Radiation Effect on MHD Steady Free Convection Flow of a Gas at a Stretching Surface with a Uniform Free Stream with Viscous Dissipation

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Abstract

This article investigates the influence of radiation effects on steady free convection flow near isothermal stretching sheet in the presence of a magnetic field and viscous dissipation. The governing equations are converted into a system of non-linear ordinary differential equations via a similarity transformation. The resulting system of nonlinear coupled ordinary differential equations are solved numerically by using the Adams-Moulton predictor-corrector method with the shooting method. The numerical results for the velocity and temperature profiles are displayed graphically showing the effect of various values of the flow parameters: radiation parameter F, free convection parameter Gr, Magnetic parameter M, Prandtl number Pr, Eckert number Ec, and the parameter of relative difference between the temperature of the sheet, and the temperature far away from the sheet r. The effect of the radiation and magnetic field parameter on the shear stress and heat flux are discussed.

Keywords: MHD, radiation, viscous dissipation, free convection, stretching sheet.

1. Introduction

The study of the boundary layer behaviour on a continuous surface is important because the analysis of such flows finds applications in different areas such as the aerodynamic extrusion of a plastic sheet, the cooling of a metallic plate in a cooling bath, the boundary layer along material handling conveyers, and the boundary layers along a liquid film in condensation processes. As examples on stretched sheets, many metallurgical processes involve the cooling of continues strips or filaments by drawing them through a quiescent fluid and in the process of drawing, these strips are stretched. Elbashbeshy [1] investigated heat

transfer over a stretching surface with variable and uniform surface heat flux subject to injection and suction. Guptha P.S. and Guptha A.S [2] studied the heat and mass transfer corresponding to the similarity solution for the boundary layer over an isothermal stretching sheet subject to blowing or suction. Sakiadis [3], first presented boundary layer flow over a moving continues solid surface with Vajravelu constant speed. and Hadyinicolaou [5] studied the convection heat transfer in an electrically conducting fluid near an isothermal stretching sheet, and they studied the effect of internal heat generation or absorption.

From the technological point of view, MHD free- convection flows have also great significance for the applications in the fields of stellar and planetary magnetospheres, chemical engineering, aeronautics. and electronics. The effects of magnetic field on flow of electrically free convection conducting fluids past a plate has been by many authors such studied as Soundalgekar [14], Singh et al.[13]. All the above investigations are restricted to MHD flow and heat transfer problems only. However, of late, the radiation effect on MHD flow and heat transfer problems have become more important, industrially. At high operating temperature, radiation effects can be quite significant

The radiative flows of an electrically conducting fluid with high temperature, in the presence of a magnetic field, are encountered in electrical power generation, astrophysical flows, solar power technology, space vehicle re-entry, nuclear engineering applications and other industrial areas. The radiation effects on boundary layer flow with and without applying a magnetic field under different situations has been studied by many investigators, for examples: Israelcookey et al. [10], Mahmoud [11], Hayat et al.[9], Cortel[6] and Sajid and Hayat[12]. The radiation effect on heat transfer of a micropolar fluid through a porous medium has been studied by Emad M. Abo-Eldahab and Ahmed F. Ghonaim studied [17]. Takhar et al. [4] studied the radiation effects on MHD free convection flow for a non gray-gas past a semi- infinite vertical plate. The radiation effects on steady free convection flow isothermal near an stretching sheet in the presence of a magnetic field is studied by Emad M AboEldahab[18]

In all the above mentioned studies, the viscous dissipation effects has been neglected. Gebhart [8] has shown that the viscous dissipation effect plays an important role in natural convection in various devices

processes on large scales (or large planets). Also, he pointed out that when the temperature is small. or when the gravitational field is of high intensity, viscous dissipations is more predominant in vigorous natural convection processes. The radiation effect on steady free convection flow near isothermal stretching sheet in the presence of a magnetic field is studied by Ahmed Y [15].

The previous work of Ahmed Y [15] neglected the viscous dissipation effect. In most of the problems, the combined effect of thermal radiation effect and viscous dissipation effect on MHD free convection flow of a Gas at a stretching surface have not been studied. Therefore, the aim of this study is to investigate the effects of radiation on steady free convection flow near an isothermal stretching sheet in the presence of a magnetic field, by taking into account the effect of viscous dissipation.

2. Mathematical Formulation

Here we consider the flow of an electrically conducting fluid, adjacent to a sheet coinciding vertical with the plane y = 0, where the flow is confined to y > 0. Two equal and opposite forces are introduced along the x-axis, so that the sheet is stretched, keeping the origin fixed. A uniform magnetic field of strength B_0 is imposed along the y-axis. The fluid is considered to be a gray, radiation absorber and emitter, but non- scattering medium. Gravity acts in the opposite direction to the positive x-axis. The radiative heat flux from the fluid in the x – direction is considered negligible in comparison to that in the y direction. The Rosseland approximation [16] is used to describe the radiative heat flux in the energy equation. See Figure 1.

Under the usual boundary layer approximation, the flow and heat transfer in



Figure1 Sketch of the physical model.

the presence of radiation are governed by the following equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= v\frac{\partial^{2} u}{\partial y^{2}} + g\beta(T - T_{\infty}) - \frac{\sigma B_{0}^{2}}{\rho}u$$
(2)
$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}$$

$$= \frac{k}{\rho c_{p}}\frac{\partial^{2} T}{\partial y^{2}} - \frac{1}{\rho c_{p}}\frac{\partial q_{r}}{\partial y} + \frac{\mu}{\rho c_{p}}\left(\frac{\partial u}{\partial y}\right)^{2}$$
(3)

where u and v are the velocity components in the x and y directions, respectively, T is the temperature, g is the acceleration due to gravity, v is the fluid kinematic viscosity, ρ is the density, σ is the electric conductivity, C_p is the specific heat at constant pressure, and q_r is the radiative heat flux.

The boundary conditions of the problem are

u = cx, v = 0, $T = T_w$ at y = 0 (4a) $u \to u_\infty$, $T \to T_\infty$ at $y \to \infty$ (4b) Where c > 0, T_w is the constant temperature of sheet, T_∞ is the temperature far away from the sheet, and u_∞ is the free stream velocity. By using the Rosseland approximation [16], we have:

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y},\tag{5}$$

Where σ^* is the Stefan- Boltzmann constant and k^* is the mean absorption coefficient. By using (5), the energy equation (3) becomes:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{4\sigma^*}{3k^*\rho c_p}\frac{\partial^2 T^4}{\partial y^2} + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2$$
(6)

Introducing the following non dimensional parameters:

$$\overline{x} = \frac{cx}{u_{\infty}}, \quad \overline{y} = \frac{cyR}{u_{\infty}}, \quad \overline{u} = \frac{u}{u_{\infty}},$$
$$\overline{v} = \frac{vR}{u_{\infty}}, \quad \theta = \frac{T - T_{\infty}}{T_{w} - T_{\infty}}$$
(7)

We can obtain the governing equation in dimensionless form as (with dropping the bars):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (8)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + Gr\theta - Mu \qquad (9)$$

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial y^2}$$

$$+ \frac{4}{3F \Pr} \left((1 + r\theta)^3 \frac{\partial^2 \theta}{\partial y^2} + 3r (1 + r\theta)^2 \left(\frac{\partial \theta}{\partial y} \right)^2 \right)$$

$$+ E_c \left[(f'')^2 + 2xg'' f'' + x^2 (g'')^2 \right] \qquad (10)$$
With the boundary conditions

 $u = x, v = 0, \theta = 1$ at y = 0, (11a) $u = 1, \quad \theta = 0$ as $y \to \infty$, (11b) where $M = \frac{\sigma B^2_0}{\rho c}$ is the magnetic parameter

$$R = \frac{u_{\infty}}{\sqrt{cv}} \text{ is the Reynolds number,}$$
$$Gr = \frac{g\beta(T_w - T_{\infty})}{cu_{\infty}} \text{ is the free}$$

convection parameter,

$$Pr = \frac{\mu c_p}{k}$$
 is the Prandtl number,
$$hk^*$$

$$F = \frac{\kappa \kappa}{4\sigma^* T_{\infty}^3} \quad \text{is the radiation}$$

parameter,

 $\mu = \rho v$ is the viscosity of the fluid, and

$$r = \frac{(T_w - T_{\infty})}{T_{\infty}}$$
 is the relative

difference between the temperature of the sheet and the temperature far away from the sheet.

$$E_c = \frac{u^2 \infty}{c_p (T_w - T_\infty)}$$
 is the Eckert

number.

Introducing the stream function ψ defined in the usual way:

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x}$$

Equation (10) can then be written as:

$$\frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2}$$

$$= \frac{\partial^3 \psi}{\partial y^3} - M \frac{\partial \psi}{\partial y} + Gr\theta \qquad (12a)$$

$$\frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} = \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial y^2}$$

$$+ \frac{4}{3F \Pr} \left[(1 + r\theta)^3 \frac{\partial^2 \theta}{\partial y^2} + 3r(1 + r\theta)^2 \left(\frac{\partial \theta}{\partial y} \right)^2 \right]$$

$$+ E_c \left[(f'')^2 + 2xg'' f'' + x^2 (g'')^2 \right] \qquad (12b)$$

And the boundary conditions (10) become:

$$\frac{\partial \psi}{\partial y} = x \quad \frac{\partial \psi}{\partial x} = 0, \ \theta = 1, \text{ at } y = 0, \ (13)$$
$$\frac{\partial \psi}{\partial y} = 1, \qquad \theta = 0 \text{ as } y \to \infty$$

Introducing,

$$\psi(x, y) = f(y) + xg(y), \qquad (14)$$

Substituting equation (14) in equations (12) and equating the coefficients of x^0 and x^1 , we obtain the coupled nonlinear ordinary differential equations:

$$f''' = f'g' - gf'' + Mf' - Gr\theta (15)$$

$$g''' = g'^{2} - gg'' + Mg' (16)$$

$$\left(3F + 4(1 + r\theta)^{3}\right)\theta'' + 3\Pr Fg\theta'$$

 $+12(1+r\theta)^{2}\theta'^{2}+3F \operatorname{Pr} Ec(f'')^{2}=0 (17)$

The primes above indicate differentiation with respect to y only. The boundary conditions (13) in view of (14) is reduced to: $f(0) = f'(0) = g(0) = g'(\infty) = \theta(\infty) = 0$

$$g'(0) = f'(\infty) = g(0) = g'(\infty) = \theta(\infty) = 0$$
(18a)
(18b)

The physical quantities of interest in this problem are the skin friction coefficient and the Nusselt number, which are defined by:

$$\tau_{w} = \mu \left(\frac{\partial u}{\partial y} \right) \Big|_{y=0} N_{u} = \frac{q_{w}}{k (T_{w} - T_{\infty})}, \quad (19)$$

where $q_w = -k \frac{\partial T}{\partial y}\Big|_{y=0}$

Using (14), the quantities in (19) can be expressed as:

$$\tau_w = \mu c R (f''(0) + xg''(0)),$$

$$N_u = \frac{cR}{u_{\infty}} \theta'(0),$$
(20)

The effect of parameters F, Gr, M, and Pr, on the functions f'', g'' and θ' at the plate surface is tabulated in Table1 for r = 0.05.

F	Gr	М	Pr	Ahmed Y[15] g"(0)	g"(0) Present	Ahmed Y[15] $\theta'(0)$	$\theta'(0)$ Present	Ahmed Y[15] <i>f</i> "(0)	f"(0) Present
1	0.5	0.1	0.73	-1.04771	-1.049	-0.224411	-0.225	0.820805	0.825
2	0.5	0.1	0.73	-1.04771	-1.049	-0.297402	-0.298	0.703769	0.705
3	0.5	0.1	0.73	-1.04771	-1.049	-0.335702	-0.336	0.656791	0.664
1	0	0.1	0.73	-1.04771	-1.049	-0.224411	-0.225	0.110292	0.117
1	0.5	0.1	0.73	-1.04771	-1.049	-0.224411	-0.225	0.820805	0.825
1	1	0.1	0.73	-1.04771	-1.049	-0.224411	-0.225	1.53188	1.533
1	0.5	0.01	0.73	-1.00398	-1.005	-0.230155	-0.231	1.12575	1.125
1	0.5	0.1	0.73	-1.04771	-1.049	-0.224411	-0.225	0.820805	0.825
1	0.5	0.5	0.73	-1.22325	-1.234	-0.204004	-0.205	0.513629	0.514

Table 1 Variation of f'', g'', θ' at the plate surface with F, Gr, M, and Pr parameters.

3. Method of Solution

The closed form solution for the equation (16) is:

$$g(y) = \frac{1}{\sqrt{M+1}} \left[1 - e^{-\sqrt{M+1}y} \right]$$
 (21)

For the purpose of numerical computation, the differential equations (15) & (17) are written in the form of a system of first order differential equations. The following transformation variables are used.

$$\begin{array}{ll} Y_1=f & ; & Y_2=f' \; ; & Y_3=f'' \; ; \\ Y_4=\theta \; ; & Y_5=\theta' \end{array}$$

With this substitution, the couple system of equations (15) & (17) can be written as a first order system as:

$$Y_{1}' = Y_{2}$$

$$Y_{2}' = Y_{3}$$

$$Y_{3}' = Y_{2}g' - Y_{3}g + MY_{2} - GrY_{4}$$

$$Y_{4}' = Y_{5}$$
 and

 Y'_5

$$=\frac{-3\Pr FgY_{5}+12r(1+r\theta)^{2}Y_{5}^{2}+3F\Pr EcY_{3}^{2}}{3F+4(1+r\theta)^{3}}$$

(22)

The boundary conditions (18) give $Y_1(0) = Y_2(0) = 0, \ Y_3(0) = \alpha$,

$$Y_4(0) = 1, Y_5(0) = \beta$$

The fundamental problem is to find the solution to differential equations (15)-(17) subject to the boundary conditions (18) for the various values of parameter α , β . In this case the choice of missing initial solution is difficult. Assuming initial values for α and β , the system is solved. An iterative shooting method, which uses globally convergent Newton Raphson method is employed. The guessed solution is changed in a systematic way until correct starting values are The fourth order Adams determined. predictor-corrector method is used to solve the system of equations.

4. Results and Discussion

The numerical results for the velocity and temperature distribution are shown in Fig 1-5 for different flow field parameters of number Pr. free Prandtl convection parameter Gr, radiation parameter F, the parameter of relative difference between the temperature of the sheet. and the temperature far away from the sheet r, and the Eckert number Ec. Fig.1 (a) displays the effect of Pr and F on the velocity distribution. It is seen from this Figure that the velocity profile f' decreases with an increase of the F and Pr. The effect of rincreases the velocity profile f', while the velocity profile f' decreases with an increase of magnetic field parameter M, as observed from Fig 1(b).

From Fig.2 it is observed that the temperature distribution decreases as Fincreases as well as Pr increases. It is also noticed that the temperature distribution increases as r increases. This is an agreement with the physical fact that the thermal boundary thickness decreases with the increasing Pr Fig.3 displays the behavior of g' with changes in the values of the magnetic field parameter M. It is seen, that the g' decreases with an increase in the magnetic field parameter M. The effects of viscous dissipation on f' and θ are illustrated in Figures 4 and 5. The effect of the Eckert number Ec on the dimensionless velocity component f' is displayed in Fig.4(a) and (4b) for different Pr values. It is clear from this Figure that the velocity of the fluid increases with an increase of the Eckert number Ec. The increasing in fluid temperature due to viscosity is observed to be more pronounced for higher values of Ec. Fig.5(a) and (5b) show the effect of Eckert number *Ec* on the temperature profile for different Pr values. It is seen from this figure that the temperature profile increases with an increase of Eckert number Ec. It can be seen that the velocity profiles as well as temperature profiles are higher when Pr = 0.73, compared with Pr = 2. It is clear from Figs 4 and 5 that the velocity and temperature of the fluid is at a higher level when viscous dissipation is considered, than if it is neglected.

The effect of variation parameter on the skin friction coefficient f''(0) is shown in Fig.6 and the heat flux $\theta'(0)$ is shown Fig.6 displays the effect of in Fig.7. magnetic field parameter M on the skin friction coefficient f''(0), with the effect of magnetic parameter M. It is noticed that the effect of increasing M is a decrease in the wall temperature gradient $\theta'(0)$, as shown in Fig 7. Variation of f'', g'', θ' at the plate surface with different values of F, Gr, M Pr and E_c are shown in Table (1)-(3) From these tables, it should be mentioned that the results obtained herein are in good agreement with the previous work of Ahmed Y[15], when $E_c = 0$, which gives validation of the present solution.

5. Conclusion

In the present work we have studied effect of viscous dissipation and the radiation on MHD free convection flow near an isothermal stretching sheet in the presence of a magnetic field. The effect of magnetic field reduces the temperature profiles and velocity profiles. It is observed that the effect of increasing viscous dissipation leads to an increase in the velocity profiles as well as temperature profiles. The velocity and temperature profiles decrease with an increase of radiation parameter F. The velocity profiles increases with an increase of the parameter difference of relative between the

temperature of the sheet, and the temperature far away from the sheet r.

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Figure 1(a) Variation of the dimensionless velocity component f' with different Pr parameters and F values



Figure 1(b) Variation of the dimensionless velocity component f' with different parameters M and r.



Figure 2 Variation of the dimensionless temperature θ with different parameter Pr, *r*, and *F*.



Figure 3. Variation of the dimensionless velocity component g' with M parameters.



Figure 4(a) Variation of the dimensionless velocity component f' with Ec parameter.



Figure (4b) Variation of dimensionless velocity components f' with Ec parameter.



Figure 5(a) Variation of the dimensionless θ with Ec parameter.



Figure 5(b) Variation of the dimensionless temperature θ with Ec parameter.



Figure 6 Variation of f''(0) with F and M parameters



Figure 7 Variation of the heat flux $\theta'(0)$ with *F* and *M* parameters.

F	Gr	М	Pr	Ahmed Y[15] g"(0)	g"(0) Present	Ahmed Y[15] $\theta'(0)$	$\theta'(0)$ Present	Ahmed Y[15] <i>f</i> "(0)	f"(0) Present
1	0.5	0.1	0.73	-1.04771	-1.049	-0.224411	-0.225	0.820805	0.825
1	0.5	0.1	2	-1.04771	-1.048	-0.480357	-0.480	0.523724	0.531
1	0.5	0.1	5	-1.04771	-1.048	-0.882528	-0.882	0.36651	0.374

Table 2 Variation of f'', g'', θ' at the plate surface with F, Gr, M, and Pr parameters.

Table 3 Variation of f''(0), g''(0), $\theta'(0)$ at the plate surface with F, Gr, M, and Pr and E_c parameters.

E_{c}	F	Gr	М	Pr	<i>g</i> ″(0)	$\theta'(0)$	<i>f</i> "(0)
0.1	1	0.5	0.1	2	-1.049	-0.470	0.536
0.5	1	0.5	0.1	2	-1.049	-0.456	0.542
1	1	0.5	0.1	2	-1.049	-0.45	0.554
1.2	1	0.5	0.1	2	-1.049	-0.331	0.586
1.5	1	0.5	0.1	2	-1.049	-0.284	0.599

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