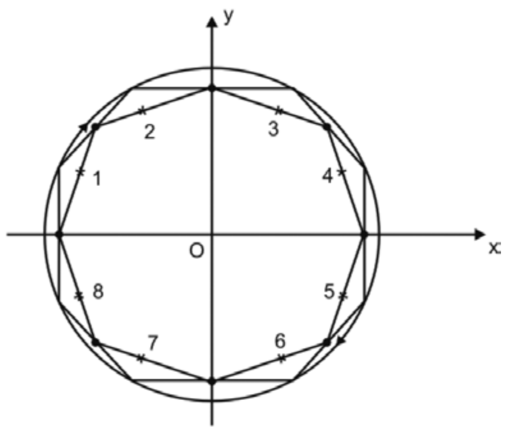

Fig. 3 Discretization of constant boundary elements.

The velocity midway between two nodes on the boundary can then be approximated by using the formula

$$\frac{L}{V} = \frac{\Phi_{k+1} - \Phi_k}{\text{Length from node } k \text{ to } k} \quad (25)$$

The method has been implemented using FORTRAN programming with 16, 32, and 64 constant boundary elements.

Where

Us = Uniform Stream Velocity.
C = Constant of integration.
γ = Euler's constant.
m = Number of nodes.
Φ = Total Potential.
ϕ = Perturbation Velocity Potential.
k = inertia coefficient.

Table 1. Comparison of computed velocity and exact velocity results for 16 constant boundary elements.

ELEMENT	X	Y	COMPUTED VELOCITY	EXACT VELOCITY	ERROR
1	-.94	.19	.39785E+00	.11841E+01	.07862E+01
2	-.80	.53	.11330E+01	.15207E+01	.03877E+01
3	-.53	.80	.16956E+01	.18973E+01	.02017E+01
4	-.19	.94	.20001E+01	.21237E+01	.01236E+01
5	.19	.94	.20001E+01	.21237E+01	.01236E+01
6	.53	.80	.16956E+01	.18973E+01	.02017E+01
7	.80	.53	.11330E+01	.15207E+01	.03877E+01
8	.94	.19	.39785E+00	.11841E+01	.07862E+01
9	.94	-.19	.39785E+00	.11841E+01	.07862E+01
10	.80	-.53	.11330E+01	.15207E+01	.03877E+01
11	.53	-.80	.16956E+01	.18973E+01	.02017E+01
12	.19	-.94	.20001E+01	.21237E+01	.01236E+01
13	-.19	-.94	.20001E+01	.21237E+01	.01236E+01
14	-.53	-.80	.16956E+01	.18973E+01	.02017E+01
15	-.80	-.53	.11330E+01	.15207E+01	.03877E+01
16	-.94	-.19	.39785E+00	.11841E+01	.07862E+01

Table 2. Comparison of computed velocity and exact velocity results for 32 constant boundary elements.

ELEMENT	X	Y	COMPUTED VELOCITY	EXACT VELOCITY	ERROR
1	-.99	.10	.19699E+00	.10503E+01	.08533E+01
2	-.95	.29	.58339E+00	.11537E+01	.05703E+01
3	-.87	.47	.94738E+00	.13237E+01	.03763E+01
4	-.77	.63	.12750E+01	.15173E+01	.02423E+01
5	-.63	.77	.15535E+01	.17022E+01	.01487E+01
6	-.47	.87	.17724E+01	.18568E+01	.00844E+01
7	-.29	.95	.19232E+01	.19669E+01	.00437E+01
8	-.10	.99	.20001E+01	.20240E+01	.00239E+01
9	.10	.99	.20001E+01	.20240E+01	.00239E+01
10	.29	.95	.19232E+01	.19669E+01	.00437E+01
11	.47	.87	.17724E+01	.18568E+01	.00844E+01
12	.63	.77	.15535E+01	.17022E+01	.01487E+01
13	.77	.63	.12750E+01	.15173E+01	.02423E+01
14	.87	.47	.94738E+00	.13237E+01	.03763E+01
15	.95	.29	.58339E+00	.11537E+01	.05703E+01
16	.99	.10	.19699E+00	.10503E+01	.08533E+01
17	.99	-.10	.19699E+00	.10503E+01	.08533E+01
18	.95	-.29	.58339E+00	.11537E+01	.05703E+01
19	.87	-.47	.94738E+00	.13237E+01	.03763E+01
20	.77	-.63	.12750E+01	.15173E+01	.02423E+01
21	.63	-.77	.15535E+01	.17022E+01	.01487E+01
22	.47	-.87	.17724E+01	.18568E+01	.00844E+01
23	.29	-.95	.19232E+01	.19669E+01	.00437E+01
24	.10	-.99	.20001E+01	.20240E+01	.00239E+01
25	-.10	-.99	.20001E+01	.20240E+01	.00239E+01
26	-.29	-.95	.19232E+01	.19669E+01	.00437E+01
27	-.47	-.87	.17724E+01	.18568E+01	.00844E+01
28	-.63	-.77	.15535E+01	.17022E+01	.01487E+01
29	-.77	-.63	.12750E+01	.15173E+01	.02423E+01
30	-.87	-.47	.94738E+00	.13237E+01	.03763E+01
31	-.95	-.29	.58339E+00	.11537E+01	.05703E+01
32	-.99	-.10	.19698E+00	.10503E+01	.08533E+01

Table 3. Comparison of computed velocity and exact velocity results for 64 constant boundary elements.

ELEMENT	X	Y	COMPUTED VELOCITY	EXACT VELOCITY	ERROR
1	-1.00	.05	.98258E-01	.10178E+01	.09195E+01
2	-.99	.15	.29383E+00	.10454E+01	.07515E+01
3	-.97	.24	.48654E+00	.10975E+01	.06109E+01
4	-.94	.34	.67460E+00	.11691E+01	.04945E+01
5	-.90	.43	.85616E+00	.12543E+01	.03981E+01
6	-.86	.51	.10295E+01	.13476E+01	.03181E+01
7	-.80	.59	.11929E+01	.14442E+01	.02513E+01
8	-.74	.67	.13447E+01	.15401E+01	.01954E+01
9	-.67	.74	.14837E+01	.16322E+01	.01485E+01
10	-.59	.80	.16084E+01	.17176E+01	.01092E+01
11	-.51	.86	.17175E+01	.17944E+01	.00769E+01
12	-.43	.90	.18102E+01	.18609E+01	.00507E+01
13	-.34	.94	.18854E+01	.19155E+01	.00301E+01
14	-.24	.97	.19424E+01	.19574E+01	.00150E+01
15	-.15	.99	.19808E+01	.19857E+01	.00049E+01
16	-.05	1.00	.20000E+01	.20000E+01	.00000E+01
17	.05	1.00	.20000E+01	.20000E+01	.00000E+01
18	.15	.99	.19808E+01	.19857E+01	.00049E+01
19	.24	.97	.19424E+01	.19574E+01	.00150E+01
20	.34	.94	.18854E+01	.19155E+01	.00301E+01
21	.43	.90	.18102E+01	.18608E+01	.00506E+01
22	.51	.86	.17175E+01	.17944E+01	.00769E+01
23	.59	.80	.16084E+01	.17176E+01	.01092E+01
24	.67	.74	.14837E+01	.16322E+01	.01485E+01
25	.74	.67	.13448E+01	.15401E+01	.01953E+01
26	.80	.59	.11929E+01	.14442E+01	.02513E+01
27	.86	.51	.10294E+01	.13476E+01	.03182E+01
28	.90	.43	.85616E+00	.12543E+01	.03981E+01
29	.94	.34	.67461E+00	.11691E+01	.04944E+01
30	.97	.24	.48654E+00	.10975E+01	.06109E+01
31	.99	.15	.29383E+00	.10454E+01	.07515E+01
32	1.00	.05	.98239E-01	.10178E+01	.09195E+01
33	1.00	-.05	.98242E-01	.10178E+01	.09195E+01

ELEMENT	X	Y	COMPUTED VELOCITY	EXACT VELOCITY	ERROR
34	.99	-.15	.29383E+00	.10454E+01	.07515E+01
35	.97	-.24	.48654E+00	.10975E+01	.06109E+01
36	.94	-.34	.67461E+00	.11691E+01	.04944E+01
37	.90	-.43	.85615E+00	.12543E+01	.03981E+01
38	.86	-.51	.10295E+01	.13476E+01	.03181E+01
39	.80	-.59	.11929E+01	.14442E+01	.02513E+01
40	.74	-.67	.13448E+01	.15401E+01	.01953E+01
41	.67	-.74	.14837E+01	.16322E+01	.01485E+01
42	.59	-.80	.16084E+01	.17176E+01	.01092E+01
43	.51	-.86	.17175E+01	.17944E+01	.00769E+01
44	.43	-.90	.18102E+01	.18608E+01	.00506E+01
45	.34	-.94	.18854E+01	.19155E+01	.00301E+01
46	.24	-.97	.19424E+01	.19574E+01	.00150E+01
47	.15	-.99	.19808E+01	.19857E+01	.00049E+01
48	.05	-1.00	.20000E+01	.20000E+01	.00000E+01
49	-.05	-1.00	.20000E+01	.20000E+01	.00000E+01
50	-.15	-.99	.19808E+01	.19857E+01	.00049E+01
51	-.24	-.97	.19424E+01	.19574E+01	.00150E+01
52	-.34	-.94	.18854E+01	.19155E+01	.00301E+01
53	-.43	-.90	.18102E+01	.18609E+01	.00507E+01
54	-.51	-.86	.17175E+01	.17944E+01	.00769E+01
55	-.59	-.80	.16084E+01	.17176E+01	.01092E+01
56	-.67	-.74	.14837E+01	.16322E+01	.01485E+01
57	-.74	-.67	.13447E+01	.15401E+01	.01954E+01
58	-.80	-.59	.11929E+01	.14442E+01	.02513E+01
59	-.86	-.51	.10295E+01	.13476E+01	.03181E+01
60	-.90	-.43	.85616E+00	.12543E+01	.03981E+01
61	-.94	-.34	.67460E+00	.11691E+01	.04945E+01
62	-.97	-.24	.48654E+00	.10975E+01	.06109E+01
63	-.99	-.15	.29384E+00	.10454E+01	.07515E+01
64	-1.00	-.05	.98246E-01	.10178E+01	.09195E+01

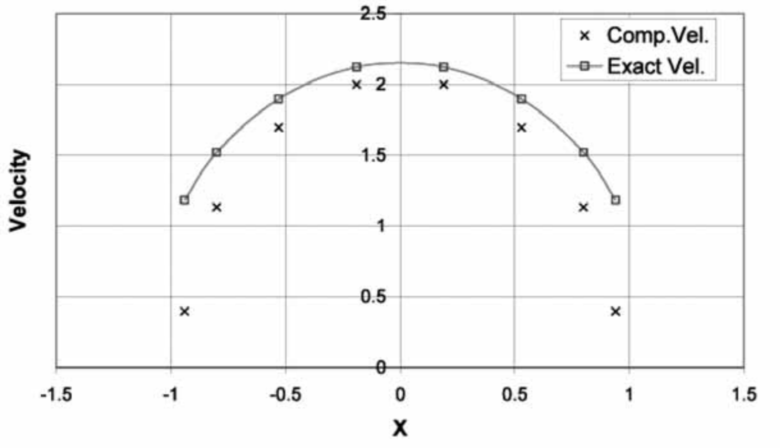

Fig. 4. Comparison of exact and computed values over the boundary of a circular cylinder for 16 constant boundary elements.

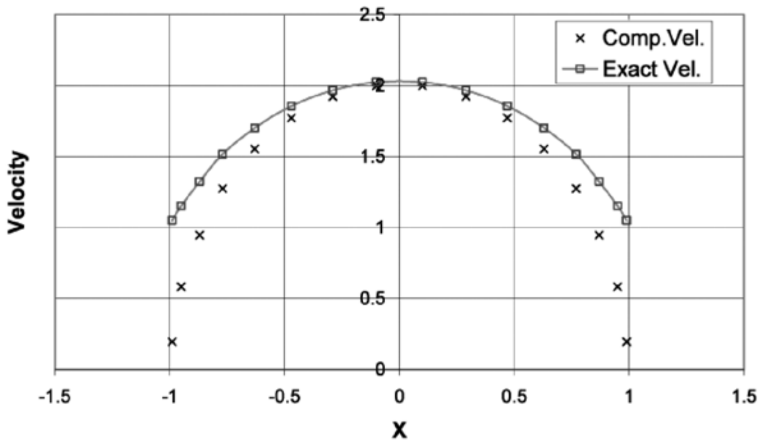

Fig. 5. Comparison of exact and computed values over the boundary of a circular cylinder for 32 constant boundary elements.

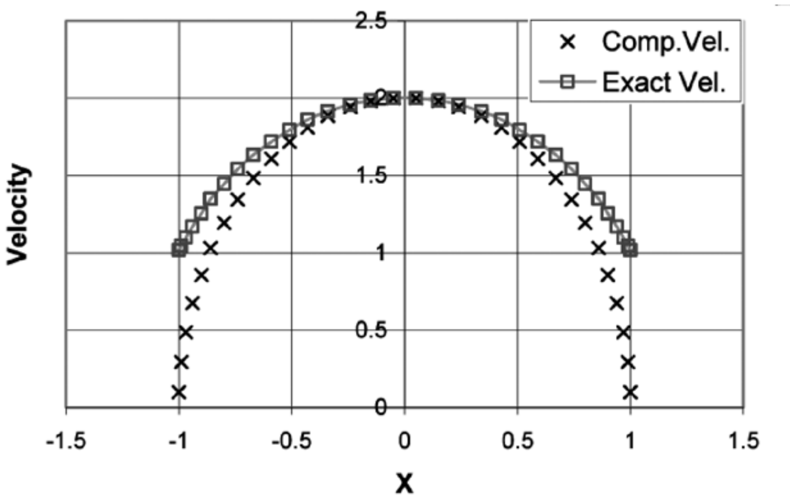

Fig. 6. Comparison of exact and computed values over the boundary of a circular cylinder for 64 constant boundary elements.

4. Conclusion

The indirect boundary element method has been applied to calculate Oseen's flow past a circular cylinder in the case of constant variation. The improvement in results gained by taking 32 and 64 constant elements can be seen from Tables 2 and 3, and Figures 5 and 6, and the improvement increases with an increase in number of boundary elements. Moreover, at the top of Figure 6, the computed results are convergent with the exact results and as we come down, these results are slightly divergent from the exact ones due to an increase of viscous effects.

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