Effects of Viscous Dissipation and Radiation on Magnetohydrodynamic Free Convection flow along a Sphere with Joule Heating and Heat Generation

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Abstract

The effects of viscous dissipation and radiation on magnetohydrodynamic(MHD) free convection flow along a sphere with joule heating and heat generation have been investigated. The governing equations are transformed into dimensionless non-similar equations by using set of suitable transformations and solved numerically by the finite difference method along with Newton's linearization approximation. We have focused our attention on the evaluation of velocity profiles, temperature profiles, shear stress in terms of local skin friction and the rate of heat transfer in terms of local Nusselt number for different values of radiation parameter, Prandlt number, heat generation parameter, magnetic parameter, joule heating parameter and viscous dissipation parameter and the numerical results have been shown graphically.

Keywords: Natural convection, Radiation, Prandtl number, Heat generation, Nusselt number, Joule heating parameter, Viscous dissipation parameter and Magnetohydrodynamic.

1. Introduction

In this paper, the description of the effects of viscous dissipation and radiation on magnetohydrodynamic free convection flow along a sphere with joule heating and heat generation has been focused on. Many researchers have studied the problems of free convection boundary layer flow over or on a various types of geometrical shapes. Amongst them Nazar et al. [1] studied free convection boundary layer on an isothermal sphere in a micropolar fluid. Soundalgekar et al. [2] have studied radiation effects on free convection flow of a gas past a semi-infinite flat plate. Akhter and Alim [3] studied effects of radiation on natural convection flow around a sphere with uniform surface heat flux. Limitations of this approximation are discussed briefly in Ö zisik [4]. The transformed boundary layer equations are solved numerically using Keller box method described by Keller [5] and later by Cebeci and Bradshaw [6] along with Newton's linearization approximation and used by Hossain et al. [7] and Alim et al. [8]. The effects of radiation and joule heating on magnetohydrodynamic free convection flow along a sphere with heat generation have been studied by Miraj et al. [9-10]. Miraj et al. [11] have also studied the effects of pressure work and radiation on natural convection flow around a sphere with heat generation. Molla et al. [12] have studied the problem of magnetohydrodynamic natural convection flow on a sphere in the presence of heat generation or absorption. Alam et al. [13] studied the viscous dissipation effects with MHD natural convection flow on a sphere in the presence of heat generation. Amin [14] also analyzed the influences of both first and second order resistance, due to the solid matrix of non-darcy porous medium, Joule heating and viscous dissipation on forced convection flow from a horizontal circular cylinder under the action of transverse magnetic field. Numerical results have been obtained in terms of local skin friction, rate of heat transfer, velocity profiles as well as temperature profiles for a selection of relevant physical parameters and shown graphically.

2. Formulation of the problem

A steady two-dimensional magnetohydrodynamic (MHD) natural convection boundary layer flow from an isothermal sphere of radius a, which is immersed in a viscous and incompressible o ptically dense fluid with heat generation and radiation heat loss is considered. It is assumed that theconstant temperature at the surface of thsphere is T_w, where $T_w > T_\infty$. Here T_∞ is the ambient temperature of the fluid, T is the temperature of the fluid in the boundary layer, g is the acceleration due to gravity and (U, V) are velocity components along the (X, Y) axes. The physical configuration considered is as shown in Fig. 1

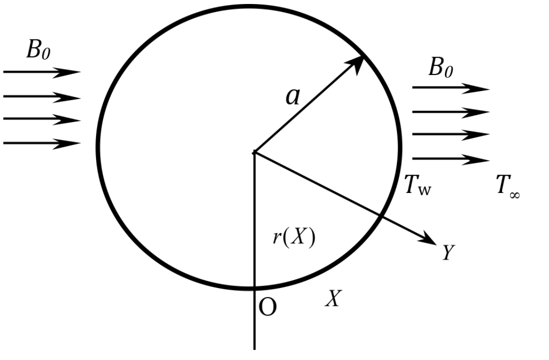

Fig. 1. Physical model and coordinate system.

According to the above assumption, the governing equations continuity, momentum and energy for steady two-dimensional laminar boundary layer flow problem under consideration can be written as :

$$\frac{\partial}{\partial X}(rU) + \frac{\partial}{\partial Y}(rV) = 0 \qquad (1)$$

$$U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \nu\frac{\partial^2 U}{\partial Y^2} + g\beta\,(T-T_\infty)\sin\left(\frac{X}{a}\right) - \frac{\sigma_0 B_0^2}{p}U \qquad (2)$$

$$U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \nu\frac{k}{pc_p}\left(\frac{\partial^2 T}{\partial Y^2} - \frac{1}{k}\frac{\partial q_r}{\partial Y}\right) + \frac{\nu}{pc_p}\left(\frac{\partial U}{\partial Y}\right)^2 + \frac{Q_o}{pc_p}(T-T_\infty) + \frac{\sigma_0 B_0^2}{p}U^2 \qquad (3)$$

With the boundary conditions

$$U = V = 0,\ T = T_\infty\ at\ Y = 0$$
$$U \to 0,\ \ T \to T_\infty\ \ as\ \ Y \to \infty \qquad (4)$$

where r(X)=α sin is the radial distance from the centre to the surface of the sphere, k is the thermal conductivity, β is the coefficient of thermal expansion, B_0 is the strength of magnetic field, σ_0 is the electrical conductivity, ν (= μ/ϱ) is the kinematic viscosity, μ is the viscosity of the fluid, ϱ is the density, cp is the specific heat due to constant pressure and Q_0 is the heat generation constant.

The above equations are non-dimensionalised using the following new variables:

$$\xi=\frac{X}{a},\ \eta=\frac{Y}{a}Gr^{\frac{1}{4}},u=\frac{Y}{a}Gr^{-\frac{1}{2}},\ v=\frac{aV}{v}Gr^{-\frac{1}{4}} \quad (5)$$

$$\theta=\frac{T-T_\infty}{T_w-T_\infty},\ Gr - g\beta(T_w-T_\infty)\frac{a^3}{v^2} \quad (6)$$

$$\theta=\frac{T_w}{T_\infty},\ \Delta=\theta w-1=\frac{T_w}{T_\infty}-1=\frac{T_w-T_\infty}{T_\infty} \quad (7)$$

where Gr is the Grashof number, θ is the nondimensional temperature function, θ_w is the surface temperature parameter and q_r is the radiation heat flux. Thus the Rosseland diffusion approximation proposed by Siegel and Howell [15] is given by simplified radiation heat flux term as:

$$q_r = -\frac{4\sigma}{3(a_r+\sigma_s)}\frac{\partial T^4}{\partial Y} \quad (8)$$

where a_r is the Rosseland mean absorption co-efficient, σ_s is the scattering co-efficient and σ is the Stefan-Boltzmann constant.

Substituting Eqs. (5), (6) and (7) in the continuity Eq. (1), the momentum Eq. (2) and the energy Eq. (3) leads to the following non-dimensional equations:

$$\frac{\partial}{\partial_\xi}(ru)+\frac{\partial_u}{\partial_\eta}(rv)=0 \quad (9)$$

$$u\frac{\partial_u}{\partial_\xi}+v\frac{\partial_u}{\partial_\eta}=\frac{\partial^2 u}{\partial\eta^2}+\theta\sin\xi-\frac{\sigma_0 B_0^2 a^2}{pv\,Gr^{\frac{1}{4}}}u \quad (10)$$

$$u\frac{\partial\theta}{\partial_\xi}+v\frac{\partial\theta}{\partial_\eta}=\frac{1}{Pr}\frac{\partial}{\theta\eta}\left[\left\{1\frac{4}{3}Rd\right.\right.$$

$$\left.\left.(1+(\theta_w-1)\theta)^3\right\}\right]+Vd\left(\frac{\partial_u}{\partial_\eta}\right)^2+Q\theta+Ju^2 \quad (11)$$

where $Pr=\frac{u\,c_p}{k}$ is the Prandtl number, $Q=\frac{Q_0 a^2}{u\,c_p Gr^{\frac{1}{2}}}$ is the heat generation parameter, $J=\frac{Q_0 B_0^2 v}{pc_p(T_w-T_\infty)}$ is the joule heating parameter, $Vd=\frac{v2Gr}{p\,a^2 c_p(T_w-T_\infty)}$ is the viscous dissipation and $Rd=\frac{4\sigma T_\infty^3}{k(a_r+\sigma_s)}$ is the radiation parameter.

With the boundary conditions (4) becomes :

$$U=V=0,\ \theta=1\ at\ \eta=0$$
$$u\to 0,\ \ \theta\to 0\ \ as\ \eta\to\infty \quad (12)$$

To solve Eqs. (10) and (11) with the help of following variables :

$$\psi=\xi\ r(\xi)f(\xi,\eta)\ ,\ \theta=\theta(\xi,\eta),\ r(\xi)=\sin_\xi \quad (13)$$

where ψ is the stream function defined by :

$$u=\frac{1}{r}\frac{\partial\psi}{\partial\eta},\ \ v=\frac{1}{r}\frac{\partial\psi}{\partial\xi} \quad (14)$$

$$u=\frac{1}{r}\frac{\partial\psi}{\partial\eta}=-\frac{1}{r}\frac{\partial}{\partial\eta}(\xi rf)=\xi\frac{\partial f}{\partial\eta}=\xi f'$$

where $f'=\frac{\partial f}{\partial\eta}$

$$v=-\frac{1}{r}\frac{\partial\psi}{\partial\xi}=-\frac{1}{r}\frac{\partial}{\partial\xi}(\xi rf)=\frac{1}{r}\left(rf+\xi r'f+\xi r\frac{\partial f}{\partial\xi}\right)$$

$$=-f-\frac{\xi}{\sin\xi}f\cos\xi-\xi\frac{\partial f}{\partial\xi}$$

Using the above values in Eq. (10), we get the following equation :

$$\frac{\partial^2 f}{\partial \eta^3}+\left(1+\frac{\xi}{\sin\xi}\cos\xi\right)f\frac{\partial^2 f}{\partial \eta^2}+\theta\frac{\sin\xi}{\xi}$$

$$-\left(\frac{\partial f}{\partial \eta}\right)^2-M\frac{\partial f}{\partial \eta}=\xi\left(\frac{\partial f}{\partial \eta}\frac{\partial^2 f}{\partial\xi\partial \eta}-\frac{\partial f}{\partial \xi}\frac{\partial^2 f}{\partial \eta^2}\right) \qquad (15)$$

where $M=\frac{\sigma_0 B_0{}^2 a^2}{\mu Gr^{1/2}}$ is the MHD parameter

Putting the values of u and v in Equation (11), we get the following equation :

$$\xi f'\frac{\partial\theta}{\partial\xi}+\left(-f-\frac{\xi}{\sin\xi}f\cos\xi-\xi\frac{\partial f}{\partial\xi}\right)\frac{\partial\theta}{\partial\eta}$$

$$=\frac{1}{Pr}\frac{\partial}{\partial\eta}\left\{\left[1+\frac{4}{3}Rd(1+(\theta_w-1)\theta)^3\right]\right\}\frac{\partial\theta}{\partial\eta}$$

$$+Vd\xi^2\left(\frac{\partial^2 f}{\partial\eta^2}\right)+Q\theta+J\xi^2\left(\frac{\partial f}{\partial\eta}\right)^2 \qquad (16)$$

Along with boundary conditions :

$$f=f'=0,\ \theta=1\ at\ \eta=0$$
$$f'\to 0,\quad \theta\to 0\quad as\ \eta\to\infty \qquad (17)$$

where primes denote the differentiation of the function with respect to η

It can be seen that near the lower stagnation point of the sphere, i.e., $\xi\approx 0$, Eqs. (15) and (16) reduce to the following ordinary differential equations:

$$f'''+2ff''-f'^{+}\theta^{-}Mf'=0 \qquad (18)$$

$$\frac{1}{Pr}\left[\left\{1+\frac{4}{3}Rd(1+(\theta_w-1)\theta)^3\right\}\theta'\right]'$$
$$+2f\theta'+Q\theta=0 \qquad (19)$$

Subject to the boundary conditions :

$$f(0)=f'(0)=0,\ \theta(0)=1$$
$$f'\to 0,\quad \theta\to 0\quad as\ \eta\to\infty \qquad (20)$$

In practical applications, the physical quantities of principle interest are the shearing stress τ_w , the rate heat transfer and the rate of species concentration transfer in terms of the skin friction coefficient C_f and Nusselt number Nu, which can be written in non-dimensional form as :

$$C_f=\frac{a^2 Gr^{-\frac{1}{4}}}{\mu\nu}\tau_w\ and\ Nu=\frac{aGr^{-\frac{1}{4}}}{k(T_w-T_\infty)}(q_c+q_r)_{Y=0} \qquad (21)$$

Where $\tau_w=\mu\left(\frac{\partial U}{\partial Y}\right)_{Y=0}$ is the shearing stress,

$q_c=-\left(\frac{\partial T}{\partial Y}\right)_{Y=0}$ is the conduction heat flux,

k being the thermal conductivity of the fluid

and q_r is the radiation heat flux.

The heat flux q_x is defined by :

$$qw=(q_c)_{Y=0}+(q_r)_{Y=0}=-k\left(\frac{\partial T}{\partial Y}\right)_{Y=0}+(qr)_{Y=0}$$

Using Eqs. (5), (6), boundary condition (20) and putting the values of τ_w and q_w in (21), we get the following equations

$$Nu=-\left(1+\frac{4}{3}Rd\theta_w{}^3\right)\theta'(\xi,0) \qquad (22)$$

$$C_f=\xi f''(\xi,0) \qquad (23)$$

The values of the velocity and temperature distribution are calculated respectively from the following relations:

$$u=\left(\frac{\partial f}{\partial\eta}\right),\quad \theta=\theta(\xi,\eta) \qquad (24)$$

We discuss the velocity distribution as well as the temperature profiles for a selection of parameter sets consisting of heat generation parameter, magnetic parameter, viscous dissipation parameter, joule heating parameter, and the Prandlt number at different position of ξ .

3. Method of Solution

To obtain the solution of the problem, the numerical method used is a finite difference method known as Keller-box method [5]. To begin with, the partial differential Eqs. (15)-(16) are first converted into a system of first order differential equations. Then these equations are expressed in finite difference forms by approximating the functions and their derivatives in terms of the centered differences and two point averages using only values at the corner of the box (or mesh rectangle). Denoting the mesh points in the (ξ ,η)-plane by ξ_i and η_j where i = 1, 2, . . . , M and j = 1, 2, . . . , N, central difference approximations are made, such that those equations involving ξ explicitly are centered at ($\xi_{i-1/2}$, $\eta_{j-1/2}$) and the remainder at (ξ_i , $\eta_{j-1/2}$), where $\frac{1}{2}\eta_{j-1/2} = (\eta_{j+}\eta_{j-1})$etc. Grid dependency has been tested and solutions are obtained with grid of optimum dimensions 182×200 in the (ξ, η) domain and non-uniform mesh size is employed to produce results of high accuracy near the coordinate $\xi = 0$, $\eta = 0$. The central difference approximations reduces the system of first order differential equations to a set of non-linear difference equations for the unknown at ξ_i in terms of their values at ξ_{i-1}. The resulting set of nonlinear difference equations are solved by using the Newton's quasi-linearization method. The Jacobian matrix has a block-tridiagonal structure and the difference equations are solved using a block-matrix version of the Thomas algorithm; further details of the computational procedure have been discussed in the book by Cebecci and Bradshow [6] and widely used by many authors including Hossain et al. [7] and Alim et al. [8].

4. Results and discussion

We have investigated the effects of viscous dissipation and radiation on magnetohydrodynamic free convection flow along a sphere with joule heating and heat generation. Solutions are obtained in terms of velocity profiles, temperature profiles, skin friction coefficient, rate of heat transfer and presented graphically for selected values of the radiation parameter (Rd = 1.00, 2.50, 4.00, 6.00, 9.00), Prandtl number (Pr = 0.72, 1.00, 3.00, 5.00, 7.00), heat generation parameter (Q = 0.00, 0.10, 0.15, 0.20, 0.25), magnetic parameter (M = 0.10, 0.50, 1.00, 2.00, 3.00) , joule heating parameter (J = 0.30, 1.00, 2.00, 3.00, 3.50) and viscous dissipation parameter (Vd = 0.10, 25.00, 50.00, 80.00, 100.00) against η at any position of ξ. The effects for different values of radiation parameter Rd the velocity profiles and temperature profiles in the case of Prandtl number Pr = 0.72, heat generation parameter Q = 0.20, magnetic parameter M = 0.50, joule heating parameter J = 0.30 and viscous dissipation parameter Vd =25.00 are shown in Fig. 2(a) and 2(b), respectively. We observed that, when the radiation parameter Rd increases, both the velocity profiles and the temperature profiles increase. With increasing values of Prandtl number the velocity profiles and the temperature profiles decrease as shown in Fig. 3(a) and 3(b), respectively. The velocity boundary layer thickness and thermal boundary layer thickness decrease for the increasing values of radiation parameter. In Fig. 4(a) and 4(b), the heat generation parameter Q increases while with radiation parameter Rd = 1.00, Prandtl number Pr = 0.72, magnetic parameter M = 0.50, joule heating parameter J = 0.30 and viscous dissipation parameter Vd = 25.00, both the velocity and temperature profiles, increase. Fig. 5(a) display results for the velocity profiles for different values of magnetic parameter M in the case of radiation parameter Rd = 1.00, Prandtl number Pr = 0.72, heat generation parameter Q = 0.10, joule heating parameter J = 0.30, and viscous dissipation parameter Vd = 25.00. It can be seen from Fig. 5(a) that as the magnetic parameter M increases, the velocity profiles decrease to the position of η = 3.33718. From that position of η velocity profiles change with the increase of magnetic parameter and all the velocity profiles cross in between η = 3.33718 and η = 4.64344. This is because of the velocity profiles, having lower peak values for higher values of magnetic parameter, tend to decrease comparatively slower along η direction than velocity profiles with higher peak values for lower values of magnetic parameter Fig. 5(b) displays results for the Temperature profiles, temperature profiles increase for increasing values of magnetic parameter. Fig. 6(a)-6(b) display results of the velocity profiles and temperature profiles, for different values of joule heating parameter J with radiation parameter Rd = 1.00, Prandtl number Pr = 0.72, heat generation parameter Q = 0.20, magnetic parameter M = 0.50 and viscous dissipation parameter Vd = 25.00. The joule heating parameter J increases, and the velocities rise the position of η = 1.23788 for J = 0.30, 1.00, 2.05, 2.00, 3.50 and from that position of η velocities fall slowly and finally approaches to zero. It is also observed from Fig. 6(b) that as the joule heating parameter J increases, the temperature profiles increases. Fig. 7(a)-7(b) display results for increasing values of viscous dissipation parameter Vd , the velocity profiles and the temperature profiles increase. We observed from Figure (b) that as the viscous dissipation parameter Vd increases, the temperature profiles increase. There are no changes of velocity boundary layer thickness and thermal boundary layer thickness.

It has been seen from Fig. 8(a) that as radiation parameter Rd increases, the skin friction coefficient C_f increases up to the position of ξ = 0.83776. From that position the skin friction coefficient C_f decreases and the rate of heat transfer Nu increases as shown in the Fig. 8(b). Fig. 9(a) shows that when the Prandtl number Pr increases, the skin friction coefficients C_f decrease up to the positionof ξ = 0.71558 From that position of ξ, skinfriction coefficients C_f change with the increase of Prandtl number Pr. It is seen from Fig. 9(b) that as Prandtl number Pr increases, the rate of heat transfer Nu increases up to the position of ξ = 0.06981 and from the point ξ = 0.27925 the rate of heat transfer decreases for Pr = 0.72, 1.00, 3.00, 5.00 and 7.00. That is, the rate of heat transfer falls slowly from the position of ξ = 0.06981 and from the point ξ = 0.27925 the rate of heat transfer quickly falls. The skin friction coefficient C_f increases and rate of heat transfer decreases for on increase of heat generation parameter Q as shown in Fig. 10(a)-10(b). In Figure 11(a), the values of magnetic parameter M increase for the while radiation parameter Rd =1.00, Prandtl number Pr = 0.72, heat generation parameter Q=0.10, joule heating parameter J = 0.30 and viscous dissipation parameter Vd =25.00, and the skin friction coefficient Cf decreases. It is observed from Fig. 11(b), that the rate of heat transfer decreases along the ξ direction from lower stagnation point to downstream. From Fig. 12(a)-12(b) we observe that the skin friction coefficient C_f increases and heat transfer coefficient decreases for increasing values of joule heating parameter J with radiation parameter Rd = 1.00, Prandtl number Pr = 0.72, heat generation parameter Q = 0.10, magnetic parameter M = 0.50, and viscous dissipation parameter Vd =25.00. Fig. 13(a) shows the skin friction coefficient C_f increases for different increasing values of viscous dissipation parameter Vd with radiation parameter Rd = 1.00, Prandtl number Pr = 0.72, heat generation parameter Q = 0.10, magnetic parameter M = 0.50, and joule heating parameter J = 0.30. Frictional force at the wall becomes much higher towards the downstream for higher values of Vd and the rate of heat transfer as shown in Fig. 13(b) gradually decreases for higher values of viscous dissipation parameter.

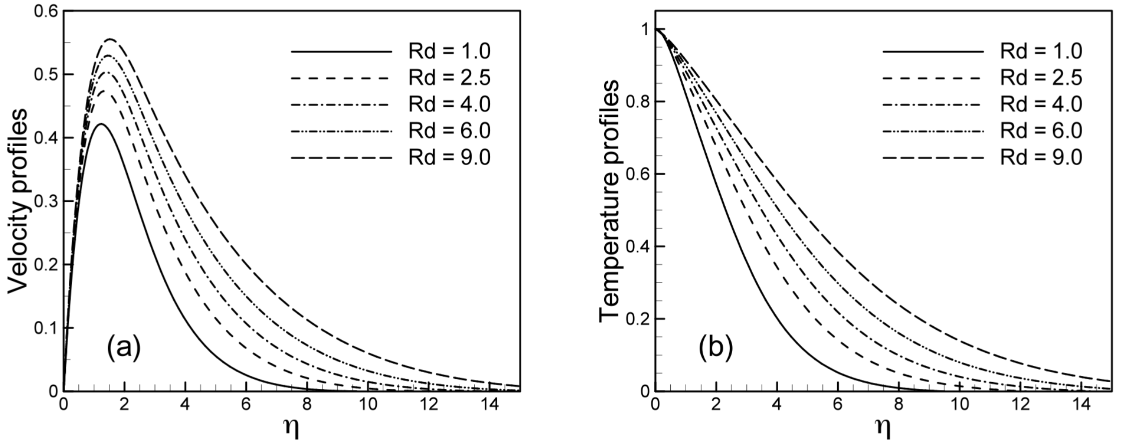

Fig. 2. (a) Velocity profiles and (b) Temperature profiles for different values of *Rd* when *Pr* = 0.72, *Q* =0.2, *M* = 0.5, *J* = 0.3 and *Vd* = 25.0

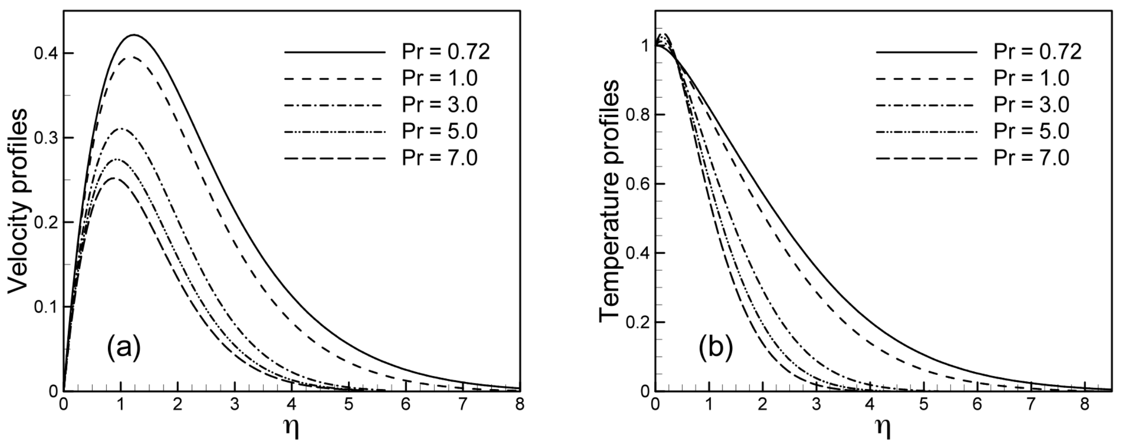

Fig. 3. (a) Velocity profiles and (b) Temperature profiles for different values of *Pr* when *Rd* = 1.0, *Q* =0.2, *M* = 0.5, *J* = 0.3 and *Vd* = 25.0.

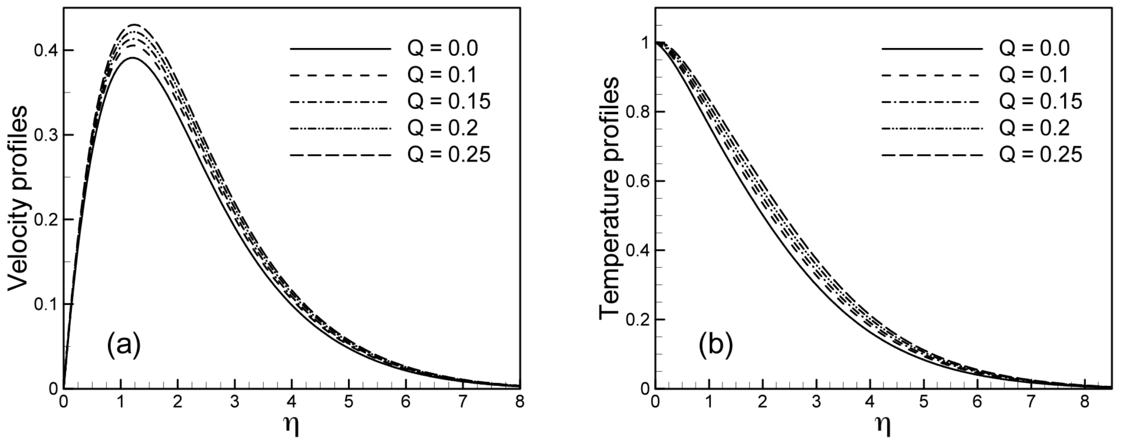

Fig. 4. (a) Velocity profiles and (b) Temperature profiles for different values of *Q* when *Rd* = 1.0, *Pr* = 0.72, *M* = 0.5, *J* = 0.3 and *Vd* = 25.0.

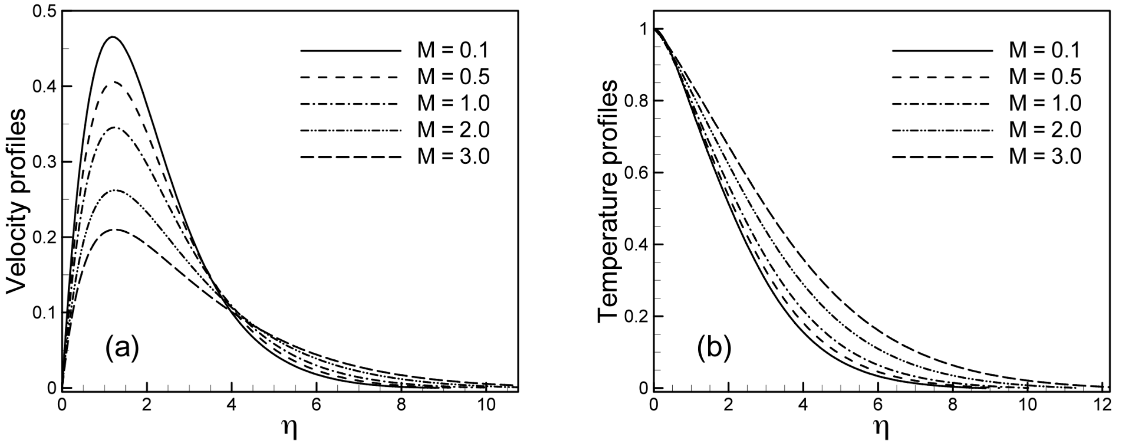

Fig. 5. (a) Velocity profiles and (b) Temperature profiles for different values of *M* when *Rd* = 1.0, *Pr* = 0.72, *Q* = 0.1, J = 0.3 and *Vd* = 25.0.

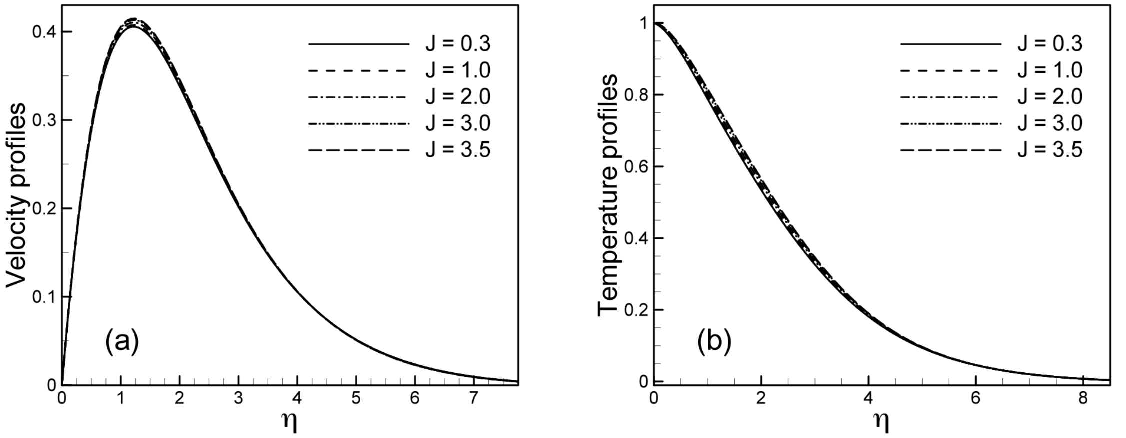

Fig. 6. (a) Velocity profiles and (b) Temperature profiles for different values of *J* when *Rd* = 1.0, *Pr* =0.72, *Q* =0.2, *M* = 0.5 and *Vd* = 25.0.

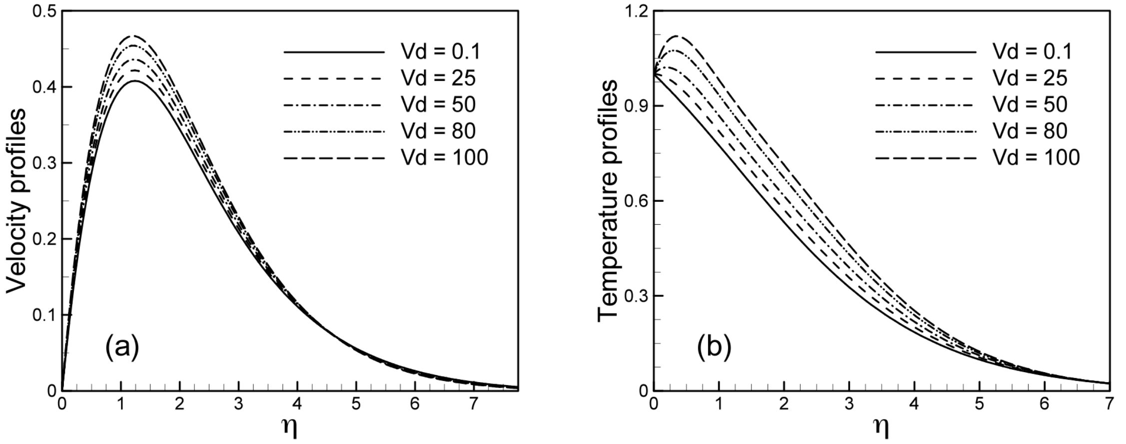

Fig. 7. (a) Velocity profiles and (b) Temperature profiles for different values of *Vd* when *Rd* = 1.0, *Pr* = 0.72, *Q* = 0.2, *M* = 0.5 and *J* = 0.3.

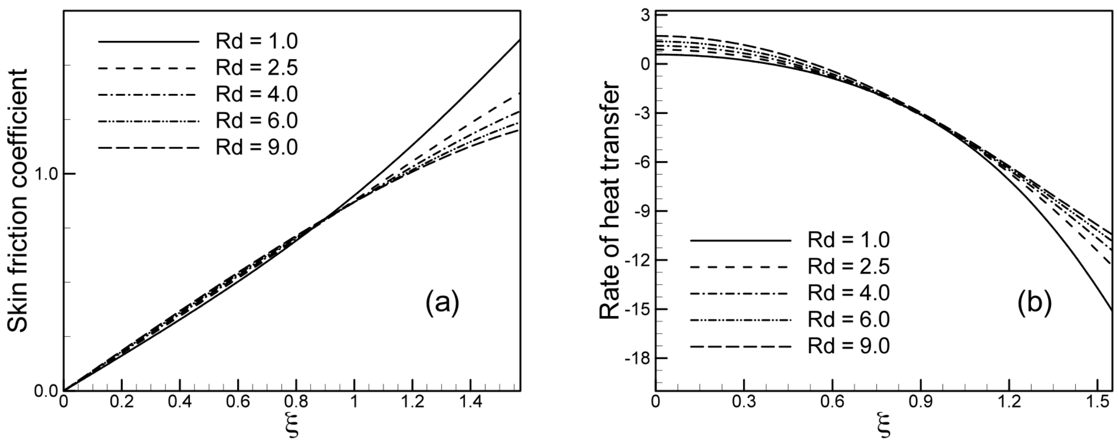

Fig. 8. (a) Skin friction coefficient and (b) Rate of heat transfer for different when Pr = 0.72, Q =0.2, M = 0.5, J = 0.3 and Vd = 25.0.

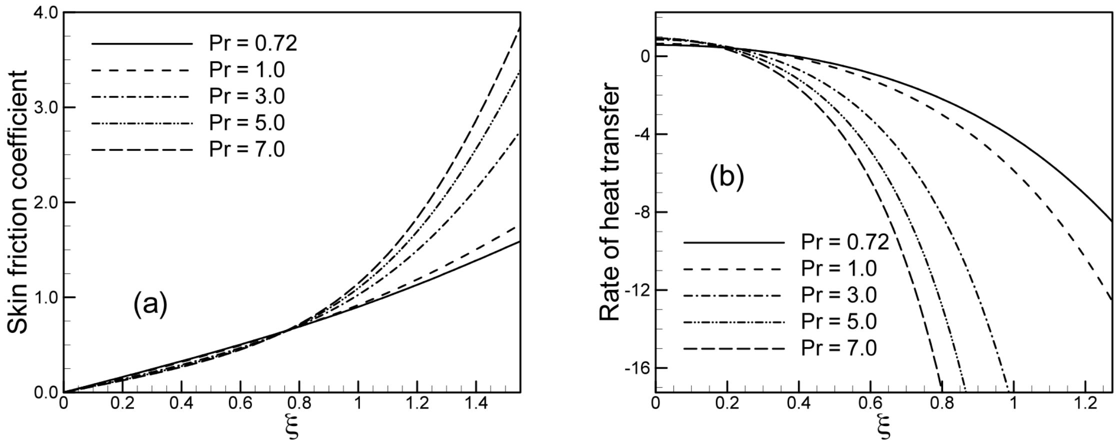

Fig. 9. (a) Skin friction coefficient and (b) Rate of heat transfer for different values of Pr when Rd = 1.0, Q =0.2, M = 0.5, J = 0.3 and Vd = 25.0

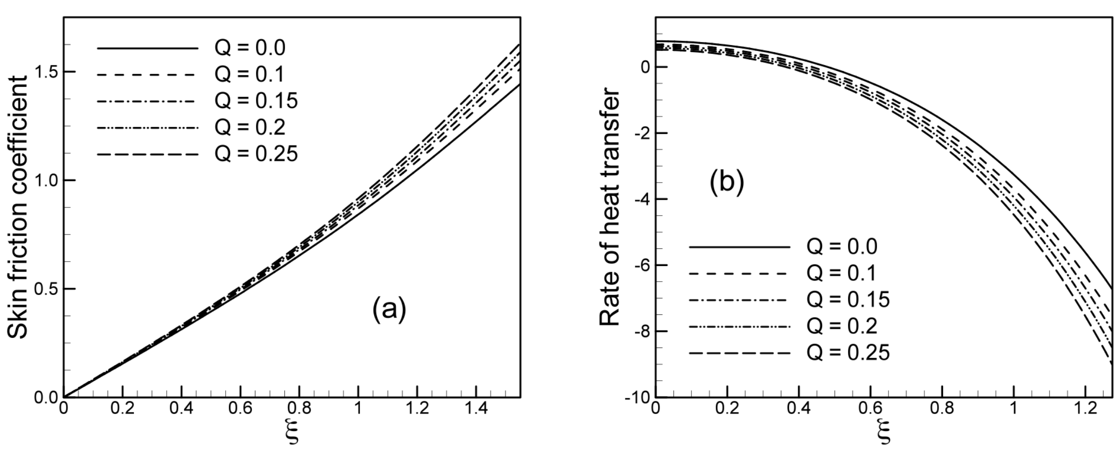

Fig. 10. (a) Skin friction coefficient and (b) Rate of heat transfer for different values of Pr when Rd = 1.0, Pr = 0.72, M = 0.5, J = 0.3 and Vd = 25.0

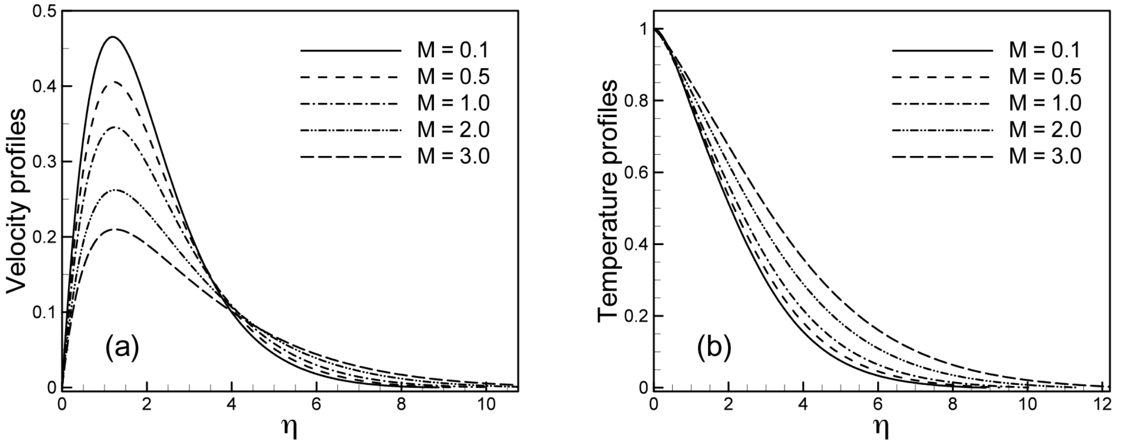

Fig. 11. (a) Skin friction coefficient and (b) Rate of heat transfer for different values of *Q* when *Rd* = 1.0, *Pr* = 0.72, *Q* = 0.1, *J* = 0.3 and *Vd* = 25.0.

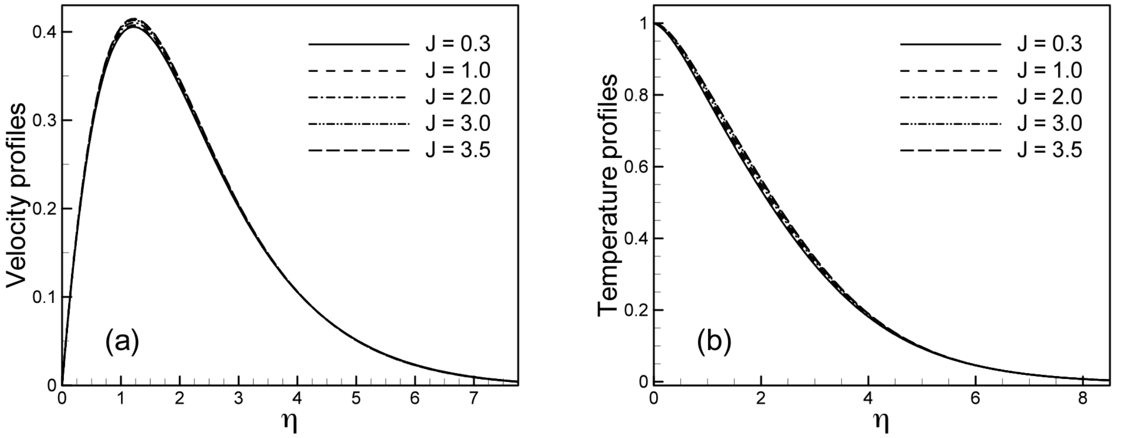

Fig. 12. (a) Skin friction coefficient and (b) Rate of heat transfer for different values of *J* when *Rd* = 1.0, *Pr* = 0.72, *Q* = 0.2, *M* = 0.5 and *Vd* = 25.0.

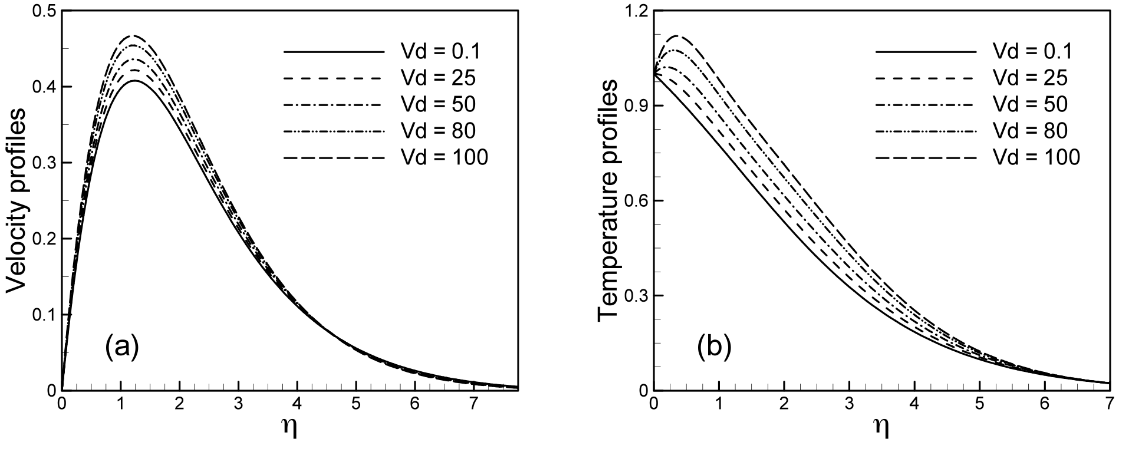

Fig. 13. (a) Skin friction coefficient and (b) Rate of heat transfer for different values of *Vd* when *Rd* = 1.0, *Pr* = 0.72, *Q* = 0.2, *M* = 0.5 and *J* = 0.3.

5. Conclusion

The effects of viscous dissipation and radiation on magnetohydrodynamic free convection flow along a sphere with joule heating. Heat generation has been investigated for different values of relevant physical parameters including the magnetic parameter M and viscous dissipation *Vd*. From the present investigation the following conclusions may be drawn:

- Velocity profiles increase for increasing values of radiation parameter *Rd*, heat generation parameter *Q*, joule heating parameter J, and viscous dissipation paramete *Vd*.
- Temperature profiles increase for increasing values of radiation parameter *Rd*, heat generation parameter *Q*, magnetic parameter *M*, joule heating parameter *J*, and viscous dissipation parameter *Vd*.
- Velocity profiles and temperature profiles decrease for increasing values of Prandlt number *Pr*.
- Skin friction coefficients *Cf* increase for increasing values of heat generation parameter *Q*, joule heating parameter *J*, and viscous dissipation parameter *Vd*. Skin friction coefficients C_f decrease for increasing values of magnetic parameter *M*.
- Rate of heat transfer Nu increases for increasing values of radiation parameter *Rd* and rate of heat transfer *Nu* decreases for increasing values of heat generation parameter *Q*, joule heating parameter *J*, and viscous dissipation parameter *Vd*.

6. References

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7. Nomenclature

a	Radius of the sphere [m]
a_r	Rosseland mean absorption co-efficient [cm^3/s]
B_0	Strength of magnetic field [A/m]
C_f	Skin-friction coefficient
C_P	Specific heat at constant pressure [$Jkg^{-1}k^{-1}$]
f	Dimensionless stream function
g	Acceleration due to gravity [ms^{-2}]
Gr	Grashof number
J	Joule heating parameter
k	Thermal conductivity [$wm^{-1}k^{-1}$]
M	Magnetic parameter
Nu	Nusselt number
Pr	Prandtl number
q_c	conduction heat flux [w/m2]
q_r	Radiative heat flux [w/m2]
q_w	Heat flux at the surface [w/m2]
Q_0	Heat generation constant
Q	Heat generation parameter
Rd	Radiation parameter
r	Radial distance from the symmetric axis to the surface [m]
T	Temperature of the fluid in the boundary layer [K]
T_∞	Temperature of the ambient fluid [K]
T_w	Temperature at the surface [K]

U Velocity component along the surface [ms-1]

V Velocity component normal to the surface [ms-1]

u Dimensionless velocity along the surface

v Dimensionless velocity normal to the surface

X Coordinate along the surface [m]

Y Coordinate normal to the surface [m]

Greek symbols

β Coefficient of thermal expansion [K^{-1}]

θ Dimensionless temperature

μ Dynamic viscosity of the fluid [$kgm^{-1}s^{-1}$]

ν Kinematic viscosity [m^2/s]

p Density of the fluid [kgm^{-3}]

σ Stephan Boltzmann constant [$js^{-1}m^{-2}k^{-4}$]

σ_0 Electrical conductivity [$mho.m^{-1}$]

σ_s Scattering coefficient [m^{-1}]

τ_w Shearing stress at the wall [N/m^2]

ξ Dimensionless coordinate along the surface

η Dimensionless coordinates normal to the surface

ψ Stream function [m^2s^{-1}]