BER Performance of UWB Waveforms Satisfying FCC Indoor and Outdoor Spectral Masks

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Abstract

In this paper, the bit error rate (BER) performance of ultra wideband (UWB) waveforms with matched filter and correlation receivers is analyzed. The rectangular passband, modulated rectangular and modulated Gaussian waveforms, which satisfy the UWB signal definition and Federal Communications Commission (FCC) indoor and outdoor limit spectral masks, are used as the transmitted UWB signals. The complex form of Friis' transmission formula is considered as the UWB free space channel. Therefore, the distortion effects caused from the channel are included. The BER performance of each waveform is shown and compared. The results are discussed in the conclusion.

Keyword: Bit error rate (BER), ultra wideband (UWB), indoor and outdoor spectral masks, antipodal and orthogonal modulations, matched filter and correlation receivers.

1. Introduction

Ultra wideband (UWB) radio technology has become an important topic for microwave communications because of its low cost and low power consumption potentials [1], [2]. The UWB radio technology differs from the conventional narrow band radio frequency (RF) and spread spectrum (SS) technologies. The UWB radio technology uses an extremely wideband of RF spectrum to transmit the data with very short pulses and power spectral density (PSD) in the range of ultra wide frequency spectrum instead of using narrow carrier frequency in traditional RF technologies. The UWB radio technology is a unique and new usage of recently legalized frequency spectrum. The UWB radio technology is specified in the frequencies ranging from 3.1 GHz to 10.6 GHz by the Federal Communications Commission (FCC) [3]. The FCC defined the UWB signal as those, which have a fractional bandwidth equal to or greater than 0.20, or occupy bandwidth equal to or greater than 500 MHz.

The PSD of a UWB signal does not exceed the FCC part 15 limits or -41.3 dBm/MHz, so that the PSD of a UWB signal is considered as noise for other radio communication systems. Therefore, the UWB radio technology can coexist with other RF communications without interference. Moreover, UWB radio technology is an ideal candidate that can be utilized for commercial, short-range, low power and low cost indoor communication systems, such as wireless personal area networks (WPANs) [4].

The Friis' transmission formula [5] is widely used to calculate the free space path loss for narrowband communications. The complex form of Friis' transmission formula is developed for UWB communications [6]-[8]. The matched filter and correlation receivers are used as the UWB receivers [9]-[11]. Although, the performances of UWB communications are analyzed [12], [13], there are no considerations about the FCC regulation of UWB signal and distortion of a UWB signal caused by a channel. After that, the rectangular waveform distorted by a UWB free space channel is used to derive the theoretical BER performance [14]. However, there are no considerations about other causal waveforms.

In this paper, the BER performance of UWB waveforms with matched filter and correlation receivers is analyzed. The rectangular passband, modulated rectangular and modulated Gaussian waveforms, which satisfy the UWB signal definition and FCC indoor and outdoor limit spectral masks [15], are used as the transmitted UWB signal. The complex form of Friis' transmission formula is considered as the UWB free space channel. The spectral density of received signal, considering the distortion caused by a free space channel, is evaluated. At the receiver, the matched filter and correlation receivers with frequency transfer functions, which satisfy the constant noise power conditions between the input and output, are used. The BER performance of each waveform is shown and compared.

This paper is organized as follows. In Sections 2 and 3, the UWB waveform models and BER performance analysis are briefly discussed, respectively. Next, the analysis results are illustrated in Section 4. Finally, the conclusions are discussed in Section 5.

2. UWB Waveform Models

For UWB waveforms, the rectangular passband, modulated rectangular and modulated Gaussian waveforms are considered as the UWB transmitted waveform v_t in the time domain and its spectral density V_t in the frequency domain. These waveforms can satisfy the FCC definition of UWB signal and FCC spectral masks for indoor and outdoor limits. The parameters obtained from maximum bandwidth, amplitude and average power optimizations, which are proposed in [15], are used.

2.1 Rectangular Passband Waveform

The rectangular passband transmitted waveform in the time domain and its spectral density function are given by:

$$v_t(t) = \frac{A}{f_b}[f_H \operatorname{sinc}(2f_H t) - f_L \operatorname{sinc}(2f_L t)] \quad (1)$$

$$V_t(f) = \begin{cases} \dfrac{A}{2f_b} & \| f | - f_c | \le \dfrac{f_b}{2} \\ 0 & \| f | - f_c | > \dfrac{f_b}{2} \end{cases} \quad (2)$$

where A is the maximum amplitude, f_b is the occupied bandwidth, f_c is the center frequency, $f_L = f_c - f_b/2$ and $f_H = f_c + f_b/2$ are the minimum and maximum frequencies. This waveform has an $A/2f_b$ constant magnitude of spectral density in $-f_H$ to $-f_L$ and f_L to f_H frequency ranges. The area of its spectral density is $\int_{-\infty}^{\infty} F(f)df = A$. This waveform has the A maximum amplitude at $t = 0$. This is the ideal case of UWB waveform . It is used to consider the upper limit of maximum bandwidth, amplitude and average power for UWB waveforms.

2.2 Modulated Rectangular Waveform

The modulated rectangular transmitted waveform in the time domain and its spectral density function are given by:

$$v_t(t) = \begin{cases} A\sin(2\pi f_c t) & |t| \le \dfrac{t_b}{2} \\ 0 & |t| > \dfrac{t_b}{2} \end{cases} \quad (3)$$

$$V_t(f) = \frac{At_b}{j2}\begin{Bmatrix} \operatorname{sinc}|t_b(f - f_c)| \\ -\operatorname{sinc}|t_b(f + f_c)| \end{Bmatrix} \quad (4)$$

where A is the maximum amplitude, f_c is the carrier frequency and t_b is the pulse width of waveform.

This waveform is modulated between the A constant amplitude and t_b width rectangular pulse and f_c carrier frequency. The sine function is used for reducing the direct current (DC) component of the modulated waveform to zero.

2.3 Modulated Gaussian Waveform

The modulated Gaussian transmitted waveform in time domain and its spectral density function are given by:

$$v_t(t) = Ae^{-(t/d)^2}\sin(2\pi f_c t) \tag{5}$$

$$V_t(f) = \frac{Ad\sqrt{\pi}}{j2}\left[e^{-\pi^2 d^2 (f-f_c)^2} - e^{-\pi^2 d^2 (f+f_c)^2}\right] \tag{6}$$

where A is the maximum amplitude of envelope waveform, f_c is the carrier frequency and d is the $1/e$ characteristic decay time.

This waveform is modulated between the A maximum amplitude and d ($1/e$ characteristic decay time Gaussian pulse) and f_c carrier frequency. The sine function is used for reducing the direct current (DC) component of the modulated waveform to zero, in the same way as the modulated rectangular waveform.

3. BER Performance Analysis

In this section, the analysis of BER performance is theoretically discussed. The UWB waveform models discussed in Sec. 2 are used as the UWB transmitted waveforms. For UWB free space channel, the complex form of Friis' transmission formula is used [6]-[8]. The transmitting (Tx) and receiving (Rx) antennas are considered to have one constant gain. The frequency transfer function of free space channel H_f can be written as :

$$H_f(f,d) = \frac{c}{4\pi|f|d}e^{-j2\pi fd/c} \tag{7}$$

where d is the transmitter-receiver (T-R) separation distance and c is the velocity of light. This equation satisfies for both positive and negative frequencies as it satisfies the causality.

The spectral density of UWB received signal V_r is calculated by using multiplication between H_f and V_t, which can be written as:

$$V_r(f,d) = H_f(f,d)\cdot V_t(f) \tag{8}$$

This spectral density of UWB received signal includes the distortion effect caused by UWB free space channel.

The matched filter and correlations receivers with frequency transfer functions, satisfying constant noise power condition between input and output , are considered. For a matched filter receiver or an optimum correlation receiver, the spectral density of template signal is the complex conjugate of V_r. For correlation or transmitted template signal receivers, the spectral density of template signal is the complex conjugate of V_t. The receiver gains satisfy the constant noise power conditions. Therefore, the frequency transfer functions of matched filter and correlation receivers, H_m and H_c, can be written as (see Appendix A) :

$$H_m(f,d) = \frac{\sqrt{2f_b}}{\sqrt{\int_{-\infty}^{\infty}|V_r(f,d)|^2 df}}V_r^*(f,d) \tag{9}$$

$$H_c(f) = \frac{\sqrt{2f_b}}{\sqrt{\int_{-\infty}^{\infty}|V_t(f)|^2 df}}V_t^*(f) \tag{10}$$

where * is the complex conjugate operator.

The spectral densities of output signal from matched filter and correlation receivers, V_m and V_c can be written as :

$$V_m(f,d) = H_m(f,d)\cdot V_r(f,d) \tag{11}$$

$$V_c(f,d) = H_c(f)\cdot V_r(f,d) \tag{12}$$

The signal-to-noise ratio (SNR) gain of these receivers is defined as the ratio between average powers of received signal at receiver output and that at receiver input for the case of constant noise power conditions between input and output. The SNR gains of matched filter and correlation receivers, G_m and G_c can be respectively written as:

$$G_m = \frac{\int_0^{\infty}|V_m(f,d)|^2 df}{\int_0^{\infty}|V_r(f,d)|^2 df} \tag{13}$$

$$G_c = \frac{\int_0^{\infty}|V_c(f,d)|^2 df}{\int_0^{\infty}|V_r(f,d)|^2 df} \tag{14}$$

The efficiency of receiver is considered by using correlation coefficients between received and template signals. The correlation coefficients of matched filter and correlation receivers, C_m and C_c can be respectively written as (see Appendix B):

$$C_m = 1 \qquad (15)$$

$$C_c = \frac{\max\left|\int_{-\infty}^{\infty} V_c(f,d)e^{j2\pi ft}\,df\right|}{\sqrt{\int_{-\infty}^{\infty}\left|V_r(f,d)\right|^2 df \cdot \int_{-\infty}^{\infty}\left|H_c(f)\right|^2 df}} \qquad (16)$$

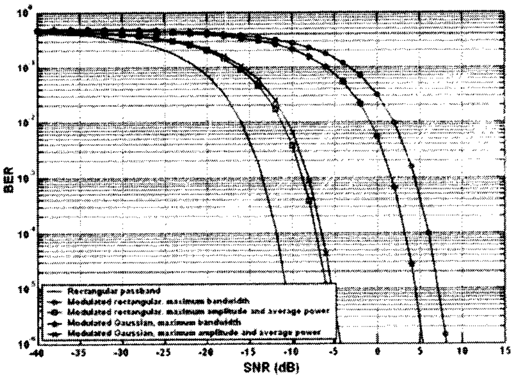

Fig. 1 BER of UWB waveform with matched filter receiver satisfying FCC indoor limit spectral mask for antipodal modulation scheme.

The UWB modulation schemes can be classified into antipodal modulation scheme such as binary pulse amplitude modulation (BPAM), and orthogonal modulation scheme such as on-off keying (OOK) and pulse position modulation (PPM), with modulation index of $\delta = 1$ [16]. In this paper, BER performance of antipodal and orthogonal modulation schemes are considered. The BER performance of matched filter and correlation receivers for antipodal modulation scheme, $B_{m,a}$ and $B_{c,a}$, in additive white Gaussian noise (AWGN) can be respectively written as [17] (see Appendix C) :

$$B_{m,a} = Q\left(\sqrt{\frac{2C_m G_m f_b S}{B_r N}}\right) \qquad (17)$$

$$B_{c,a} = Q\left(\sqrt{\frac{2C_c G_c f_b S}{B_r N}}\right) \qquad (18)$$

The BER performance of matched filter and correlation receivers for orthogonal modulation scheme, $B_{m,o}$ and $B_{c,o}$, in AWGN can be written as [17] (see Appendix C):

$$B_{m,o} = Q\left(\sqrt{\frac{C_m G_m f_b S}{B_r N}}\right) \qquad (19)$$

$$B_{c,o} = Q\left(\sqrt{\frac{C_c G_c f_b S}{B_r N}}\right). \qquad (20)$$

where B_r is the bit rate, S/N is the SNR at input of receiver and

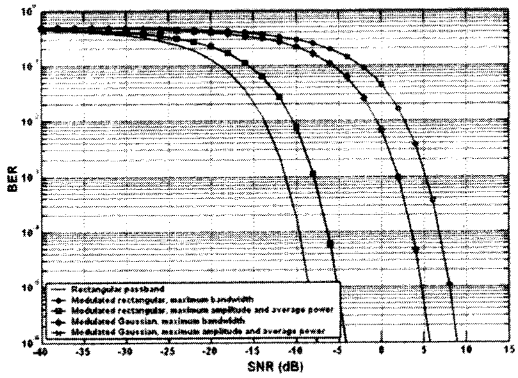

Fig. 2 BER of UWB waveform with correlation receiver satisfying FCC indoor limit spectral mask for antipodal modulation scheme.

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-t^2/2}dt, \quad x \geq 0$$

The SNR is normalized by average power of rectangular bandpass waveform, which is the theoretical maximum average power of UWB signal, for considering FCC spectral masks. Moreover, the distortion effect caused from UWB channel, occupied bandwidth and bit rate are also included to analyze the BER performance.

4. Analysis Results

In this section, the analysis results of BER performance are shown. The BER performance of transmitted waveforms, which are the rectangular bandpass, modulated rectangular and modulated Gaussian waveforms, are analyzed. The TR separation distance is set to be 10 m, while the bit rate is set to be 110 Mbps. These parameters are based on the IEEE 802.15.3a [18]. The parameters of each waveform obtained from the maximum bandwidth, amplitude and average power optimizations, satisfying FCC spectral masks for indoor and outdoor limits, are used [14].

Figures 1 to 4 show the BER performances of matched filter and correlation receivers for UWB waveforms satisfying FCC indoor limit spectral mask. The BER performances for antipodal modulation scheme are shown in Figs. 1 and 2, and those for orthogonal modulation scheme are shown in Figs. 3 and 4.

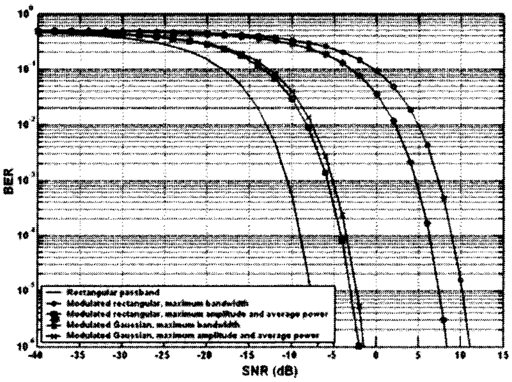

Fig. 3 BER of UWB waveform with matched filter receiver satisfying FCC indoor limit spectral mask for orthogonal modulation scheme.

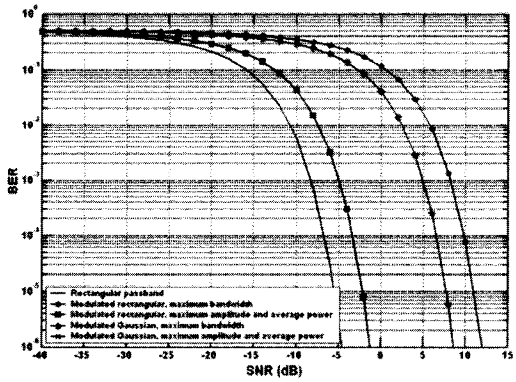

Fig. 4 BER of UWB waveform with correlation receiver satisfying FCC indoor limit spectral mask for orthogonal modulation scheme.

From these figures, we can see that the BER performances of rectangular passband waveforms are the lowest of all cases. The BER performances of waveform obtained from maximum amplitude and average power optimizations are less than that obtained from maximum bandwidth optimizations. That is because the waveforms obtained from maximum bandwidth optimizations give much less average power and slightly more bandwidth compared with that obtained from maximum amplitude and average power optimizations [15]. For comparison between modulated rectangular and Gaussian waveforms, the BER performance of modulated rectangular and Gaussian waveforms with maximum amplitude and average power

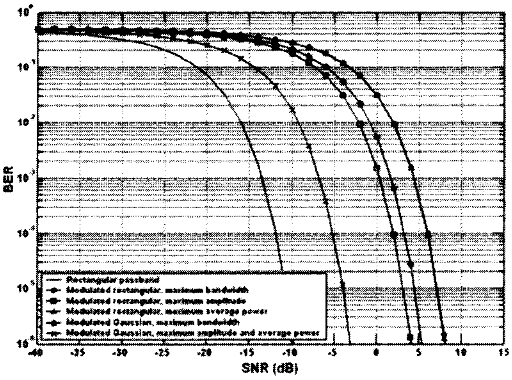

Fig. 5 BER of UWB waveform with matched filter receiver satisfying FCC outdoor limit spectral mask for antipodal modulation scheme.

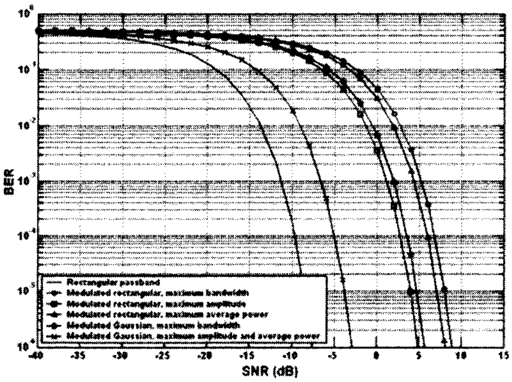

Fig. 6 BER of UWB waveform with correlation receiver satisfying FCC outdoor limit spectral mask for antipodal modulation scheme.

optimizations are almost the same and are better than the others.

The BER performances of matched filter and correlation receivers for UWB waveforms satisfying FCC outdoor limit spectral mask are shown in Figs. 5 to 8. The BER performances for antipodal modulation scheme are shown in Figs. 5 and 6, and those for orthogonal modulation scheme are shown in Figs. 7 and 8.

The BER performances of rectangular passband waveforms are also the lowest. The

BER performances for FCC outdoor limit spectral mask are average more than that for FCC indoor limit spectral mask. From the results, the average BER performances of all FCC indoor and outdoor limit spectral masks at -10 dB SNR are 0.14 and 0.22, respectively. For comparison between modulated rectangular and

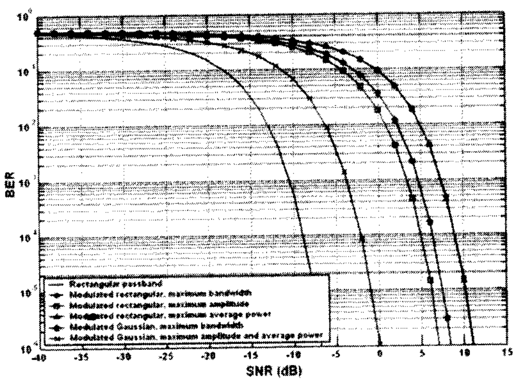

Fig. 7 BER of UWB waveform with matched filter receiver satisfying FCC outdoor limit spectral mask for orthogonal modulation scheme.

Gaussian waveforms, the BER performances of modulated Gaussian waveform with maximum amplitude and average power optimizations are clearly better than the others. That is because the side lobe spectrum of modulated Gaussian waveform is less than that of modulated rectangular waveforms and the FCC outdoor spectral mask is very strict compared with the FCC indoor spectral mask. Therefore, the modulated Gaussian waveform is appropriate for this case.

For considering the modulation scheme, the BER of antipodal modulation scheme is less than that of orthogonal modulation scheme because the threshold distance of antipodal modulation scheme is wider than that of orthogonal modulation scheme.

5. Conclusion

In this paper, the BER performance of UWB waveforms with matched filter and correlation receivers is analyzed. The rectangular passband, modulated rectangular and modulated Gaussian waveforms, which satisfy the UWB signal definition and FCC indoor and outdoor limit spectral masks with antipodal and orthogonal modulation schemes, are used as the transmitted UWB signal. From the results, the BER of rectangular bandpass waveform with matched filter receiver is the ideal theoretical bound, which is the minimum BER performance case. The modulated rectangular and Gaussian waveforms with maximum amplitude and

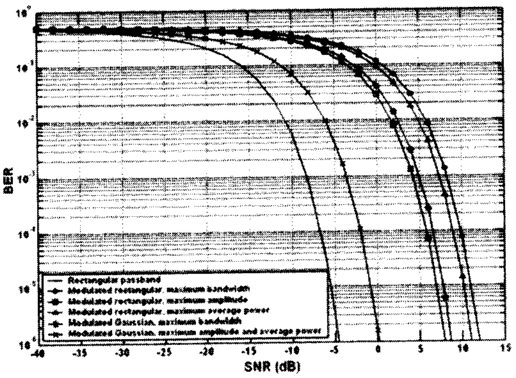

Fig. 8 BER of UWB waveform with correlation receiver satisfying FCC outdoor limit spectral mask for orthogonal modulation scheme.

average power optimizations are appropriate for FCC indoor limit spectral mask, while the modulated Gaussian waveform with maximum amplitude and average power optimizations is appropriate for FCC outdoor limit spectral mask.

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Appendix A: Derivations of Receiver Frequency Transfer Functions

Frequency transfer functions of matched filter and correlation receivers are derived, which satisfy constant noise power conditions between input and output. Noise is considered as AWGN with zero-mean and PSD of $N_0/2$ W/Hz. Therefore, the input noise power N_i can be written as:

$$N_i = N_0 f_b \tag{A-1}$$

The output noise power of matched filter and correlation receivers, $N_{o,m}$ and $N_{o,i}$ are:

$$N_{o,m} = \frac{N_0}{2}\int_{-\infty}^{\infty}\left|H_m(f,d)\right|^2 df \qquad \text{(A-2)}$$

$$N_{o,c} = \frac{N_0}{2}\int_{-\infty}^{\infty}\left|H_c(f,d)\right|^2 df \qquad \text{(A-3)}$$

Set $N_{o,m}$ and $N_{o,i}$ equal to N_i, the constant noise power conditions between input and output of matched filter and correlation receivers can be respectively derived as:

$$\int_{-\infty}^{\infty}\left|H_m(f,d)\right|^2 df = 2f_b \qquad \text{(A-4)}$$

$$\int_{-\infty}^{\infty}\left|H_c(f,d)\right|^2 df = 2f_b \qquad \text{(A-5)}$$

The general form of frequency transfer function of matched filter receiver with output maximum SNR is [19]:

$$H_m(f,d) = A_m V_r^*(f,d)e^{-j2\pi f t_m} \qquad \text{(A-6)}$$

where arbitrary constant A_m and t_m are magnitude and delayed time, respectively.

Set $t_m = 0$ and solve A_m, which satisfied constant noise power condition (A-4). The frequency transfer function of matched filter receiver can be derived as:

$$H_m(f,d) = \frac{\sqrt{2f_b}}{\sqrt{\int_{-\infty}^{\infty}|V_r(f,d)|^2\, df}} V_r^*(f,d) \qquad \text{(A-7)}$$

In practice, the received signal is unknown. The correlation receiver, which the transmitted signal is used instead of received signal, is considered. Therefore, the frequency transfer function of correlation receiver can be derived as:

$$H_c(f) = \frac{\sqrt{2f_b}}{\sqrt{\int_{-\infty}^{\infty}|V_t(f)|^2\, df}} V_t^*(f) \qquad \text{(A-8)}$$

Appendix B: Derivations of Correlation Coefficients

The correlation coefficient between received and template signals is used to consider the efficiency of receiver. The correlation coefficients of matched filter and correlation receivers are respectively defined as:

$$C_m = \frac{\max|r_{v_r,v_r}(\tau)|}{\sqrt{\max|r_{v_r}(\tau)|\cdot\max|r_{v_r}(\tau)|}} \qquad \text{(B-1)}$$

$$C_c = \frac{\max|r_{v_r,v_t}(\tau)|}{\sqrt{\max|r_{v_r}(\tau)|\cdot\max|r_{v_t}(\tau)|}} \qquad \text{(B-2)}$$

where v_r is the received signal waveform, r_a and $r_{a,b}$ respectively are the autocorrelation of signal a and cross-correlation between signal a and b, which are defined as [19]:

$$r_a(\tau) = \int_{-\infty}^{\infty} a^*(t)a(t+\tau)dt \qquad \text{(B-3)}$$

$$r_{a,b}(\tau) = \int_{-\infty}^{\infty} a^*(t)b(t+\tau)dt \qquad \text{(B-4)}$$

Substitute the definitions of autocorrelation and cross-correlation, (B-3) and (B-4), in (B-1). The correlation coefficient of matched filter receiver can be derived as:

$$C_m = 1 \qquad \text{(B-5)}$$

From this result, the correlation coefficient of matched filter receiver is always equal to 1. That is the ideal case, which the template signal is identical with received signal. Therefore, the energy of signal with matched filter receiver is optimum.

For considering the correlation coefficient of correlation receiver, evaluate the impulse response of correlation receiver h_c by using the inverse Fourier transform of (10) and assume the transmitted signal is the real value signal. This obtains:

$$h_c(t) = \frac{\sqrt{2f_b}}{\sqrt{\int_{-\infty}^{\infty}\left|V_t(f)\right|^2 df}} v_t(-t) \qquad \text{(B-6)}$$

Use the correlation function properties and Perseval's theorem for energy signal, which are [19]

$$E_a = \max\ |r_a(\tau)| = \int_{-\infty}^{\infty}\left|A(f)\right|^2 df \qquad \text{(B-7)}$$

where E_a is the energy of signal a and A is the spectral density of signal a.

Use relation between cross-correlation and convolution functions for the real value signal. This obtains [19]:

$$\int_{-\infty}^{\infty} v_r^*(t)h_c(-t+\tau)dt = v_r(t)\otimes h_c(t) = v_c(t) \quad \text{(B-8)}$$

where $\otimes$ is the convolution operator and v_c is the output signal from correlation receiver, which is obtained by using the inverse Fourier transform of its spectral density. That is [19]:

$$v_c(t) = \int_{-\infty}^{\infty} V_c(f,d)e^{j2\pi ft}df \quad \text{(B-9)}$$

Substitute (B-3), (B-4), (B-6), (B-7), (B-8) and (B-9) in (B-2). The correlation coefficient of correlation receiver can be derived as:

$$C_c = \frac{\max\left|\int_{-\infty}^{\infty} V_c(f,d)e^{j2\pi ft}df\right|}{\sqrt{\int_{-\infty}^{\infty}|V_r(f,d)|^2 df\cdot\int_{-\infty}^{\infty}|H_c(f)|^2 df}} \quad \text{(B-10)}$$

The efficiency of correlation receiver is equal to 1, when the transmitted signal is identical with received signal. There is no distortion of signal. If there is more distortion of signal, the correlation coefficient is decreased. The energy of signal with correlation receiver is proportional to the correlation coefficient.

Appendix C: Derivations of BER Performance

In this appendix, the BER performances of antipodal and orthogonal modulation schemes are derived for UWB communications. For antipodal modulation scheme, the geometric representation of binary signals is the one-dimensional vector and can be written as [17]:

$$s_1 = \sqrt{E_b} \quad \text{(C-1)}$$

$$s_2 = -\sqrt{E_b} \quad \text{(C-2)}$$

where E_b is the average energy per bit, s_1 and s_2 are vector representations of bits '1' and '0', respectively.

The received signals of bits '1' and '0', r_1 and r_2, from matched filter or correlation receivers with no distortion of signals respectively are [17]:

$$r_1 = \sqrt{E_b} + n \quad \text{(C-3)}$$

$$r_2 = -\sqrt{E_b} + n \quad \text{(C-4)}$$

where n is the AWGN with zero-mean and its PSD or variance of $N_0/2$ W/Hz. Therefore, the probability distribution functions (PDFs) of received signals that s_1 and s_2 are transmitted, $p(r\,|\,s_1)$ and $p(r\,|\,s_2)$, are [17]:

$$p(r\,|\,s_1) = \frac{1}{\sqrt{\pi N_0}}e^{-(r-\sqrt{E_b})^2/N_0} \quad \text{(C-5)}$$

$$p(r\,|\,s_2) = \frac{1}{\sqrt{\pi N_0}}e^{-(r+\sqrt{E_b})^2/N_0} \quad \text{(C-6)}$$

In this case, the decision conditions compare r with zero-threshold. If $r>0$, the decision is s_1 and if $r<0$, the decision is s_2. The probabilities of bit errors that s_1 and s_2 are transmitted, $P(e\,|\,s_1)$ and $P(r\,|\,s_2)$, are [17]:

$$P(e\,|\,s_1) = \int_{-\infty}^{0} p(r\,|\,s_1)dr = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \quad \text{(C-7)}$$

$$P(e\,|\,s_2) = \int_{0}^{\infty} p(r\,|\,s_2)dr = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \quad \text{(C-8)}$$

The BER performance of antipodal modulation scheme is the average probability of error. This is [17]:

$$B_a = \frac{1}{2}P(e\,|\,s_1) + \frac{1}{2}P(e\,|\,s_2)$$

$$= Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \quad \text{(C-9)}$$

The BER performance can be expressed in the term of threshold distance between signals s_1 and s_2. For this case, the threshold distance d_{12} is equal to $2\sqrt{E_b}$. Therefore, the BER performance B can be written as [17]:

$$B = Q\left(\sqrt{\frac{d_{12}^2}{2N_0}}\right) \quad \text{(C-10)}$$

For orthogonal modulation scheme, the geometric representation of binary signals is a two-dimensional vector and can be written as [17]:

$$s_1 = (\sqrt{E_b}, 0) \quad \text{(C-11)}$$

$$s_2 = (0, \sqrt{E_b}) \quad \text{(C-12)}$$

For this case, the threshold distance d_{12} is equal to $\sqrt{2E_b}$. Substitute $d_{12} = \sqrt{2E_b}$ in (C-10). The BER performance of orthogonal modulation scheme B_0 can be written as [17]:

$$B_o = Q\left(\sqrt{\frac{E_b}{N_0}}\right) \quad \text{(C-13)}$$

Note that these BER performance formulas, (C-9) and (C-13) assume that the signals have no distortion and E_b / N_0 in these formulas Are at output side of receiver.

For UWB communications, the distortion of signal is necessary to consider. The result of signal distortion is to decrease the energy per bit of signal with correlation coefficient. The energy per bit that considers distortion $E_{b,d}$ is given by:

$$E_{b,d} = CE_b \quad \text{(C-14)}$$

where C is the correlation coefficient. Therefore, E_b in (C-9) and (C-13) is substituted with CE_b for considering the distortion of signal.

From the relation between SNR at output side of receiver S_o / N_o and E_b / N_0:

$$\frac{S_o}{N_o} = \frac{B_r E_{b,d}}{f_b N_0} \quad \text{(C-15)}$$

and the relation between S / N and S_o / N_o:

$$\frac{S}{N} = \frac{S_o}{GN_o} \quad \text{(C-16)}$$

where G is the SNR gain.

Substitute the relations (C-15) and (C-16) in (C-9) and substitute E_b in (C-9) with CE_b. The BER performance of matched filter and correlation receivers for antipodal scheme in AWGN can be respectively derived as:

$$B_{m,a} = Q\left(\sqrt{\frac{2C_m G_m f_b S}{B_r N}}\right) \quad \text{(C-17)}$$

$$B_{c,a} = Q\left(\sqrt{\frac{2C_c G_c f_b S}{B_r N}}\right) \quad \text{(C-18)}$$

Substitute the relations (C-15) and (C-16) in (C-13) and substitute E_b in (C-13) with CE_b. The BER performance of matched filter and correlation receivers for orthogonal scheme in AWGN can be respectively derived as:

$$B_{m,o} = Q\left(\sqrt{\frac{C_m G_m f_b S}{B_r N}}\right) \quad \text{(C-19)}$$

$$B_{c,o} = Q\left(\sqrt{\frac{C_c G_c f_b S}{B_r N}}\right) \quad \text{(C-20)}$$