Mathematics of Call Blending in Call Centres

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Abstract

In a call centre, one of the problems is to set the agents for the varying arrival rates. Call blending can give a solution in this case. An agent working on inbound calls and outbound calls seems more effective than an agent handling only inbound calls. The advantage is that it makes the scheduling of the agents easier; it is possible to use the same number of agents the whole day while the average productivity of the agents is increased. Call blending can be used to decrease the waiting time. Thus call blending offers a way to increase resource utilization in a call centre. The main objective in call blending is to maximize the productivity, and meet demands. In this paper we assume certain policy to be optimal for call blending.

1. Introduction

This paper describes a certain aspect of a call centre called call blending. There are two kinds of calls in a call centre. The first type of calls is phone calls at the call centre, referred to as inbound calls. The call centre itself also contacts other people by e-mail, faxes, and phone calls. These calls are referred to as outbound calls. In most call centres, agents, the employees who make inbound and outbound calls in call centres, exclusively handle either inbound or outbound calls. In this system, it could be the case that agents working on inbound calls have no work to do because no phone calls arrive. This approach seems quite expensive because unnecessary agents are appointed, hence, cost is more. The problem with the inbound calls is the fact that arrivals per unit time fluctuate over a day. A simple solution to both problems is to mix the inbound and outbound calls. This is what call blending is about. Only one agent can work on both inbound and outbound calls. The advantage is that it makes the setting of the agents easier; it is possible to use the same amount of agents the whole day while increases the average productivity of the agents. Thus call blending offers a way to increase resource utilization in a call centre.

2. Model Description

The exact model formulation is as follows. There are two types of calls, inbound calls and outbound calls. The service time of these calls are independently exponentially distributed.

Notation

- λ = number of arrivals per time unit
- μ_1 = inverse of the mean service time for inbound calls
- μ_2 = inverse of the mean service time for outbound calls
- s = number of available agents
- x = number of inbound calls in the system
- y = number of outbound calls in the system
- c = threshold, i.e., $x+y \geq c$ should be satisfied
- δ = probability to do nothing when finished serving a job at level $x + y = c + 1$

The call centre is assumed to have an infinite number of phone lines available for inbound calls that cannot be served yet. We also assume that there are an infinite number of outbound calls available. The number of agents available is represented by *s*. The long term average waiting time of inbound calls should be below a constant α. Waiting excludes the service time. The objective for the outbound calls is to serve as many jobs as possible, while the time constraint for the inbound calls is still met.

The following control actions are possible. The moment an agent finishes service, or, more generally at any moment that an agent is available, he can take one of the following three actions:

- Start serving an inbound call, if one or more phone calls are waiting;
- Start serving an outbound call;
- Remain idle.

Model assumptions summarized

(i) Inbound calls arrive according to a Poisson process with parameter λ; (ii) The duration of inbound calls is exponentially distributed with parameter $\mu_{1;}$ (iii) The duration of inbound calls is exponentially distributed with parameter $\mu_{2;}$ (iv) Customers do not abandon; (v) An infinite number of outbound calls is present; (vi) An infinite number of phone lines is available.

3. Solution for the model

Stationary Probabilities for $\delta = 1$ and $c=s$

3.1 Case (i)

Consider $y=c=s$ *and* $x+y \geq s$.

Note that there are no inbound calls in service at this level ($x=0$).

The following equilibrium equation holds:

$$\left(\lambda + s\mu_2\right) q_{x,s} = \lambda q_{x-1,s} \quad \text{--- (1)}$$

The solution for this level is:

$$q_{x,s} = \left(\frac{\lambda}{\lambda + s\mu_2}\right) q_{x-1,s}$$

$$q_{x-1,s} = \left(\frac{\lambda}{\lambda + s\mu_2}\right)^{x-1} q_{0,s}$$

$$\therefore q_{x,s} = \left(\frac{\lambda}{\lambda + s\mu_2}\right)^{x} q_{o,s}$$

For $x \geq s-y, \quad y = s$ --- (2)

We define $\gamma_s := \left(\frac{\lambda}{\lambda + s\mu_2}\right)$ and

$$B_{s,0} := q_{0,s} \quad \text{--- (3)}$$

∴ The solution becomes

$$q_{x,s} = B_{s,0}\left(\gamma_s\right)^x \quad \text{--- (4)}$$

3.2 Case (ii)

Now consider $y = s - 1$, and $x + y \geq s$

The equilibrium equation is as follows:

$$\left(\lambda + \mu_1 + (s-1)\mu_2\right) q_{x,s-1} = \lambda q_{x-1,s-1} + \mu_1 q_{x+1,s-1} + s\mu_2 q_{x,s} \quad \text{--- (5)}$$

Solve the homogeneous equation with respect to level *s-1*

Suppose that:

$$q_{x,s-1} = \left(\gamma_{s-1}\right)^{x-1} q_{1,s-1} \quad \text{--- (6)}$$

With γ_{s-1} to be determined.

Then (4) becomes:

$$\left(\lambda + \mu_1 + (s-1)\mu_2\right)\left(\gamma_{s-1}\right)^{x-1} q_{1,s-1} = \lambda\left(\gamma_{s-1}\right)^{x-2} q_{1,s-1} + \mu_1\left(\gamma_{s-1}\right)^{x} q_{1,s-1} \quad \text{--- (7)}$$

$$\left(\gamma_{s-1}\right)^{x-2} q_{1,s-1} \,`\left[\begin{array}{l}\lambda + \mu_1\left(\gamma_{s-1}\right)^2 - \\ \left(\lambda + \mu_1 + (s-1)\mu_2\right)\left(\gamma_{s-1}\right)\end{array}\right] = 0$$

Since $\gamma_{s-1} > 0$

$$\mu_1\left(\gamma_{s-1}\right)^2 - \left(\lambda + \mu_1 + (s-1)\mu_2\right)\left(\gamma_{s-1}\right) + \lambda = 0$$

--- (8)

Due to the fact that the states we investigate with the previous equation can and will be reached in finite time, one of the solutions of γ_{s-1} greater than 1 is not useful:

$$\gamma_{s-1} = \frac{\left(\lambda+\mu_1+(s-1)\mu_2\right) - \sqrt{\left(\lambda+\mu_1+(s-1)\mu_2\right)^2 - 4\mu_1\lambda}}{2\mu_1} \quad \text{--- (9)}$$

Now we have the homogeneous solution:

$$q_{x,s-1} = \left(\gamma_{s-1}\right)^{x-1} q_{1,s-1}$$

For $x \geq 1, y = s-1$

For a general solution we need a particular solution to the inhomogeneous equation.

A particular solution is:

$$q_{x,y} = B_{1,s-1}\left(\gamma_s\right)^x \quad \text{--- (10)}$$

With $B_{1,s-1}$ the second constant for this level $y = s-1$

($B_{0,s-1}$ is the first constant for this level used in the general solution for the homogeneous part).

$$\left(\lambda+\mu_1+(s-1)\mu_2\right)\left(\gamma_s\right)^x B_{1,s-1} = \lambda B_{1,s-1}\left(\gamma_s\right)^{x-1} + \mu_1 B_{1,s-1}\left(\gamma_s\right)^{x+1} + s\mu_2 B_{0,s}\left(\gamma_s\right)^x$$

$$B_{1,s-1}\left[\begin{array}{l}\lambda\left(\gamma_s\right)^{x-1} - \begin{pmatrix}\lambda+\mu_1+ \\ (s-1)\mu_2\end{pmatrix}\left(\gamma_s\right)^x \\ +\mu_1\left(\gamma_s\right)^{x+1}\end{array}\right] + s\mu_2 B_{0,s}\left(\gamma_s\right)^x = 0$$

$$B_{1,s-1}\left(\gamma_s\right)^{x-1}\left[\begin{array}{l}\lambda - \begin{pmatrix}\lambda+\mu_1+ \\ (s-1)\mu_2\end{pmatrix}\left(\gamma_s\right)+ \\ \mu_1\left(\gamma_s\right)^2\end{array}\right] + s\mu_2 B_{0,s}\left(\gamma_s\right)^x = 0$$

--- (11)

From (1):

$$\lambda q_{x-1,s} - \left(\lambda+s\mu_2\right)q_{x,s} = 0$$

$$\lambda B_{1,s-1}\left(\gamma_s\right)^{x-1} - \left(\lambda+s\mu_2\right)B_{1,s-1}\left(\gamma_s\right)^x = 0$$

$$B_{1,s-1}\left(\gamma_s\right)^{x-1}\left[\lambda - \left(\lambda+s\mu_2\right)\left(\gamma_s\right)\right] = 0$$

--- (12)

Now (10) – (11):

$$B_{1,s-1}\left(\gamma_s\right)^{x-1}\left[\begin{array}{l}-\left(\mu_1-\mu_2\right)\left(\gamma_s\right)+ \\ \mu_1\left(\gamma_s\right)^2\end{array}\right] + s\mu_2 B_{0,s}\left(\gamma_s\right)^x = 0$$

$$\left(\gamma_s\right)^x\left[\begin{array}{l}B_{1,s-1}\begin{pmatrix}-\left(\mu_1-\mu_2\right)+ \\ \mu_1\left(\gamma_s\right)\end{pmatrix}+ \\ s\mu_2 B_{0,s}\end{array}\right] = 0$$

$$B_{1,s-1} = \frac{-s\mu_2 B_{0,s}}{-\left(\mu_1-\mu_2\right)+\mu_1\left(\gamma_s\right)} = \frac{s\mu_2}{\left(\mu_1-\mu_2\right)-\mu_1\left(\gamma_s\right)}B_{0,s} \quad \text{--- (13)}$$

For the general solution we have to multiply the homogeneous solution with a constant *C*. We define:

$$B_{0,s-1} = Cq_{1,s-1} \quad \text{--- (14)}$$

The general solution for level $y = s-1$ is now as follows:

$$q_{x,s-1} = B_{0,s-1}\left(\gamma_{s-1}\right)^{x-1} + B_{1,s-1}\left(\gamma_s\right)^x \quad \text{--- (15)}$$

3.3 Case (iii)

Now consider $0 < y < s$ and $x + y \geq s$ The equilibrium equation is as follows:

$$\left(\left(s-y\right)\mu_1+\lambda+y\mu_2\right)q_{x,y} = \lambda q_{x-1,y}+\left(s-y\right)\mu_1 q_{x+1,y} + \left(y+1\right)\mu_2 q_{x,y+1} \quad \text{--- (16)}$$

Solve the homogeneous equation with respect to level y

Suppose that:

$$q_{x,y}=\left(\gamma_y\right)^{x-(s-y)}q_{s-y,y} \quad \text{--- (17)}$$

With γ_y to be determined. Then:

$$\left(\left(s-y\right)\mu_1+\lambda+y\mu_2\right)\left(\gamma_y\right)^{x-(s-y)}q_{s-y,y} = \lambda q_{s-y,y}\left(\gamma_y\right)^{x-1-(s-y)} + q_{s-y,y}\left(s-y\right)\mu_1\left(\gamma_y\right)^{x+1-(s-y)}$$

$$q_{s-y,y}\left[\lambda\left(\gamma_y\right)^{x-1-(s-y)} - \begin{pmatrix}(s-y)\mu_1\\ +\\ \lambda+y\mu_2\end{pmatrix}\left(\gamma_y\right)^{x-(s-y)} + \left(s-y\right)\mu_1\left(\gamma_y\right)^{x+1-(s-y)}\right]=0$$

$$q_{s-y,y}\left(\gamma_y\right)^{x-1-(s-y)}\left[\lambda-\begin{pmatrix}(s-y)\mu_1+\\ \lambda+y\mu_2\end{pmatrix}\left(\gamma_y\right)+\left(s-y\right)\mu_1\left(\gamma_y\right)^2\right]=0 \quad \text{--- (18)}$$

Since $\left(\gamma_y\right) > 0$

$$\left(s-y\right)\mu_1\left(\gamma_y\right)^2 - \left(\left(s-y\right)\mu_1+\lambda+y\mu_2\right)\left(\gamma_y\right)+\lambda=0 \quad \text{--- (19)}$$

Due to the fact that the each state we investigate in the previous equation can and will be reached in finite time, one of the solutions is greater than 1 and is therefore, not useful.

The solution for γ_y is:

$$\gamma_y=\frac{\begin{pmatrix}(s-y)\mu_1\\ +\\ \lambda+y\mu_2\end{pmatrix}-\sqrt{\begin{pmatrix}(s-y)\mu_1+\\ \lambda+y\mu_2\end{pmatrix}^2-4\left(s-y\right)\mu_1\lambda}}{2\left(s-y\right)\mu_1} \quad \text{--- (20)}$$

Now we have the homogeneous solution:

$$q_{x,y}=\left(\gamma_y\right)^{x-(s-y)}q_{s-y,y}$$

For $x \geq s-y$, $0 < y < s$

For the general solution we need a particular solution to the inhomogeneous equation.

As a particular solution:

$$q_{x,y}=\sum_{i=y+1}^{s}B_{y,i-y}\left(\gamma_i\right)^{x-(s-i)} \quad \text{--- (21)}$$

After substitution of this particular solution we see that we get a combination of linear equations in $\left(\gamma_i\right)$ which can be solved separately.

We solve for $i = y+1, s$ separately.

So we solve $s-y$ equations.

Let us assume $j = i-y$.

So, values for j are $j = 1, 2 \ldots s-y$.

$$B_{y,j}\left(\gamma_{y+j}\right)^{x-(s-y-j)-1}\lambda - B_{y,j}\left(\gamma_{y+j}\right)^{x-(s-y-j)}\begin{pmatrix}(s-y)\mu_1+\\ \lambda+y\mu_2\end{pmatrix} + B_{y,j}\left(\gamma_{y+j}\right)^{x-(s-y-j)+1}\left(s-y\right)\mu_1 + \left(y+1\right)\mu_2 B_{y+1,j-1}\left(\gamma_{y+j}\right)^{x-(s-y-j)}=0$$

$$B_{y,j}\left(\gamma_{y+j}\right)^{x-(s-y-j)-1}\begin{bmatrix}\lambda-\begin{pmatrix}(s-y)\mu_1+\\ \lambda+y\mu_2\end{pmatrix}\left(\gamma_{y+j}\right)+\\ (s-y)\mu_1\left(\gamma_{y+j}\right)^2\end{bmatrix}+(y+1)\mu_2B_{y+1,j-1}\left(\gamma_{y+j}\right)^{x-(s-y-j)}=0 \quad \text{--- (22)}$$

$$B_{y,j}\left(\gamma_{y+j}\right)^{x-(s-y-j)-1}\begin{bmatrix}\lambda-\begin{pmatrix}(s-y-j)\mu_1+\\ \lambda+(y+j)\mu_2\end{pmatrix}\left(\gamma_{y+j}\right)+\\ (s-y-j)\mu_1\left(\gamma_{y+j}\right)^2\end{bmatrix}=0 \quad \text{--- (23)}$$

$$B_{y,j}\left(\gamma_{y+j}\right)^{x-(s-y-j)-1}\left[-j(\mu_1-\mu_2)\left(\gamma_{y+j}\right)+j\mu_1\left(\gamma_{y+j}\right)^2\right]+(y+1)\mu_2B_{y+1,j-1}\left(\gamma_{y+j}\right)^{x-(s-y-j)}=0$$

$$\left(\gamma_{y+j}\right)^{x-(s-y-j)}\left\{\begin{bmatrix}-j(\mu_1-\mu_2)\\ +\\ j\mu_1\left(\gamma_{y+j}\right)\end{bmatrix}B_{y,j}+(y+1)\mu_2B_{y+1,j-1}\right\}=0$$

$$B_{y,j}=\frac{-(y+1)\mu_2B_{y+1,j-1}}{-j(\mu_1-\mu_2)+j\mu_1\left(\gamma_{y+j}\right)}=\frac{(y+1)\mu_2}{j(\mu_1-\mu_2)-j\mu_1\left(\gamma_{y+j}\right)}B_{y+1,j-1} \quad \text{--- (24)}$$

If we add the solutions of the linear equations for each γ_i we get a particular solution. For the general solution we need to add the homogeneous solution multiplied with a constant C_y and the particular solution. We define $B_{y,0}=C_yq_{s-y,y}$ --- (25)

The general solution for level $0 < y \le s$ is now as follows:

$$q_{s,y}=\sum_{i=y}^{s}B_{y,i-y}\left(\gamma_i\right)^{x-(s-i)} \quad \text{--- (26)}$$

3.4 Case (iv)

Now consider $y = 0$ and $x + y \geq s$.

We consider the equations that hold for $0 < y < s$ and $x + y \geq s$.

We notice that the solution also holds for $y = 0$. Until now we have only $s + 1$ unknown constants, which can be solved with the remaining equations for $x + y = s$.

3.5 Case (v)

Now consider $y = s$ and $x + y = s$.

The equilibrium equation for this case is as follows:

$$\lambda q_{0,s}=\mu_1q_{1,s-1}$$

So $$q_{1,s-1}=\frac{\lambda q_{0,s}}{\mu_1} \quad \text{--- (27)}$$

3.6 Case (vi)

Now consider:

$0 < y < s$ and $x + y = s$

The equilibrium equations are as follows

$$\left(\lambda+(s-y)\mu_1\right)q_{s-y,y}=(s-y)\mu_1q_{s-y+1,y}+(s-y+1)\mu_1q_{s-y+1,y-1}+(y+1)\mu_2q_{s-y,y+1}$$

So

$$q_{s-y+1,y}=\frac{\left(\lambda+(s-y)\mu_1\right)q_{s-y,y}-(s-y)\mu_1q_{s-y+1,y}-(y+1)\mu_2q_{s-y,y+1}}{(s-y+1)\mu_1} \quad \text{--- (28)}$$

Now it is possible for each stationary probability to be expressed in terms of one constant.

We also know that: $\sum_{x=0}^{\infty}\sum_{y=0}^{s} q_{x,y} = 1$ --- (29)

This determines the last constant. Now all $q_{x,y}$ are determined for the particular case that $c = s$ and $\delta = 1$.

4. Numerical examples

Assume that we have the following parameters:

$$\lambda = 0.5, \mu_1 = 0.4, \mu_2 = 0.2, s = 5$$

Note that the service time for the outbound calls is twice as large as the service time for the inbound calls.

Assume that we do not apply call blending. In that case we have 6 probabilities to divide the group. We can use the Erlang delay formula to calculate the expected waiting time in each of these cases. The throughput can be determined by multiplying the number of agents for service outbound calls with μ_2. Then, for each of the cases, the following results hold.

Now assume that we do not apply call blending for this case. The productivity for the inbound calls equals the productivity minus the throughput for the outbound calls. Now note that the productivity remains the same, regardless of the policy. This was to be expected because the inbound calls that arrive have to be done in each case.

We can start comparing the case with no call blending and with call blending. Assume that we did the calculation taking minutes as units. If we do not apply call blending and assign only zero or one agent to the inbound calls, then the waiting time becomes very large (infinite). If we assign two agents to the inbound calls, we still have to wait more than 1 minute and 30 seconds.

Table 1 Expected Waiting time, Productivity, and Throughput

Inbound agents	Outbound agents	Expected Waiting time	Productivity	Throughput
0	5	INF	1	1
1	4	INF	1	0.8
2	3	1.602	0.85	0.6
3	2	0.222	0.65	0.4
4	1	0.038	0.45	0.2
5	0	0.006	0.25	0

If we assign three agents to the inbound calls we get an acceptable waiting time of about 13 seconds. In this case the throughput equals 0.4. Now we consider the call blending case. If we use the threshold of 3, we get an average waiting time of 7 seconds and the throughput of 0.45. So in the call blending case, the waiting time is smaller and the throughput is bigger. By applying the randomization technique, the throughput can be set to 0.40 as in the case of no call blending, but with lower waiting times. It is also possible to maximize the throughput while the expected waiting time equals the waiting time in the case with no call blending. This will result in a higher throughput.

Future Extension : for $\delta = 1$, $c < s$, $\delta < 1$, and find the throughput, expected waiting time, and productivity

5. Conclusion

We showed only one case in our paper. The result holds in many more cases. Only if the outbound calls last much longer, there is an advantage in waiting time or throughput, but not in both of them. Note however, that the outbound calls in this case last twice as long as the inbound calls. This research gives better results for cost savings.

6. References

[1] S.Bhulai and G.M.Koole, A Queuing model for Call Blending in Call Centres to appear in IEEE Transactions on Automatic Control.

[2] A Queueing model for call blending in call centers, Sandjai Bhulai and Ger Koole. Vrije University, De Boelelaan 1081a, 1081 HV Amsterdam, The Netherlands, 2000.

[3] G.M.Koole and A.Mandelbaum, Queuing model of Call Centres an introduction, 2001.

[4] Queuing Models of Call Centers: An Introduction, Ger Koole, Vrije University, De Boelelaan 1081a, 1081 HV Amsterdam, The Netherlands, 2002.

[5] Telephone Call Centers: Tutorial, Review, and Research Prospects, Noah Gans, Ger Koole, Avishai Mandelbaum, The Wharton School, University of Pennsylvania, Philadelphia, PA 19104, U.S.A., 2003.

[6] A note on profit maximization and monotonicity for inbound call centers, Ger Koole & Auke Pot, Department of Mathematics, Vrije Universities Amsterdam, The Netherlands 24th January 2006.