Comparative Analysis of a Quantum Signal Processing Based Pomf Receiver with Adaptive Filters

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Abstract

In this paper a comparative analysis with adaptive filter receivers is performed over projected orthogonal matched filter (POMF) receivers. Here an algorithm has been designed for a projected orthogonal matched filter receiver based on quantum signal processing. The algorithm has been applied to a speech signal and the performance is estimated using probability theories. The algorithm describes the measurement vector orthogonality and imposes an inner product constraint. A new method is developed for choosing a set of measurement vectors that best represent the signals of interest which have a specified inner product structure. The POMF receiver performs correlation of the received signal with that of the original signal. Whitening is performed on the correlated signal. The resultant signal obtained will be the same as that of the transmitted speech signal. The performance of the POMF receiver is compared with matched filter, RLS and LMS adaptive filter receivers. The analysis suggests that over a wide range of channel parameters, the POMF receiver can outshine both the Matched Filter and the Adaptive Filters.

Keywords: Hilbert Space, Singular value Decomposition, Quantum Mechanics, LMS Adaptive filters, Inner Product Constraint

1. Introduction

The introduction of Quantum mechanical concepts, which rely on estimation almost entirely, on some signal processing algorithms are implemented with various techniques. Various signal processing approaches have been developed to examine the behavior of physical systems. One such signal processing technique is quantum signal processing (QSP) and it is aimed at developing new or modified existing signal processing algorithms by using the principles of quantum mechanics and some of its interesting axioms and constraints. Quantum signal processing is an organized framework, which can be implemented by exploiting the different mathematical structure of quantum mechanics such as vector algebra, analytical geometry, functional analysis and calculus of variations etc. It implied as bridgework between quantum measurement and signal processing algorithms. [1, 2 and 3] .The present work is concentrated on developing a quantum signal processing framework for possible application in speech analysis. Computationally efficient approximations of matched filter receivers and other adaptive filters have been developed in [4, 5 and 12]. The problem of designing a receiver with linear approach is pervasive in signal processing and communication applications. The optimal linear receiver for this problem is straightforward to derive, and is the well-known Projected OMF (POMF) receiver [6]. However, as discussed in a recent series of papers [7, 8], when the additive noise is white and Gaussian and the signals have equal prior probabilities, it is well known [9, 10 and 11] that a receiver which maximizes the probability of correct detection, consists of a demodulator comprised of a bank of correlators with correlating signals equal to the transmitted set, followed by a detector which chooses as the detected signal, the one for which the output of the correlator is maximum. This demodulator, is referred to as a matched filter (MF) receiver. If the noise is not Gaussian, then the MF receiver does not necessarily maximize the probability of correct detection. However, it is still used as the receiver of choice in many applications, since the optimal detector or non-Gaussian noise is typically nonlinear ([9] and references therein), and depends on the noise distribution which may not be known. One justification often given for its use is that if a signal is corrupted by Gaussian or non-Gaussian additive white noise, then a filter matched to that signal maximizes the output signal-to-noise ratio (SNR) from all linear filters [12]. The choice of terminology here is based on the interpretation of the receivers in terms of orthogonal signals. We show that if the transmitted signals give a strong symmetry property called geometric uniformity [13, 14 and 15], then the POMF receivers maximize the total output SNR, subject to the constraint that the outputs of the demodulator are uncorrelated on the appropriate space. This provides some additional justification for this class of receivers.

2. Quantum signal processing constraints

One of the important elements of quantum mechanics is that the measurement vectors are constrained to be orthonormal. For imposing such constraints, the measurement vectors are restricted to have a certain inner product structure as in quantum mechanics. We develop methods for choosing a set of measurement vectors that "best" represent the signals of interest and have a specified inner product structure. Specifically, we construct measurement vectors q_i with a given inner product structure that are closest in a least square (LS) sense to a given set of vectors s_i, so that the vectors q_i are chosen to minimize the sum of the squared norms of the error vectors $e_i = q_i - s_i$. These techniques are referred to as LS inner product shaping. This LS inner product shaping leads to new processing techniques in diverse areas, including frame theory, detection, covariance shaping, linear estimation, and multi-user wireless communication, which often exhibit improved performance over traditional methods.

Underlying the development of QSP in the signal space view points towards signal processing in which signals are regarded as vectors in an abstract Hilbert space referred to as the signal space. A complex vector space V over the complex numbers C is a set of elements called vectors, together with vector addition and scalar multiplication by elements of C such that V is closed under both operations. We will assume throughout that all vector spaces are complex. We now add geometric structure to a vector space in the form of an inner product relation between pairs of vectors, which also includes a distance measure or metric on the space. The inner product on the vector space V_i denoted $<x, y>$, is a mapping from V to C, and is said to be Hilbert Space. Two vectors x, y are said to be orthogonal in V if $<x, y>=0$. This situation is denoted as $x \perp y$. A linear transformation T: V→V is called orthogonal linear transformation if it preserves the inner product. That is, all pairs of vectors x and y are in the inner product space V. This means that T preserves the angle between x and y, and that the lengths of T_x and x are equal. The word normal is sometimes also used in place of orthogonal. However, normal can also refer to unit vectors. In particular, orthonormal refers to a collection of vectors that are both orthogonal and normal (of unit length). So, using the term normal to mean "orthogonal" is often avoided. In some contexts, two things are said to be orthogonal if they are mutually exclusive.

3. Problem Formulation

Suppose that one of m signals $\{S_k(t), 1 \le k \le m\}$ is received over an additive noise channel with equal Probability, where the signals lie in a real Hilbert space H with inner product $\langle x(t)y(t)\rangle = \int_{-\infty}^{\infty} x(t)y(t)dt$. We assume that the signals are normalized. The received signal r(t) is also assumed to be in H and is modeled as $r(t)= s_i(t)+n(t)$. For one value i, n (t) is a stationary white noise process with zero mean and spectral density σ, and otherwise unknown distribution. The receiver we design consists of the correlation demodulator that cross-correlates the received signal r(t) with the m signals $\{q_i(t) \in U, 1 \leq i \leq m\}$ so that $a_i = \langle q_i(t), r(t) \rangle$, where the signals $\{q_i(t)\}$ are to be determined. The declared detected signal is $s_i(t)$, where i=argmax a_i. The difference between the modified receivers and the MF receiver lies in the choice of signals $q_i(t)$.

If we choose the signals $q_i(t)=s_i(t)$, then the resulting demodulator is equivalent to an MF demodulator. If the noise is not Gaussian, then the MF receiver does not necessarily minimize the probability of detection error. It is still used as the receiver of choice in many applications, since the optimal receiver for non-Gaussian noise is typically non-linear, and requires knowledge of the noise distribution. For a correlation demodulator, we would like to choose the signals $q_i(t)$ so that when the noise is non-Gaussian, the resulting detector leads to improved performance over MF detection. Drawing from the quantum detection problem, we propose imposing an inner product constraint on the signals, which as we show is equivalent to imposing a constraint on the correlation between the demodulator outputs. Building upon our results regarding optimal QSP measurements, we then develop a new class of correlation receivers that, like the MF, depend only on the transmitted signals, and can lead to improved performance over the MF for some classes of non-Gaussian noise, with essentially negligible loss of performance for Gaussian noise.

In the system shown, the correlation between the outputs a_i of the correlation demodulator is proportional to the inner products between the signals $q_i(t)$.

$$Cov(a_i, a_k) = E(\langle q_i(t), n(t)\rangle\langle n(t), qk(t)\rangle)\sigma^2 \langle q_i(t), q_k(t)\rangle. \quad (1)$$

In our modification of the MF demodulator we propose shaping the correlation of the outputs prior to detection. Thus, we propose choosing the signals $q_i(t)$ to have a specified inner product structure, so that the outputs a_i have the desired correlation. Here we choose the signals $q_i(t)$ so that the outputs a_i are uncorrelated. We briefly consider the more general case in which the signals $q_i(t)$ are chosen to have an arbitrary inner product structure. The subsequent development considers separately which transmitted signals are linearly independent or linearly dependent.

If the signals $s_i(t)$ are linearly independent, then we may decorrelate the outputs a_i by choosing the signals $q_i(t)$ to be orthonormal. The resulting demodulator is referred to as Orthogonal Matched Filter (OMF) demodulator, and the overall detector is referred to as an OMF detector.

If the signals $s_i(t)$ are linearly dependent, so that they span an n-dimensional subspace U, then there are at most n orthonormal signals in U, and we cannot choose the correlating signals to whiten the outputs a_i in the conventional space. Instead, we choose the correlating signals as projections of a set of orthonormal signals in a larger space containing U, i.e., we choose the correlating signals to form a normalized tight frame for U. As we show, the outputs a_i are then uncorrelated on a space formed by the transmitted signals. The resulting demodulator is a Projected Orthogonal Matched Filter demodulator or POMF detector.

4. Implementation of Projected Orthogonal Matched Filter Receiver

Suppose now the transmitted signals $\{s_i, 1 \leq i \leq m\}$ are linearly dependent, and span an n-dimensional subspace, where n<m. As in the case of linearly independent signals, we can choose the signals $q_i(t)=g_i(t)$ to be orthonormal and to minimize the LS error. Since $\langle g_i(t), g_i(t)\rangle = 1$ for any signals $g_i(t)$, minimizing the LS error is equivalent to maximizing:

$$\sum_{i=1}^{m} \langle g_i(t), s_i(t)\rangle = \sum_{i=1}^{m} \langle g_i^{u}(t), s_i(t)\rangle \quad (2)$$

where the signals form a normalized tight frame for U. For any normalized frame for U, maximizing is equivalent to minimizing:

$$\sum_{i=1}^{m} \langle g_i^{u}(t) - s_i(t), g_i^{u}(t) - s_i(t)\rangle \quad (3)$$

Thus, when the signals $s_i(t)$ are linearly dependent, choosing a set of orthogonal signals to maximize SNR is equivalent to choosing a normalized tight frame for U to minimize the LS error. Furthermore, if the transmitted signal is

$s_i(t)$, then the ith output of the correlation demodulator with signals $g_i(t)$ is:

$$\langle Tx, Ty\rangle = \langle x, y\rangle. \qquad (4)$$

$$a_i = \langle g_i(t), r(t)\rangle = \langle g_i^u(t), s_i(t)+n(t)\rangle + \langle g_i^{u\perp}(t), n(t)\rangle = r_i + n_i \qquad (5)$$

Since r_i and n_i are uncorrelated, n_i does not contain any linear information that is relevant to the detection of $s_i(t)$.Therefore, in the case of linearly dependent signals $s_i(t)$, we propose choosing the signals $q_i(t)$ to be a normalized tight frame for U, which we denote by $q_i(t)=f_i(t)$, that minimizes the LS error.

Thus we seek the signals $\{f_i(t),\ 1\leq i\leq m\}$ corresponding to F to minimize:

$$E_{LS} = \sum_{i=1}^{m} \langle s_i(t) - f_i(t), s_i(t) - f_i(t)\rangle \qquad (6)$$

Subject to the constraint:

$$FF^* = P_u \qquad (7)$$

5. Results and Discussion

Most useful parameters in speech processing are found in the frequency domain. There are several ways to extract the spectral information of speech. When the audio file contains a single channel (mono), the block's output is an M-by-1 matrix containing one frame (M consecutive samples) of mono audio data. With the use of a periodogram block a nonparametric estimate of the power spectrum of the speech signal is computed. With the use of a transpose block, the vector input signals are treated as (Mx1) matrices as output.

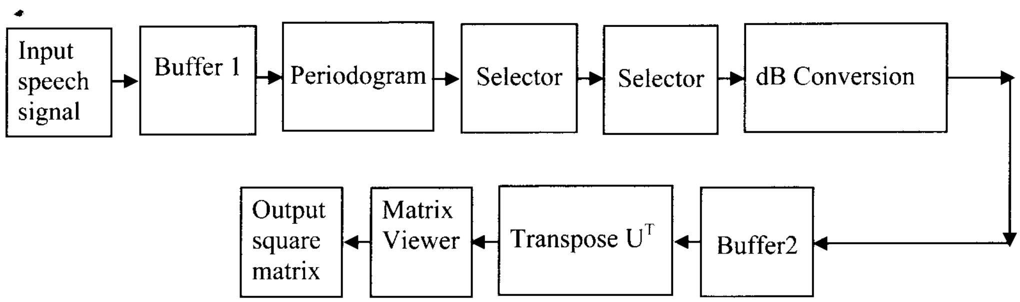

Fig 1 Feature Vector Generation

The signal is obtained as a column vector. This column vector is converted into a square matrix. Now a Hilbert transform is performed on this matrix so that the numerical values of the signal can be obtained. An FFT is performed on the signal so that the spectral values of the signal can be obtained. As the concept of QSP is satisfied, now the spectral matrix is converted into an orthogonal matrix using the Gram-Schmidt orthogonalization procedure. In the orthogonal matrix, white noise with zero mean and unit standard deviation is added to the signal. Input is a continuous speech signal given through a microphone. This signal is plotted with its amplitude with respect to time in Figure 2. The plot for the signal input to the receiver is plotted in Figure 3.

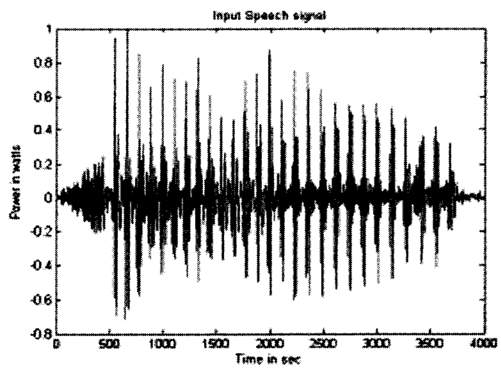

Fig 2 Input Speech Signal

The received signal is subjected to SVD (Singular Value Decomposition) and matrix based results are obtained. The Eigenvector of the signals is obtained. Figure 4 explains the correlation process between the signals at the input of the receiver and correlating signal.

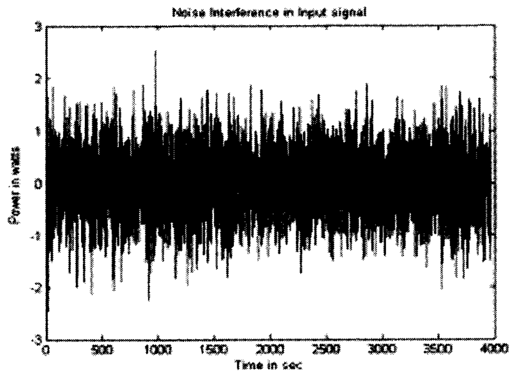

Fig 3 Input Speech Signal with Noise

Whitening is performed on the signal. An Inverse FFT is processed to the resultant matrix and is converted to a column vector which is the output of the receiver as shown in Figure 5.

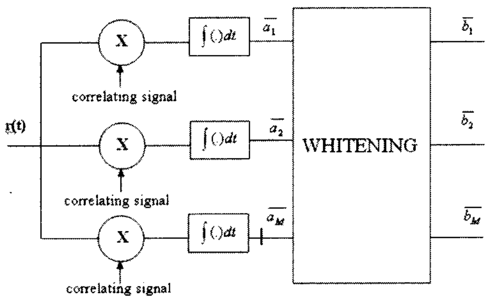

Fig 4 projected orthogonal Matched Filter Receiver

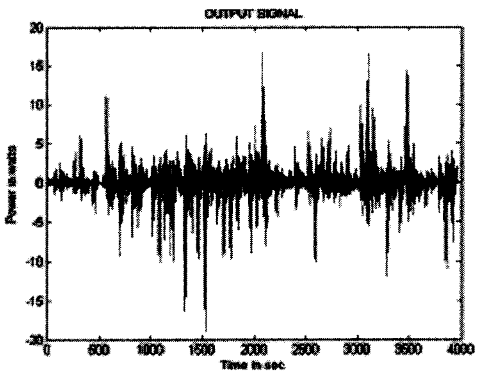

Fig 5 Output Signal

We derive exact and approximate expressions for the probability of error with the knowledge of signal to noise ratio for different receivers. The probability of the signal detection and probability of error prediction is approximately one and zero respectively, as shown in Figure 6 & Figure 7 for the POMF receiver.

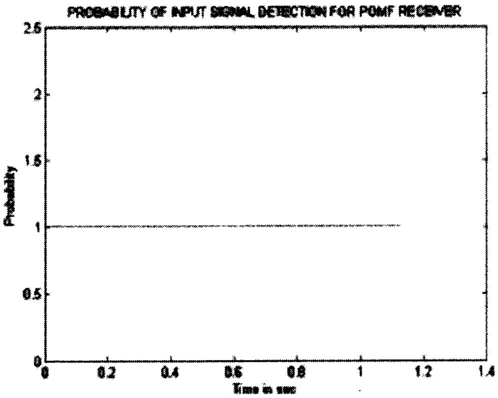

Fig 6 Probability of input signal detection for POMF receiver

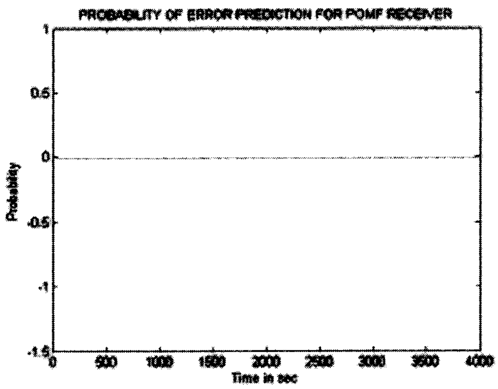

Fig 7 Probability of input signal prediction for POMF receiver

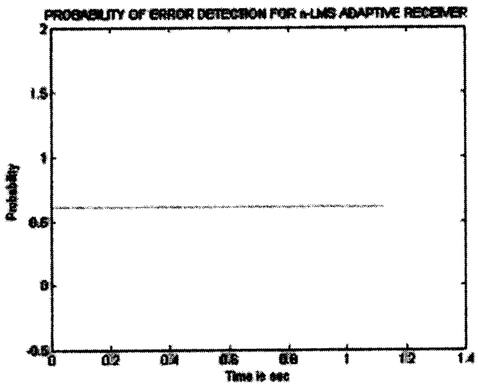

Fig 8 Probability of input signal Detection for n-LMS adaptive receiver

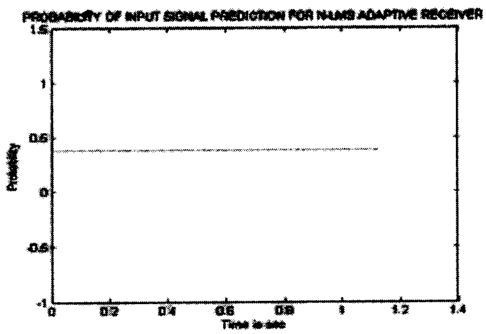

Fig 9 Probability of input signal Prediction for n-LMS adaptive receiver

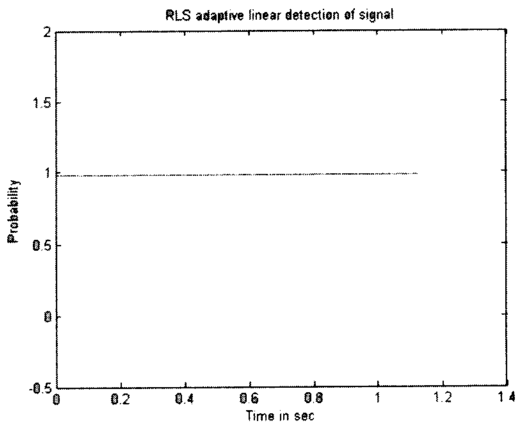

Fig 10 Probability of input signal Detection for RLS-adaptive receiver

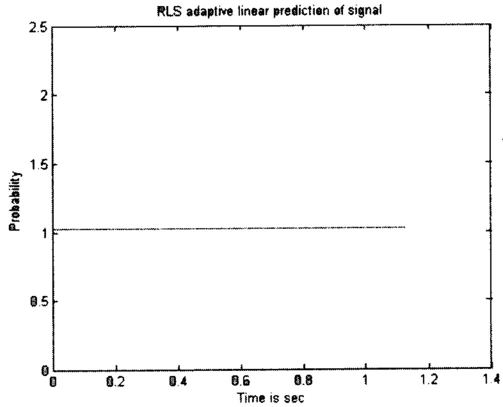

Fig 11 Probability of input signal Prediction for RLS-adaptive receiver

A recursive least square adaptive receiver(RLS), normalized LMS adaptive filter receiver and matched filter receiver can be designed for estimating the instantaneous state of linear system perturbed by white noise. The results shows that the probability of signal detection and probability of error prediction is less than one and greater than zero respectively as in Figure 8 and Figure 9 for n-LMS adaptive filter.

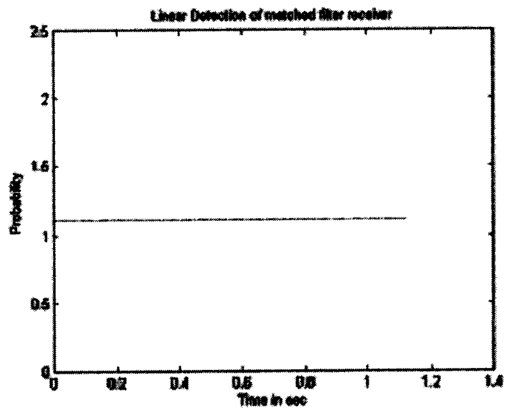

Fig 12 Probability of input signal Detection for Matched filter receiver

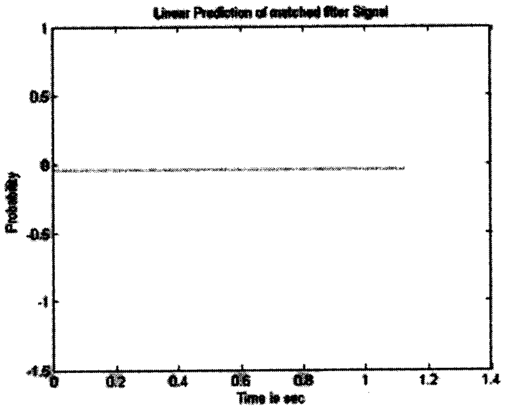

Fig 13 Probability of input signal prediction for Matched filter receiver

The analysis of RLS-adaptive filter receiver shows that probability of signal detection and probability of error prediction is less than one and greater than zero respectively as in Figure 10 and Figure 11. The results for matched filter receiver give the probability of signal detection and probability of error prediction are greater than one and less than zero respectively as in Figure 12 and Figure 13.

	POMF receiver	n-LMS adaptive receiver	RLS adaptive receiver	Matched Filter receiver
Probability of Error Detection	1.0	0.65	0.92	1.12
Probability of Error Prediction	0.0	0.485	0.12	-0.13

Table 1 Analysis of various receivers with Probability of error.

6. Conclusion

A comparative study has been made with adaptive and matched filter receivers. A speech signal is the input to the system which is received at the receiver output. This result suggests that the receiver output shows a steep probability in signal detection and a well-drop probability in the error prediction of the signal. The implementation of this receiver seems simple in the design of the hardware part so that future analysis can be made on the implementation of the receiver in orthogonal CDMA systems.

7. Reference

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