Finite Element Time Domain Approach for Solving the Reflection of Electromagnetic Waves from Anisotropic Media

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Abstract

This paper aims to present a finite element time domain method for analyzing inhomogeneous anisotropic and multi-layered media whose material properties are in tensor forms. Also, the media are exposed to an obliquely incident Gaussian pulse. Formulation of this problem is in second order partial differential equations that consist of only two components of unknown fields in term of transverse electric field. Because of an open boundary problem, using absorbing boundary conditions to truncate the infinite domain into a finite domain is appropriate. To find out solutions of unknown electric fields by the finite element time domain method, two steps are taken into consideration. In the spatial domain, the methodology is based on the application of Galerkin's method with quadratic shape functions in order to form a system of a second-order differential equation. In the time domain, this differential equation can be solved by Newmark's method to become linear system equations at each time step. This method is unconditionally stable for certain values of parameters. After solving the linear system equations, we get transverse electric fields at corresponding nodes and these fields can be transformed into the fields in the frequency domain by Fourier transform. Computational results indicate that the accuracy of this method is comparable with an analytical method, and reflection coefficients of varying frequencies from inhomogeneous anisotropic media. In addition, our results are close to results of the previous research in the past.

Keywords: finite element time domain (FETD), oblique incidense, multi-layered media.

1. Introduction

The problem of plane electromagnetic wave penetration into multi-layered anisotropic media is an important issue in shielding design or in military applications. Many factors influence this phenomenon such as material properties, number of layers, polarization, frequency and angle of wave. Much research in the past has developed to improve methods to study these factors. In the analytical method, reflection of electromagnetic waves from inhomogeneous anisotropic media was studied by Titchener and Willis [1]. The anisotropic material properties in derived equations of this method were defined in terms of permittivity and permeability tensors, not defined in term of conductivity tensor. Chiu and Chen [2] considered the plane-wave shielding performance of an anisotropic laminated composite cylindrical shell whose material was regarded as lossy medium. In numerical methods the finite difference time domain (FDTD) or the finite element time domain (FETD) approach, has been widely used in solving transient problems. Schneider and Hudson from research [3] proposed the FDTD for analysis of propagating electromagnetic wave in multi-layer anisotropic media, but the materials in this approach considered only both permittivity and conductivity tensors. Ming et al. [4] presented the computational results of the reflection and transmission in composite materials by using FDTD and an equivalent transmission line circuit. This method did not allow for the permeability tensor. Research by Onder and Kuzuoglu [5] investigated the reconstruction of permittivity and conductivity whose materials are in scalar form, in the time domain by using descent methods. Research by [6]-[8] has improved the FETD algorithm to obtain an unconditionally stable solution. Recent work [9]-[11] has developed new structures or methods to study a phenomenon of electromagnetic waves.

In this paper we propose the FETD formulation for analysis of electromagnetic waves penetrating into inhomogeneous anisotropic and multi-layered media. Two numerical examples based on the transient problems are presented to demonstrate the efficiency and accuracy of the method.

2. The Finite Element Formulation

Basic Equations

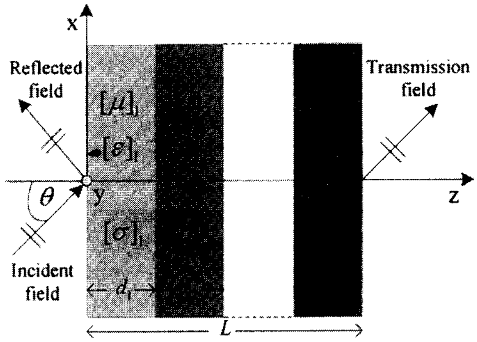


Fig.1 Geometry of multi-layered anisotropic media with a obliquely incident plane wave.

A time-harmonic plane wave propagating in an isotropic medium is obliquely incident on anisotropic medium at angle θ with respect to the z-axis as shown in Fig.1.This phenomenon usually results in the reflection and transmission of waves. Each layer with thickness of d is considered as a homogenous and anisotropic material defined by the permeability, permittivity and conductivity tensors. At the $z=0$ plane, the incident and reflected fields are expressed as:

$$\tilde{\mathbf{E}}^{inc} = \mathbf{E}_i e^{j(k_0 x \sin\theta + k_0 z \cos\theta)} \quad (1)$$

$$\tilde{\mathbf{H}}^{inc} = \mathbf{H}_i e^{j(k_0 x \sin\theta + k_0 z \cos\theta)} \quad (2)$$

$$\tilde{\mathbf{E}}^{ref} = \mathbf{E}_r e^{j(k_{rx} x + k_{ry} z)} \quad (3)$$

$$\tilde{\mathbf{H}}^{ref} = \mathbf{H}_r e^{j(k_{rx} x + k_{ry} z)} \quad (4)$$

where $\mathbf{E}_i$, $\mathbf{H}_i$, $\mathbf{E}_r$, and $\mathbf{H}_r$ are constant vectors of incident fields and reflected fields, and k_0 is a free space wave number. For the region $0 < z < L$, the fields within each medium are still plane waves and can be represented as:

$$\tilde{\mathbf{E}} = \mathbf{E}_a e^{j(k_x x + k_z z)} \quad (5)$$

$$\tilde{\mathbf{H}} = \mathbf{H}_a e^{j(k_x x + k_z z)} \quad (6)$$

where $\mathbf{E}_a$ and $\mathbf{H}_a$ are constant vectors of the transmission fields.

Enforcing the continuity of the tangential fields across the boundary interface, which is at the $z=0$ plane in (1) to (6), leads to a result of phase:

$$k_0 \sin\theta = k_{rx} = k_x . \quad (7)$$

From equation (7), this shows that the electric and magnetic fields are dependent on the x variable with $e^{jk_0 x \sin\theta}$. Then we can express $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{H}}$ as:

$$\tilde{\mathbf{E}}(x,z) = (\tilde{E}_x(z)\mathbf{a}_x + \tilde{E}_y(z)\mathbf{a}_y + \tilde{E}_z(z)\mathbf{a}_z)e^{jk_0 x \sin\theta} \quad (8)$$

$$\tilde{\mathbf{H}}(x,z) = (\tilde{H}_x(z)\mathbf{a}_x + \tilde{H}_y(z)\mathbf{a}_y + \tilde{H}_z(z)\mathbf{a}_z)e^{jk_0 x \sin\theta} \quad (9)$$

Substituting (8) and (9) into the Maxwell's curl equations, eliminating magnetic field $\tilde{\mathbf{H}}$ and then transforming from frequency domain equations into time domain equations leads to the results:

$$\frac{\partial^2 [E_t]}{\partial z^2} = \frac{\partial}{\partial t}\left(\frac{\partial [S][E_t]}{\partial z}\right) + \frac{\partial^2 ([T][E_t])}{\partial t^2} + \frac{\partial ([C][E_t])}{\partial t} \quad (10)$$

where $[E_t]$ consists of electric components in x and y direction, respectively, and matrices $[S]$, $[T]$ and $[C]$ can be expressed as:

$$[S] = [G_{ee}] + [G_{eh}][G_{hh}][G_{eh}]^{-1} \quad (11)$$

$$[T] = [G_{eh}][G_{he}] - [G_{eh}][G_{hh}][G_{eh}]^{-1}[G_{ee}] \quad (12)$$

$$[C] = [G_{eh}][P] . \quad (13)$$

The set of equations in (11)-(13) are identical to those derived by Titchener and

Willis [1] via an analytical method. From (10), we can divide into two equations:

$$\frac{\partial^2 E_x(z,t)}{\partial z^2} = s_{11}\frac{\partial}{\partial z}\left(\frac{\partial E_x(z,t)}{\partial t}\right) + s_{12}\frac{\partial}{\partial z}\left(\frac{\partial E_y(z,t)}{\partial t}\right) + t_{11}\frac{\partial^2 E_x(z,t)}{\partial t^2} + t_{12}\frac{\partial^2 E_y(z,t)}{\partial t^2} + c_{11}\frac{\partial E_x(z,t)}{\partial t} + c_{12}\frac{\partial E_y(z,t)}{\partial t} \quad (14)$$

$$\frac{\partial^2 E_y(z,t)}{\partial z^2} = s_{21}\frac{\partial}{\partial z}\left(\frac{\partial E_x(z,t)}{\partial t}\right) + s_{22}\frac{\partial}{\partial z}\left(\frac{\partial E_y(z,t)}{\partial t}\right) + t_{21}\frac{\partial^2 E_x(z,t)}{\partial t^2} + t_{22}\frac{\partial^2 E_y(z,t)}{\partial t^2} + c_{21}\frac{\partial E_x(z,t)}{\partial t} + c_{22}\frac{\partial E_y(z,t)}{\partial t} \quad (15)$$

where s_{ij}, t_{ij} and c_{ij} $(i,j=1,2)$ are elements in $[S]$, $[T]$ and $[C]$, respectively.

Boundary Conditions

To solve the solutions of electric and magnetic fields in anisotropic media by using (14) and (15), we have to find boundary conditions that are suitable for an open boundary problem. Using absorbing boundary conditions to truncate the infinite domain into a finite domain is simple. Basically, there are two polarizations of fields to analyze the problem in Fig.1: parallel and perpendicular polarizations. Then, the boundary conditions for the parallel polarization in Fig.1 can be represented as:

$$\frac{\partial E_x(z,t)}{\partial z} = -\frac{2\cos\theta}{c_0}\frac{\partial E_x^{inc}}{\partial t} + \frac{\cos\theta}{c_0}\frac{\partial E_x(z,t)}{\partial z} \quad \text{at } z=0 \quad (16)$$

$$\frac{\partial E_x(z,t)}{\partial z} = -\frac{\cos\theta}{c_0}\frac{\partial E_x(z,t)}{\partial t} \quad \text{at } z=L \quad (17)$$

where c_0 is the speed of light in free space, and the E_x^{inc} is the incident field in x direction. Subsequently, the boundary conditions for the perpendicular polarization can be derived by changing E_x and E_x^{inc} in (16) and (17) into E_y and E_y^{inc} respectively.

Shape functions

In the finite element method (FEM), the solution domain is subdivided in small regions called 'elements'. For instance, in one-dimensional applications the domain can be subdivided into finite lines such as line element. The points defining the line elements are the nodes or degrees of freedom.

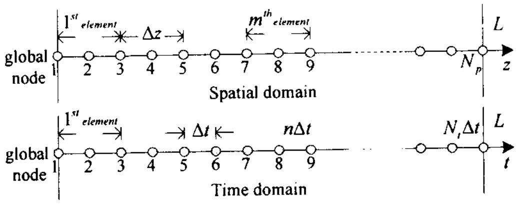


Fig.2 Subdividing into line elements in the spatial domain and time domain.

As shown in Fig.2, we split the domain of the problem in Fig.1 into two parts: a spatial domain part and a time domain part. The first figure illustrates each line element composed of three nodes, and has the total number of unknown nodes N_p. The Δz is the space increment of the finite difference node. The other figure gives the time step Δt between two nodes, and the total number of the time steps N_t. To obtain an accuracy of the finite element analysis, quadratic shape functions are taken into consideration [12]. These functions are comprised of 3 unknown parameters in each element and are defined over these subintervals and can be given by:

$$N_1^m = \frac{(z-z_2^m)(z-z_3^m)}{(z_1^m - z_2^m)(z_1^m - z_3^m)} \quad (18)$$

$$N_2^m = \frac{(z-z_1^m)(z-z_3^m)}{(z_2^m - z_1^m)(z_2^m - z_3^m)} \quad (19)$$

$$N_3^m = \frac{(z-z_1^m)(z-z_2^m)}{(z_3^m - z_1^m)(z_3^m - z_2^m)} \quad (20)$$

where

$z_1^m = (m-1)\Delta z$, $z_2^m = (m-\frac{1}{2})\Delta z$ and $z_3^m = m\Delta z$.

Additionally, these functions correspond to the nodes of the element. For instance, the N_1^m is equal to 1 at node z_1^m and to 0 at the other nodes. More information is discussed in [5] and [12]. Then, trial functions of electric fields in each element can be expressed as the sum of these shape functions and unknown parameters

$$E_x(z,n\Delta t) = \sum_{i=1}^{3} N_i^m(z)\phi_i^m(n\Delta t) \quad (21)$$

$$E_y(z,n\Delta t) = \sum_{i=1}^{3} N_i^m(z)\psi_i^m(n\Delta t) \quad (22)$$

where, $\phi_i^m(n\Delta t)$ and $\psi_i^m(n\Delta t)$ are the unknown parameters of E_x and E_y at time $n\Delta t$, respectively.

Galerkin method

Performing the inner product of basis function $U_p(z)$ with Eq.(14) and Eq.(15), integrating over the interval $0<z<L$, and imposing the boundary conditions according to Eq.(16) and Eq.(17) leads to a system of a second-order differential equation:

$$[T]\frac{d^2[X]}{dt^2} + ([S]+[C])\frac{d[X]}{dt} + [K][X] = [F] \quad (23)$$

where

$$[X] = \begin{bmatrix} \{\phi\} \\ \{\psi\} \end{bmatrix},\quad [T] = \begin{bmatrix} [T_{\phi\phi}] & [T_{\phi\psi}] \\ [T_{\psi\phi}] & [T_{\psi\psi}] \end{bmatrix},$$

$$[S] = \begin{bmatrix} [S_{\phi\phi}] & [S_{\phi\psi}] \\ [S_{\psi\phi}] & [S_{\psi\psi}] \end{bmatrix},\quad [C] = \begin{bmatrix} [C_{\phi\phi}] & [C_{\phi\psi}] \\ [C_{\psi\phi}] & [C_{\psi\psi}] \end{bmatrix}$$

$$[K] = \begin{bmatrix} [K_{\phi\phi}] & [0] \\ [0] & [K_{\psi\psi}] \end{bmatrix}, \text{ and } [F] = \begin{bmatrix} \{F_\phi\} \\ \{F_\psi\} \end{bmatrix}.$$

Here the column matrix $[X]$ represents the E_x and E_y at the corresponding nodes, respectively. The matrices $[T]$, $[S]$, $[C]$, and $[K]$ are square matrices with size $2N_p \times 2N_p$, and are time-independent matrices. The column matrix $[F]$ denotes incident fields. More information of these matrices will be shown in the appendix.

3. The Time Stepping Formulation

Equation (23) is the system of a second-order differential equation, which can be solved by Newmark's method [7]. This method begins by expanding $[X]$ and $[\dot{X}]$ (equivalent to $d[X]/dt$) at time t^{n+1} and then rearranging equations as:

$$[X]^n = \beta[X]^{n+1} + (1-2\beta)[X]^n + \beta[X]^{n-1} \quad (24)$$

$$[\dot{X}]^n = \frac{[X]^{n+1} - [X]^{n-1}}{2\Delta t} \quad (25)$$

$$[\ddot{X}]^{n+1} = \frac{[X]^{n+1} - 2[X]^n + [X]^{n-1}}{(\Delta t)^2} \quad (26)$$

where the superscript for matrices indicates the values at a particular time instant, and β is a parameter that can be chosen to obtain stability. Generally, the value of β is 1/4. Substituting (24) - (26) into (23) and rewriting in linear system equations as:

$$[A][X]^{n+1} = [B] \quad (27)$$

where

$$[A] = \frac{[T]}{(\Delta t)^2} + \frac{[S]}{2\Delta t} + [K]\beta \quad (28)$$

$$[B] = \left(\frac{2[T]}{(\Delta t)^2} - [K](1-2\beta)\right)[X]^n + \left(-\frac{[T]}{(\Delta t)^2} + \frac{[S]}{2\Delta t} - [K]\beta\right)[X]^{n-1} + [f].^{n+1} \quad (29)$$

Here, the column matrix $[f]$ denotes the incident fields that assume the Gaussian incident pulse. Equation (27) is solved at every time step for the nodes to calculate the electric field at those nodes. The time-stepping procedure is required to replace the values $[X]^n$ and $[X]^{n-1}$ from the field values at previous time steps to obtain values $[X]^{n+1}$ at the next time step or at time instant t^{n+1}. After solving the equation (27) at every time step, we obtain both electric fields at corresponding nodes in the time domain, and these fields can be transformed into fields in the frequency domain by Fourier transform. Once this procedure is achieved, the amplitude and phase of the fields is obtained.

4. Computational Results

In this section, the finite element time domain method described in the sections 2 and 3 has been applied to solve the reflection and transmission coefficients of laminated sheets based on Fig.1. As the problem space is limited to one-dimension geometries, we can compute these coefficients with a simple FORTRAN program using a personal computer with coprocessor chip in a few minutes.

The source used in all computations is the Gaussian incident pulse obtained from [4] and is defined as follow: a unit amplitude (E_0) 1 V/m, a pulse width (τ) 0.05 nS, and a pulse delay (t_0) 0.3 nS. This pulse delay means that the Gaussian will reach its maximum at $t_0 = 0.3$ nS.

Two-layer anisotropic media

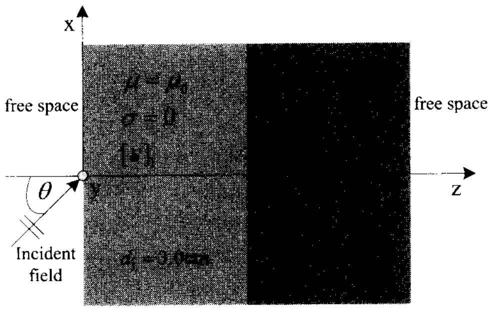


Fig.3 Two-layer anisotropic media

Let us consider the case of two-layer anisotropic media, as shown in Fig.3. In this case, we aim to test the accuracy of our method with exact solutions. Hence, to compare our results with a previously published analytical method that has been extensively studied by Titchener and Willis [1], it was necessary to write source codes of their method and run programs in order to find the values of the reflection coefficients for comparison. The geometry and parameters in this example are chosen to be the same as those of that research. The thickness of each layer is 0.03 m and it is composed of 45 elements. The length of the free space for each side of this case is 0.06 m and consists of 90 elements. Consequently, there are 270 elements and 1080 unknown electric fields in both the x and y directions. With the specification $\Delta z = 2c_0\Delta t$, we get the Δz equal to 0.33 mm and the Δt equal to 0.554 nS.

Table 1 shows that the magnitude of reflection coefficients of R_{11}, where R_{11}, found in [12]-[14] is a division of reflected field by incident field. This is compared with the results from research [1] via the analytical method. The frequencies of the incident field are chosen to be 0.9, 1.8, and 2.7 GHz; the angles of incident field are 30°, 45° and 60°, respectively. The percentage error of the R_{11} is defined as $100 \cdot (S_1 - S_2)/S_2$ where S_1 and S_2 are the analytical and numerical solutions, respectively. It should be noted that the percentage of error for all frequencies is less than 1.8, and a good accuracy has been achieved in all angles of incident field, and that the magnitudes of R_{11} vary with the angles of incident field and frequencies.

Table 1 Comparison of the reflection coefficient R_{11} by analytical method with FETD.

	Magnitude of R_{11}		
$\theta = 30^\circ$	FETD	Research*	%Error
0.9 GHz	0.6046	0.6023	0.3818
1.8 GHz	0.2798	0.2848	1.755
2.7 GHz	0.3300	0.3285	0.4566
$\theta = 45^\circ$			
0.9 GHz	0.6196	0.6220	0.3922
1.8 GHz	0.1370	0.1348	1.6015
2.7 GHz	0.3300	0.3290	0.3039
$\theta = 60^\circ$			
0.9 GHz	0.5666	0.5703	0.6487
1.8 GHz	0.5478	0.5452	0.4768
2.7 GHz	0.2172	0.2168	0.1845

*: research from Titchener and Willis.

Three-layer carbon fiber media

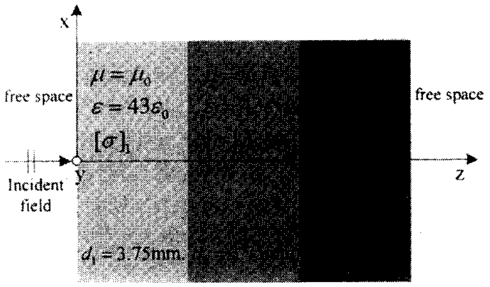
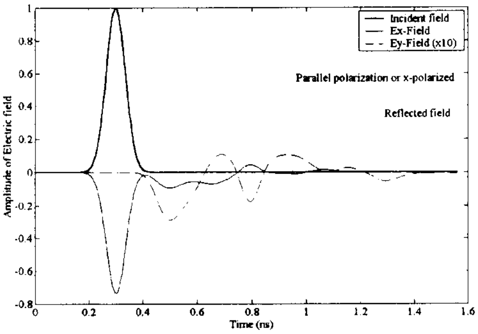
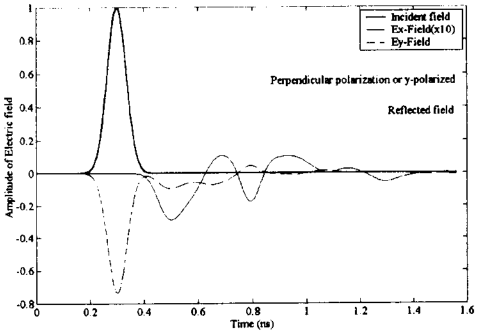


Fig.4 Three-layer anisotropic media

Let us consider the case of three-layer carbon fiber media whose conductivities are in tensor form and the other material properties are in scalar form, as shown in Fig.4. In this case, the geometry and parameters are selected to be the same as those of research [3]. The thickness of each layer is 3.75 mm. The length of the free space for each side of the geometry is 37.5 mm. Then, there are 230 elements, 30 of them are on three layers of the anisotropic materials, and the other 200 are on the free space or the outside boundary. The Δz and the Δt are 0.1875 mm and 0.3125 pS, respectively.



(a) parallel polarization



(b) perpendicular polarization.

Fig.5 Incident field and both reflected fields in front of anisotropic media.

The graphs of the Gaussian incident pulse for both polarizations and the reflected fields on a node that interfaces between the free space and the materials, is plotted in Fig.5, with respect to time. In this figure, the Gaussian pulse is propagating in the positive z direction. After 1200 time steps (0.375 nS) there are several reflected pulses or reflected fields visible in the problem space. The reflected pulses are propagating in the negative z direction, while the transmitted pulses are propagating in the positive z direction. Also they are attenuating along the z direction. For parallel polarization,the amplitude of E_x is higher than that of E_y and this result is opposite for perpendicular polarization.

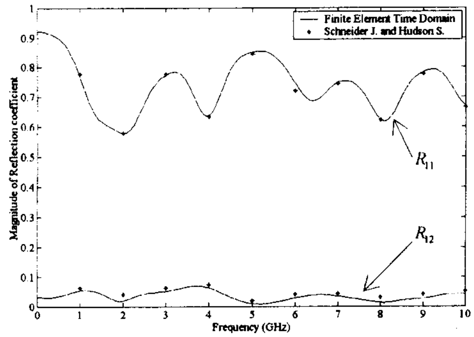
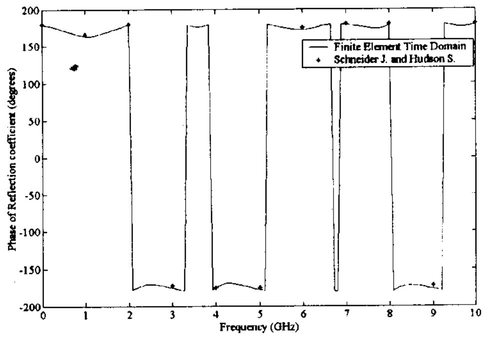
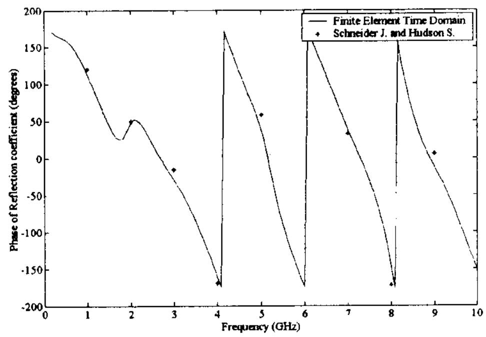


Fig.6 Magnitude of reflection coefficient R_{11} and R_{12} based on Fig.4 (solid line for FETD and asterisk for FDTD by Schneider and Hudson).

Fig.6 illustrates the magnitude of the reflection coefficient for the R_{11} and R_{12} as a function of frequencies, once the incident field with the parallel polarization is applied. The results of FETD method, marked as a line, are compared with that of FDTD method results, marked as an asterisk, by Schneider and Hudson. As seen from the graph, our results agree with results from the FDTD method. Note that the magnitude of the reflection coefficient for the R_{12} in all frequencies is low. This reflection coefficient is less than 0.1 because of cross polarization between incident field and reflected field.



(a) Phase of R_{11}.



(b) Phase of R_{12}.

Fig.7 Phase of reflection coefficient R_{11} and R_{12} based on Fig.4 (solid line for FETD and asterisk for FDTD by Schneider and Hudson).

The comparison of phase for R_{11} and R_{12} is shown in Fig.7. From the graph, the computed results based on our method are close to results of the FDTD method by Schneider and Hudson. The results give some interesting oscillations at 180 degrees.

5. Conclusions

In this paper, the finite element time domain for the analysis of propagating electromagnetic wave in the inhomogeneous anisotropic and multi-layered media, that is comprised of three tensor forms, is presented to investigate the accuracy of the calculations, and the reflection coefficients. By this FETD, all numerical solutions are performed by means of Galerkin's method in the spatial domain and Newmark's method in the time domain. As a result, the linear system equation is solved at every time step for the nodes to compute the electric fields at those nodes. Calculations are carried out in two transient problems. It is clear that this method provides the accuracy and efficiency of computing the problem involving reflection and transmission coefficients. Also, it seems more powerful and flexible than an analytical method or FDTD method. The approach used in this paper can be applied to other transient problems of electromagnetics.

6. Appendix

The matrices $[T]$, $[S]$, $[C]$, and $[K]$ in the section 2 are the assemblage of individual coefficient matrices such as $[T_{\phi\phi}]$, $[T_{\phi\psi}]$, $[T_{\psi\phi}]$, and $[T_{\psi\psi}]$. These matrices can be expressed as:

$$[T_{\phi\phi}] = \int_0^L t_{11} U_i(z) U_j(z) dz$$

$$[T_{\phi\psi}] = \int_0^L t_{12} U_i(z) U_j(z) dz$$

$$[T_{\psi\phi}] = \int_0^L t_{21} U_i(z) U_j(z) dz$$

$$[T_{\psi\psi}] = \int_0^L t_{22} U_i(z) U_j(z) dz$$

$$[S_{\phi\phi}] = \int_0^L s_{11} U_i(z) \frac{dU_j(z)}{dz} dz + \frac{\cos\theta}{c_0}[U_i(L)U_j(L) + U_i(0)U_j(0)]$$

$$[S_{\phi\psi}] = \int_0^L s_{12} U_i(z) \frac{dU_i(z)}{dz} dz$$

$$[S_{\psi\phi}] = \int_0^L s_{21} U_i(z) \frac{dU_i(z)}{dz} dz$$

$$[S_{\psi\psi}] = \int_0^L s_{22} U_i(z) \frac{dU_j(z)}{dz} dz + \frac{\cos\theta}{c_0}[U_i(L)U_j(L) + U_i(0)U_j(0)]$$

$$[C_{\phi\phi}] = \int_0^L c_{11} U_i(z) U_j(z) dz$$

$$[C_{\phi\psi}] = \int_0^L c_{12} U_i(z) U_j(z) dz$$

$$[C_{\psi\phi}] = \int_0^L c_{21} U_i(z) U_j(z) dz$$

$$[C_{\psi\psi}] = \int_0^L c_{22} U_i(z) U_j(z) dz$$

$$[K_{\phi\phi}] = \int_0^L \frac{dU_i(z)}{dz} \frac{dU_j(z)}{dz} dz$$

$$[K_{\psi\psi}] = \int_0^L \frac{dU_i(z)}{dz} \frac{dU_j(z)}{dz} dz$$

$$\{f_\phi\} = \frac{2\cos\theta}{c_0} \frac{dE_x^{inc}}{dt}[1,0,0,...,0_p]^T$$

$$\{f_\psi\} = \frac{2\cos\theta}{c_0} \frac{dE_y^{inc}}{dt}[1,0,0,...,0_p]^T .$$

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