Non-Darcy Natural Convection Flow over a Vertical Wavy Surface in Porous Media Including the Magnetic Field Effect

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Abstract

The problem of the non-Darcian natural convection flow of an electrically conducting fluid along a vertical wavy surface embedded in a fluid-saturated porous medium in the presence of a uniform normal magnetic field is investigated. We consider the boundary-layer regime where the Darcy-Rayleigh number is very large, (i.e. $Ra \to \infty$). In particular, the wavy surface is considered in the following form $\hat{\delta}(\hat{x}) = \hat{a}\sin^2(\pi\,\hat{x}/\ell)$. Appropriate transformations are employed to transform the complex wavy surface to a smooth surface. Then, the obtained boundary layer equations are solved numerically using the Runge-Kutta integration scheme with the Newton-Raphson shooting method. Extensive computations are presented for a wide range of the wave amplitudes a, the magnetic field parameter M, and the Grashof number Gr. Graphical results for the velocity, temperature, and the local Nusselt number profiles are illustrated and discussed for various physical parametric values.

Keywords: Wavy surface, Non-Darcy, MHD, Free convection, Porous media.

1. Introduction

Natural convection heat transfer by a fluid moving through a porous medium is a phenomenon of great interest. This is due to the increasing need in understanding complicated transport for applications in diverse fields which include geophysical and geothermal engineering, cooling of nuclear reactors, heat exchanger design, petroleum extraction, in-site combustion of oil shale, solid matrix heat exchangers, building insulation, solar power collectors, control of pollutant spread in ground water and many more. Nield and Bejan [1] have presented a comprehensive review of the studies of convective heat transfer mechanisms through porous media. Most of the existing works in ref. [1], depending on Darcy or non-Darcy convective flow over heated bodies embedded in fluid-saturated porous medium, are concerned with flat plates, for instance, Nakayama and Koyama [2], Plumb and Huenefeld [3], Cheng and Minkowycz [4].

Previous studies have centered on those cases where the thermal boundary conditions allow the use of similarity transformation to reduce the governing equations to a system of ordinary differential equations. In general, this means that the heated surface is a plane. However, surfaces are sometimes intentionally roughened to enhance heat transfer. Roughened surfaces are encountered in several heat transfer devices such as flat-plate solar collectors and flat-plate condensers in refrigerators. The Natural convection heat transfers from vertical wavy surfaces, such as sinusoidal surfaces have been studied. Yao [5] studied the natural convection heat transfer from an isothermal vertical wavy surface by using an extended Prandtl's transposition theorem and a finite-difference scheme. He proposed a simple transformation to study the natural convection heat transfer from isothermal vertical wavy surfaces, such as sinusoidal surfaces, in Newtonian fluids. Cheng [6-7] reported the phenomenon of natural convection heat and mass transfer near a vertical wavy surface with constant wall temperature and concentration in a porous medium, for the two cases of Darcy and non-Darcy law models. Rees and Pop [8-10] carried out some studies to analyze natural convection from vertical and horizontal wavy surfaces embedded in a porous medium with employing the Darcy law model. Rees and Pop [11] examined the combined effect of spatially stationary surface waves and the presence of fluid inertia on the free convection induced by a vertical heated surface embedded in a fluid medium for the non-Darcy flow model. Rathish Kumar et al. [12-14] presented a series of studies about the effects of phase of the wave surface on the natural convection in a porous enclosure. They found that the effects of the phase of the wavy surface on the flow and temperature fields are important.

Based on the above brief review, it is of interest in this article to analyze the natural convection problem along a vertical wavy surface embedded in electrically conducting fluid saturated porous media for the case of non- Darcy flow model in the presence of a transverse magnetic field. The applied magnetic field is assumed to be uniform and the magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected. Moreover, it is assumed that there is no external electric field.

2. Structure of Theoretical Model

We consider a vertical surface which exhibits steady transfer waves of amplitude, $\hat{a}$, and wavelength, 2ℓ, and which is embedded in a homogeneous fluid saturated porous medium. Fig. 1 shows the schematic diagram for the problem under consideration, in particular, the wavy surface profile is given by:

$$\hat{y} = \hat{\delta}(\hat{x}) = \hat{a}\sin^2(\pi\hat{x}/\ell). \tag{1}$$

The temperature of the wavy surface is held at constant value T_w and a uniform ambient temperature T_∞. The governing equations for the problem under consideration are based on the balance laws of mass, linear momentum, and energy modified to include the porous medium Darcian and non-Darcian effects. Based on the Boussinesq approximation, these equations can be written as:

$$\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{\partial \hat{\upsilon}}{\partial \hat{y}} = 0, \tag{2}$$

$$\left(1+\frac{K\sigma B^2}{\mu}+\frac{\tilde{K}}{\nu}\left|\hat{V}\right|\right)\hat{u} = -\frac{K}{\mu}\frac{\partial \hat{p}}{\partial \hat{x}}+\frac{K g\beta}{\nu}\left(T-T_\infty\right), \tag{3}$$

$$\left(1+\frac{\tilde{K}}{\nu}\left|\hat{V}\right|\right)\hat{\upsilon} = -\frac{K}{\mu}\frac{\partial \hat{p}}{\partial \hat{y}}, \tag{4}$$

$$\hat{u}\frac{\partial T}{\partial \hat{x}} + \hat{\upsilon}\frac{\partial T}{\partial \hat{y}} = \alpha\left(\frac{\partial^2 T}{\partial \hat{x}^2}+\frac{\partial^2 T}{\partial \hat{y}^2}\right). \tag{5}$$

where $\hat{u}$ and $\hat{\upsilon}$ are the velocity components along the $\hat{x}$ and $\hat{y}$ directions, respectively, and $\hat{u} = \frac{\partial\hat{\Psi}}{\partial\hat{y}}$, $\hat{\upsilon} = -\frac{\partial\hat{\Psi}}{\partial\hat{x}}$. T is the temperature; α, β are the thermal diffusivity of the saturated porous medium and thermal expansion coefficients of the fluid; K is the permeability of the porous medium and $\tilde{K}$ is a material parameter which may be thought of as a measure of the inertial impedance of the matrix. $\hat{V} = (\hat{u}, \hat{\upsilon})$ is the velocity flux vector. Properties ν, μ and ρ are the effective kinematic viscosity, the dynamic viscosity, and density of the fluid, respectively, g is the acceleration due to gravity, σ, B are the electrical conductivity and the applied magnetic flux density.

Both the permeability K, and the material parameter $\tilde{K}$ can be determined from the widely-known correlations proposed by Ergun[15]:

$$K = \frac{d^2 \varepsilon^3}{150(1-\varepsilon)^2}, \qquad \tilde{K} = \frac{1.75\, d}{150(1-\varepsilon)}. \qquad (6)$$

where d denotes the particle diameter and ε the porosity. Darcy's law is recovered when $\tilde{K} = 0$. When the term $(\tilde{K}/\nu)\sqrt{\hat{u}^2 + \hat{\upsilon}^2}$ is near or greater than 1 at any point of the flow field, the nonlinear term is important. The boundary conditions to be considered are:

$$\begin{aligned} &\hat{y} = \hat{\delta}(\hat{x}): \quad && \hat{\upsilon} = 0, \quad && T = T_w, \\ &\hat{y} \to \infty: && \hat{u} \to 0, && T \to T_\infty. \end{aligned} \qquad (7)$$

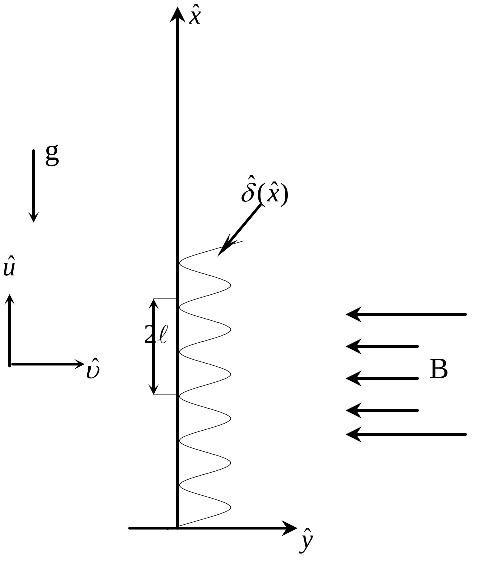

Figure 1. The physical model depicting transverse surface waves

The previous equations can be converted to non-dimensional form by considering the following new variables:

$$\begin{cases} (\tilde{x}, \tilde{y}, \tilde{a}, \tilde{\delta}) = \left(\dfrac{\hat{x}, \hat{y}, \hat{a}, \hat{\delta}}{\ell}\right), \\ \tilde{\Psi} = (\alpha\, Ra)^{-1}\, \hat{\Psi}, \\ \theta = (T - T_\infty)/(T_w - T_\infty). \end{cases} \qquad (8)$$

Using the above transformation and after eliminating the pressure between equations (3) and (4), then equations (3)-(5) turn into the following:

$$(1 + Gr\, Q)\left(\frac{\partial \tilde{u}}{\partial \tilde{y}} - \frac{\partial \tilde{\upsilon}}{\partial \tilde{x}}\right) + M\, \frac{\partial \tilde{u}}{\partial \tilde{y}} + Gr\, Q^{-1} \times \left(\tilde{u}^2 \frac{\partial \tilde{u}}{\partial \tilde{y}} + 2\tilde{u}\,\tilde{\upsilon}\frac{\partial \tilde{\upsilon}}{\partial \tilde{y}} - \tilde{\upsilon}^2 \frac{\partial \tilde{\upsilon}}{\partial \tilde{x}}\right) = \frac{\partial \theta}{\partial \tilde{y}}, \quad (9)$$

$$\tilde{u}\, \frac{\partial \theta}{\partial \tilde{x}} + \tilde{\upsilon}\, \frac{\partial \theta}{\partial \tilde{y}} = Ra^{-1}\left(\frac{\partial^2 \theta}{\partial \tilde{x}^2} + \frac{\partial^2 \theta}{\partial \tilde{y}^2}\right). \qquad (10)$$

where

$$Q = \sqrt{\tilde{u}^2 + \tilde{\upsilon}^2}, \qquad Gr = K\tilde{K}\, g\, \beta(T_w - T_\infty)/\nu^2$$

$$M = K\sigma B^2/\mu, \qquad Ra = K\, g\, \beta(T_w - T_\infty)\ell/\nu\alpha$$

are the non-dimensional velocity, Grashof number, the magnetic field parameter, and the Darcy-Rayleigh number, respectively.

The effect of the wavy surface can be transferred from the boundary conditions into the governing equations by means of the coordinate transformation given by:

$$\begin{cases} x = \tilde{x}, \\ y = (\tilde{y} - \tilde{\delta})Ra^{1/2}, \\ \Psi = Ra^{1/2}\tilde{\Psi}. \end{cases} \qquad (11)$$

Applying the above transformation into Equations (9)-(10) with $Ra \to \infty$, we get the governing equations as:

$$\left(1 + \dot{\delta}^2 + M\right)\frac{\partial u}{\partial y} + Gr\left(1 + \dot{\delta}^2\right)^{3/2}\frac{\partial u^2}{\partial y} = \frac{\partial \theta}{\partial y}, \quad (12)$$

$$\left(1 + \dot{\delta}^2\right)\frac{\partial^2 \theta}{\partial y^2} = u\, \frac{\partial \theta}{\partial x} + \upsilon\, \frac{\partial \theta}{\partial y}. \qquad (13)$$

Here $\left(\dot{\delta} = \dfrac{d\,\delta}{d\,x}\right)$ and is subject to the corresponding boundary conditions:

$$y = 0, \qquad \upsilon = 0, \qquad \theta = 1, \qquad (14a)$$

$$y \to 0, \qquad u \to 0, \qquad \theta \to 0. \qquad (14b)$$

For solving the equations (12), (13) along with the boundary conditions (14), let us introduce the following group of transformations:

$$\begin{cases} \eta = x^{-1/2}\left(1+\dot{\delta}^2\right)^{-1} y, \\ \Psi = x^{1/2}\ f(x,\eta). \end{cases} \qquad (15)$$

Thus, we get the boundary layer equations as follows:

$$\left(1+\frac{M}{1+\dot{\delta}^2}\right) f'' + 2\,Gr\left(1+\dot{\delta}^2\right)^{-1/2} f'\,f'' = \theta', \quad (16)$$

$$\theta'' + \frac{1}{2} f\,\theta' = x\left(f'\frac{\partial \theta}{\partial x} - \theta'\frac{\partial f}{\partial x}\right). \qquad (17)$$

The boundary conditions are:

$$\eta = 0, \quad f = 0, \quad \theta = 1, \qquad (18a)$$

$$\eta \to \infty, \quad f' \to 0, \quad \theta \to 0. \qquad (18b)$$

The result of practical interest in many applications is the heat transfer coefficient. The heat transfer coefficient is expressed in terms of the local Nusselt number, and is given by:

$$Nu_x = \frac{\hat{x}\,\hat{q}_w}{k\left(T_w - T_\infty\right)}. \qquad (19)$$

where $\hat{q}_w$, is the wall heat flux on the wavy surface, and is defined by:

$$\hat{q}_w = -k\,\underline{n}\cdot\hat{\nabla}\mathrm{T}. \qquad (20)$$

and $\underline{n} = \left(-\dot{\delta}\big/\sqrt{1+\dot{\delta}^2}\ ,\ 1\big/\sqrt{1+\dot{\delta}^2}\right)$ is the unit vector normal to the wavy surface. k is the effective porous medium thermal conductivity.

Employing transformations (8), (11), (15) and (20) we get the local Nusselt number from the following expressions:

$$Nu_x\,Ra^{-1/2} = -\frac{\sqrt{x}}{\sqrt{1+\dot{\delta}^2}}\,\theta'\left(x,0\right). \qquad (21)$$

The total tare of heat transfer between the leading edge and a streamwise location $\hat{x} = \hat{X}$ is given by:

$$q = \int_0^{\hat{X}} -k\left(n.\hat{\nabla}T\right)_{\hat{y}=\hat{\delta}}\ \frac{d\,\hat{S}}{d\,\hat{x}}\ d\,\hat{x}.$$

where $\hat{S}$ is the distance along the wavy surface. In terms of the non-dimensional variables this expression becomes

$$q = -k\left(T_w - T_\infty\right)Ra^{1/2}\int_0^{\hat{X}} \frac{\theta'\left(x,0\right)}{\sqrt{x}}\ d\,x. \qquad (22)$$

3. Results and Discussion

The obtained partial differential Equations (16), and (17), subject to the boundary conditions (18), are solved by numerical integration using the Runge-Kutta fourth-order with Newton-Raphson shooting technique with a systematic guessing of $f'(x,0)$, $\theta'(x,0)$ for a range of values of the governing physical parameters, the amplitude-wavelength a, Grashof number Gr, and the magnetic field parameter M. The results are shown graphically.

In order to check the accuracy of the solution, we compare the values of $-\theta'(0,0)$ and $f'(0,0)$ obtained by the present study with the solution reported by Plumb and Huenefeld [3]. Table 1 shows the values of $-\theta'(0,0)$ and $f'(0,0)$ for the case of a flat plate and no magnetic field effect, for various values of the Grashof number Gr. It is clearly shown in Table 1 that the present results are in excellent agreement with the solutions reported by Plumb and Huenefeld [3].

Figure 2 and 3 illustrate that as the magnetic field parameter increases, it is observed that the velocity decreases, while the temperature increases. The effect of the Grashof number is depicted in Figure 4 and 5. It is observed that as the Grashof number increases, the velocity decreases and the temperature increases.

Table 1. Comparison of $-\theta'(0,0)$ and $f'(0,0)$ with $M = 0$ and $a = 0$ (Flat plate)

	[3]		Present study	
Gr	$-\theta'(0,0)$	$f'(0,0)$	θ'	f'
0.0	0.44390	1.00000	0.4445	1.000
0.01	0.44232	0.99020	0.4434	0.990
0.1	0.42969	0.91608	0.4301	0.916
1.0	0.36617	0.61803	0.3679	0.618
10.0	0.25126	0.27016	0.2523	0.270
100.0	0.15186	0.09512	0.1521	0.095

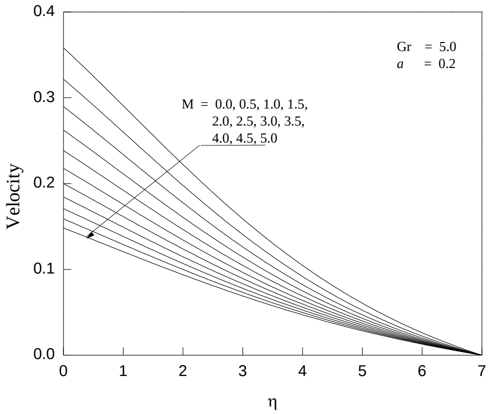

Figure 2. Influences of the magnetic field parameter on the non-dimensional velocity profiles

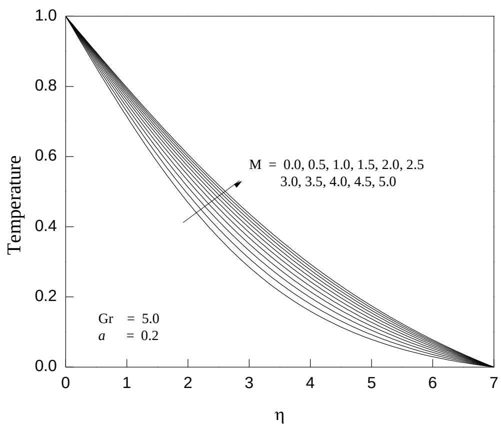

Figure 3. Influences of the magnetic field parameter on the non-dimensional temperature profiles

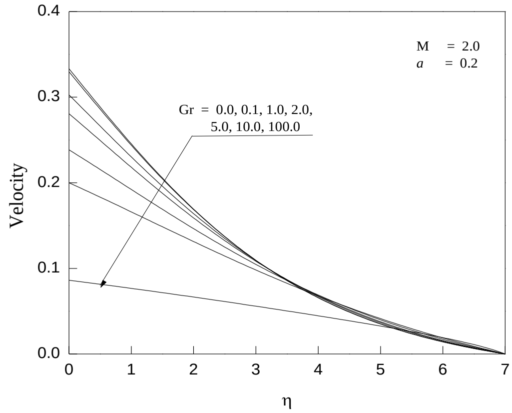

Figure 4. The effect of Grashof number on the non-dimensional velocity profiles

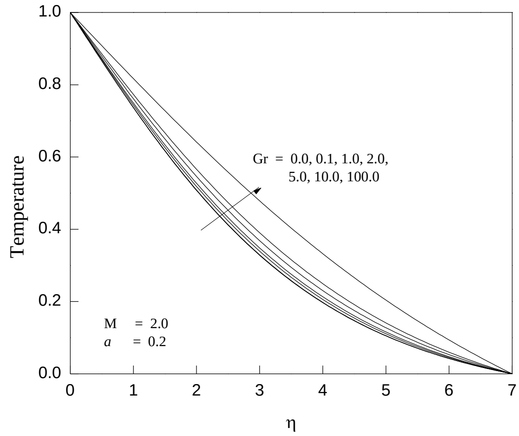

Figure 5. The effect of Grashof number on the non-dimensional temperature profiles

Figure 6 shows the axial distribution of the heat transfer coefficient in terms of the local Nusselt number $Nu_x \left(x\, Ra\right)^{-1/2}$ as a function of axial coordinate x for various values of the surface wave amplitude (a = 0.0, 0.1, 0.2, 0.3, 0.4 and 0.5). It is observed that this quantity varies periodically in the direction of x when $a \neq 0$ (wavy surface). Also, one can see that increasing the amplitude wavelength ratio a tends to increase the amplitude of the local Nusselt number and decrease the global rate of heat transfer as compared with the limiting case of a vertical smooth surface.

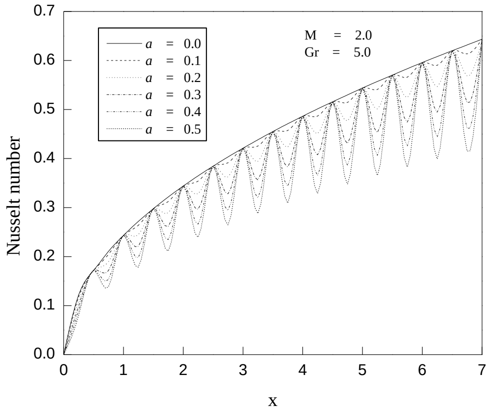

Figure 6. The axial distribution of the local Nusselt number for selected values of wave amplitudes

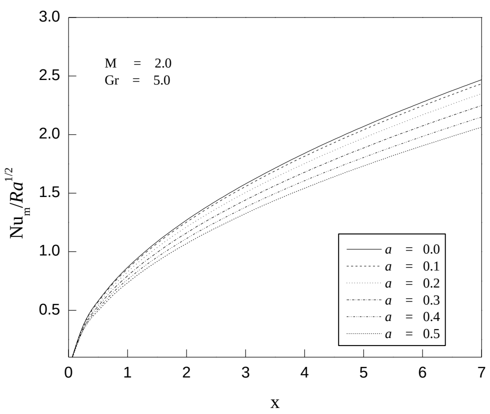

Figure 7. The average Nusselt number for Selected values of wave amplitudes

Figure 7 depicts the global rate of heat transfer (Mean Nusselt number) $Nu_m\, Ra^{-1/2}$ for various values of amplitude wavelength ratio (a = 0.0, 0.1, 0.2, 0.3, 0.4 and 0.5), with Gr = 5.0, and M = 2.0. The increase of the amplitude wavelength ratio, on the global rate of heat transfer tends to decrease the natural convection heat transfer rates as compared with the limiting case of a vertical smooth surface. It is clearly shown that increasing amplitude wavelength ratio tends to increase the thermal boundary layer thickness and hence decrease the global rate of heat transfer (Mean Nusselt number).

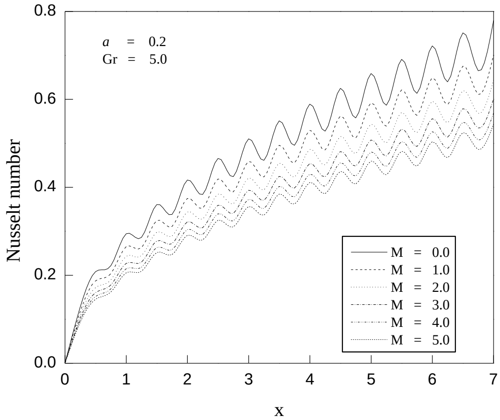

Figure 8. The axial distribution of the local Nusselt number for selected values of the magnetic field

Figure 8 displays the effect of the magnetic field parameter on the local Nusselt number as a function of x. From this Figure we observe that as M increases, the rate and amplitude of heat transfer decreases, and make greater fluctuations of $Nu_x\,(x\,Ra)^{-1/2}$ with increasing x. Figure 9 shows the global rate of heat transfer (Mean Nusselt number) $Nu_m\, Ra^{-1/2}$ for various values of magnetic field (M = 0.0, 1.0, 2.0, 3.0, 4.0, and 5.0), while a = 0.2 and Gr = 5.0. It is clearly shown that increasing the magnetic parameter tends to decrease the global rate of heat transfer. The influence of the Grashof number (Gr = 0.1, 1.0, 2.0, 5.0, 10.0 and 100.0) on the local and mean Nusselt number is illustrated in Figure 10 and 11. We observe that while Gr increases, the local and global rates of heat transfer decreases.

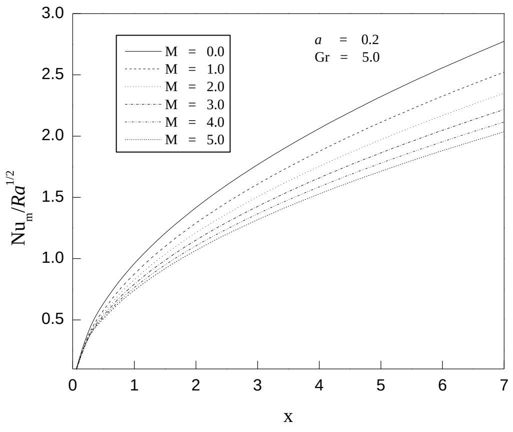

Figure 9. The average Nusselt number for selected values of the magnetic field

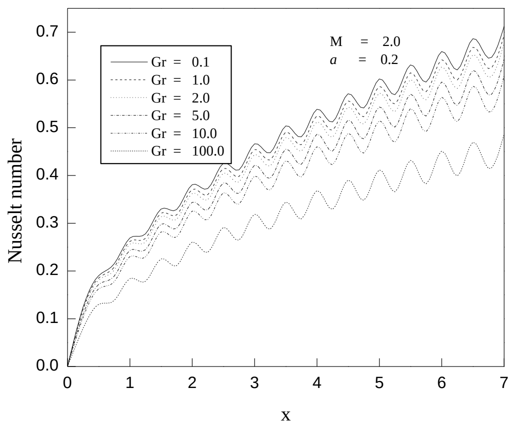

Figure 10. The axial distribution of the local Nusselt number for selected values of Grashof number

4. Conclusions

The effect of magnetic field on non-Darcian natural convection heat and fluid flow along a vertical wavy surface was studied. The governing equations were solved numerically to obtain the wall heat transfer rates for the wavy surface of the form $\hat{y} = \hat{a}\sin^2(\pi\hat{x}/\ell)$. It was found that the magnetic field retards the heat transfer process by decreasing the local Nusselt number and increasing the fluid temperature. In addition, the velocity field was strongly affected by the presence of the magnetic field. The wall heat transfer rate was found to vary according to the slope of the wavy surface $\dot{\delta}$. The axial distribution of the local Nusselt number varies periodically with a wavelength equal to the wavy surface. Increasing amplitude of wavelength leads to greater fluctuations of the local Nusselt number. Local Nusselt number along a wavy surface in the presence of the magnetic field was found to be much lower than those of flat plate values. Finally, increasing *Gr* leads to smaller fluctuations of the local Nusselt number.

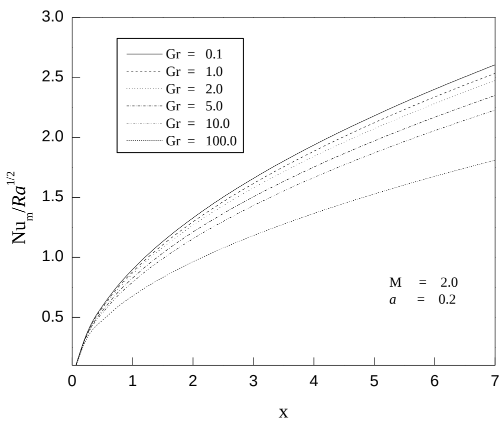

Figure 11. The average Nusselt number for selected values of Grashof number

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