Analysis of Integrated Linear Models of a Drainage Network System for the Inner Bangkok Metropolitan Administration

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Abstract

The objective is focused on the drainage system in the area of Inner Bangkok, the capital of Thailand. Networks of water flow in the canals and are summarised to optimise the energy costs, maintenance, and installation costs. This paper formulates the drainage system of five sub-networks into mathematical models with a principal of Kirchhoff's laws. Operational research techniques are then used to find the optimal flow paths which lead to a suitable number of pumps in each station. However, some nodes or pumping stations are excluded from the models. Fluid mechanics is applied to measure the flow pressure in those nodes. The results reflected improved expenses compared with the past. Finally, the investigation on the change of flow channel characteristics such as depth of canal by silt and flow directions are summarised on the effect of the pump capacity via studies of sensitivity and the design of experiments.

Keywords: Drainage, Network, Kirchhoff, Fluid Mechanics, Linear Programming, Sensitivity Analysis

1. Introduction

Bangkok, the capital of Thailand, is located on the lower basin of the Chao Phraya river and is close to the gulf of Thailand. It therefore is a flooding area by nature, especially Inner Bangkok. The area is the passageway of water from the upper and eastern parts of the country. Due to rapid urbanisation of Bangkok, watercourses such as ponds, wells, canals, and ditches were filled up and replaced by buildings and other structures. The existing drainage network system then cannot work properly. Bangkok always has problems of flooding under the circumstances of heavy rain and a high level of tide. The study area is focused only on the inner area of the Bangkok Metropolitan Administration (BMA).

BMA governors have proposed various master plans on this area to protect from the related damages. They also carried out feasibility studies for flood protection and a drainage network system. The BMA drainage system consists of pipe networks including natural opened channels, called "Khlongs". Khlongs are mainly connected to the Chao Phraya river. Khlongs have regulators and pumps to control water level and their discharge. Each area can be connected to others by regulators and pumping stations. The direction of drainage in the network is mostly directly to the Chao Phraya river. The khlong depth is in the range of 1-2.5 meters and the width is approximately 8-12 meters. The total length of khlongs is approximately 100 kilometers. The Flood Control Center of BMA is responsible for operations of important gates and regulators. During the rainy season, regulators will be operated according to the tidal level in the Chao Phraya River while regulators in the west will be closed all the time to prevent tidal intrusion. The flooding causes can be summarised as follows. Firstly, the operated water level before raining cannot be set because the tide can propagate to the north of the inner area. Secondly, regulators cannot be closed to stop tidal effects because of heavy boat traffic. Finally, there are some problems from the management of all pumps and regulators. The downstream pumps and regulators will be operated to currently cover the provincial boundary. Flooding in the upstream with different provincial boundary can occur. Sufficient numbers of pumps and regulators at all the stations are required under the current situation. To evaluate, various improvement alternatives are then determined: such as balancing construction of structures to prevent inflow from outer areas, installing pumping stations for drainage of floods, accelerating the development and improvement of the Flood Control Center.

In this research the determination of the proper number of pumps in each station is proposed in a mathematical model. The objective is to minimise total operating costs. This model is initiated by transforming the problem into a network. Kirchhoff's laws are then used to formulate a linear programming model which is originally developed from a mine ventilation system [1]. Operational research techniques are selected to find the preferable solutions [2]. Some constrained nodes or pumping stations of the network will be excluded from the model if they have no connection with others. The mechanics of fluids are then used to find the flow pressure which needs to be pumped. In this study there are five sub-networks so that all models including their constrained nodes, which are not included in the models, have to be finally integrated to fulfill the answer of the BMA drainage system.

2. Related Algorithms

2.1 Network Model

This research focuses on an analysis of an engineering network model under a consideration of minimal cost objective function. Flow cost will be used to determine the drainage direction. The aims are to reduce flow cost and to manage the installation and pumping performance in order to achieve the lowest cost of purchasing and maintenance. The network can depict the nature of the problem. The network itself can be formulated into a linear programming (LP) model and the operational research algorithms for LP can then be used to solve problems with computing tools such as Excel Solver.

The network is a special type of model of the general linear programme [3-6]. The class of network programmes includes such problems as transportation, assignment, shortest path and maximal flow problems. It is an important model because many actual situations are depicted and recognised as networks. The representation of the network model is much more understandable than other general linear programmes. When engineering problems have been possibly modelled as a network, various algorithms exist to determine the optimal solutions.

The network model is a combination of the nodes and arcs or branches. The nodes represent the pumping stations in this drainage management problem. Arcs or branches are the directed line components of the model. Every network model has a linear programming model, which is a model with linear expressions describing the objective function and resource constraints. The branch passes from its origin node to its terminal node. Branch flow is conserved at the nodes. It is implied that the total flow entering a node must equal the total flow leaving the node. These branch flows are normally the decision variables for the network programming problems. However, there is a limitation of flow in an arc by the lower and upper bounds (sometimes called capacity). The objective function of the minimal total arc cost will select the best flow in each branch, or by means of a sufficient number of pumps.

2.2 Fluid Mechanics Calculations

Water flow in canals is categorised as open channel flow. The open channel has a surface contacted by air pressure [7, 8]. The flow of water on this open channel is affected by the surface slope. The proper solution related to the open channel seems to be difficult when compared with closed channels. Flow energy of the open channel flow is related to the kinetic energy and average velocity [9, 10, 11]. The pressure head of the pressure energy equals the depth of water surface to the bottom of the channel. Elevation energy measures the distance from the bottom to a centre line (datum) of channel. Flow velocity, flow rate and hydraulic radius can be measured from Manning's formulas [12, 13]. Pressure loss (HL) is the right angle flow force to an area. The measurement of HL depends on Newton's laws or the equalisation of action and reaction. Finally, natural pressure (HN) can be easily defined from the relationship of the density and the height of channels.

2.3 Kirchhoff's Law

An electrical network is a system of interconnected electrical elements. Closed paths in the network are called meshes while junctions of paths are called nodes. Currents and potential differences in the networks follow Kirchhoff's laws [14]. Because charge is conserved, if $\sum I$ represents the net current out of any region, the charge Q in that region satisfies the following statement. In steady state, nothing changes with time (t) and in particular dQ/dt = 0 so that $\sum I = 0$. The mentioned equation is a statement of Kirchhoff's current law. It is worth stressing that it is strictly applicable in steady state only. Kirchhoff's current law, expressed by the equation, $\sum I = 0$, is not relevant to networks comprising only single mesh. As the number of meshes in the network increases, they rapidly become more cumbersome to solve by straightforward application of Kirchhoff's laws. They play an important role to find ways of simplifying the analysis of networks.

2.4 Sensitivity Analysis

Sensitivity analysis is very useful in consideration of the changeability of problem parameters. It involves a speculation on alternative scenarios and estimating the accuracy of the typical model solution under various circumstances e.g. optimistic, pessimistic or most likely future estimates of the input parameters. This is an important procedure to check the quality of a given model, as well as a powerful tool to check the robustness and reliability of its analysis. While explaining the impact of source uncertainties and framing assumptions, guides can be included to allow the more experienced practitioners to readily access the current specific application.

3. Bma Drainage Network

BMA drainage network system will send the rainfalls to canals which are connected as a network [15, 16]. The control system is responsible for pumps located in various stations around Inner Bangkok. These would enhance the performance of water drainage including the drainage speed. According to the previous reason, operational research techniques are applied to locate the preferable positions and also specify the performance of pumps to minimise the total operating cost [17].

The drainage system can be depicted as a network. Flow channels are called branches and the connected points or pumping stations are called nodes. The numerical values on each branch show the drainage performance of each station. The installation of pumps concerns the capacity of water needed to be drained under the consideration of conservation of flow, which is the water flow out of any node, equal to the flow into that node. C_p denotes annual energy and maintenance cost. The annual cost of power and maintenance is then directly related to power (watts) of pumps. C_j is the installation and purchase cost per pump.

On the drainage problem, the pump location and drainage control equipment are determined to achieve minimal total operating cost [9, 10]. All the constraints have to satisfy Kirchhoff's conditions. Since the flow distribution in the network is known, Kirchhoff's current law conditions can be dropped. The problem for closed mesh can be originally formulated as:

Minimise $Z = \sum_{j=1}^{L} C_p Q_j HP_j + \sum_{j=1}^{L} C_j$

subject to

$$\sum_{j=1}^{b} b_{ij}(HL_j - HP_j - HN_j) = 0 \qquad ; i = 1, 2, \ldots, m$$

$$HP_j \geq 0 \qquad ; j = 1, 2, \ldots, b$$

where;

Q_j = flow rate in branch j

HL_j = pressure loss in branch j

HP_j = pump pressure in branch j

HN_j = natural pressure across branch j or atmosphere pressure

n = number of nodes in the network

m = number of chords in the network = b-n+1

b = number of branches in the network

b_{ij} = fundamental mesh matrix

L = j: branch j is allowed to have a pump

The model above is very close to an ordinary linear programming model except the objective function Z is a nonlinear function. The previous model can be converted via binary variables Y_j and letting [1]:

$C_pQ_j = a_j$

$Y_j = 1, HP_j > 0$
0, otherwise.

Formulation of BMA Drainage Network System

According to the present status and characteristics of the Inner Bangkok, it is reasonable to construct a 5 sub-network drainage system. In each sub-network, the nodes can be categorised into two classes. Nodes with classes of O and I denote nodes excluded and included in the mathematical model, respectively. Nodes and their branches included in the model will be explained later in the context of Kirchhoff's law. Sub-network C, the pump pressure for nodes 1-4, excluded from the model, can use the fluid mechanics calculation. The proposed drainage system, of I-class nodes only, consists of 5 sub-networks; A, B, C, D, and E as shown in Figures 1-5.

All sub-networks will be determined by their fundamental mesh matrices. A hypothetical drainage sub-network: C is chosen to demonstrate the applicability of algorithms. There are 4 of 17 nodes, excluded from the model. As a result, there are 13 nodes, no. 5-17, in sub-network C to form 15 branches, b_1 - b_{15}. By using Kirchhoff's flow and energy conservation laws, this sub-network can be categorised into Maxwell's loops 1-4 as shown in Figures 6-9.

On all k sub-networks, the branches of all nodes included in the model will bring the corresponding fundamental mesh matrices B, b_{ijk}, (Figures 10-14). The problems for all k sub-networks, k = A, B, C, D, and E, can then be formulated as:

$$\text{Minimise } Z = \sum_{j=1}^{L} a_{jk}HP_{jk} + \sum_{j=1}^{L} C_{jk}Y_{jk}$$

subject to

$$\sum_{j=1}^{b} b_{ijk}(HL_{jk} - HP_{jk} - HN_{jk}) = 0 \quad ; i = 1, 2, \ldots, m$$

$$HP_{jk} \le d_{jk}Y_{jk} \quad ; j \in L$$

$$HP_{jk} \ge 0 \quad ; j = 1, 2, \ldots, b$$

$$Y_{jk} = (0, 1) \quad ; j \in L$$

L = j: branch j is allowed to have a pump

B = fundamental mesh matrix, $[b_{ijk}]$;

a_{jk} = energy and maintenance cost in branch j and network k

b_{ijk} = 1 if branch j is contained in mesh i and has the same direction

b_{ijk} = -1 if branch j is contained in mesh i and has the opposite direction

b_{ijk} = 0 if branch j is not contained in mesh i

d_{jk} = upper boundary of pump (HP_{jk})

The model above is called a mixed integer programming model [2]. If HP_{jk} >0 then Y_{jk} must be 1, while if HP_{jk} = 0 then Y_{jk} must be 0 since Y_{jk} =1 produces the highest level of Z. The natural pressure across branch j and sub-network k or atmosphere pressure (HN_{jk}) and pressure loss in branch j and sub-network k (HL_{jk}) are summarised. However, there are some stations or nodes having no connected arcs. The HN_{kl} and HL_{kl} for these l pumping stations are excluded from the mathematical models.

4. Results and Sensitivity Analysis

In the study of the drainage network of the Inner BMA there are five sub-networks with a consideration of Kirchhoff's law. The complexity of the models is reduced via Mesh or Maxwell's loops. All closed loops are independent. The formulation of a linear programming problem has been used to find the decision variables of the network. Under the conditions of the objective function and constraints, the model solutions will be determined via Excel Premium Solver.

From the experimental results, alternatives of optimal pressure for each branch can be transformed into motor power of pumps. Submersible motor pumps affect annual energy, maintenance and installation costs. The model can simultaneously determine the proper locations including minimal operating cost. The experimental summarisation will be finally collected and adjusted to enhance the efficiency of the BMA drainage network system.

The experimental data shows the motor power required by pumping stations, on both classes I and O, of the sub-networks. The total operating cost can be measured and compared with the data before and after an application of the mathematical model, e.g. for sub-network C in Table 1. The total operating cost consists of annual energy and maintenance costs (C_p) including an installation cost (C_j). The power of a submersible motor pump can be transformed into the number required in each pumping station. The standard specification of pumps can be defined differently from manufacturers. Model number designations are used to meet the requirements of the BMA governor. These are the depth of head, pumping rate and motor power. The number of pumps and the recommended models in each station for the sub-network C are summarised in Table 2. Overall expenses for pumps for five sub-networks before and after the application of the models are approximately 148,000,000 and 81,000,000 Bahts (Thai currency), respectively.

The procedures to determine the sensitivity of the outcomes of an alternative to changes in the parameters of this drainage problem can be summarised as follows. If a small change in a parameter results in relatively large changes in the outcomes, the outcomes would be said to be sensitive to that problem parameter. This may mean that the parameter has to be determined very accurately in further applications. A Design of Experiment (DOE) is applied to study the sensitivity of the drainage problem. It is a structured, organised method for determining the relationship between factors affecting a process and the output of that process. In this case a general factorial design is used to study the effects of problem parameters. For the drainage problem there are 3 parameters. They consist of the flow rate, height and channel slope.

From the experiments, the proper level of motor powers can be determined via Kirchhoff's voltage law. The objective is to minimise total operating cost. However, the physical parameters might affect the optimal solution. It could only be applied under a current circumstance. If the parameters have to be changed from the nature of open flow channel, for example, a decrease in the depth of channels and a change of flow direction, the forecasting of the number of pumps and their locations might be sensitive. The study of these conditions is then used in the drainage network problem. The statistical hypotheses are set to determine the effects on these problem parameters. They consist of a null hypothesis; there is no effect of the problem parameter (τ_i) on the response (flow rate) or $\tau_i = 0$ and an alternative hypothesis of $\tau_i \neq 0$. The results of the sensitivity of problem parameters are then concluded, including an analysis of variance (ANOVA).

From ANOVA (P-value < 0.005), without a consideration of higher order effects, there are significant parameters, the depth and width of open channels, to the water flow rate. If these levels of problem parameters are changed, the optimal solutions of the mathematical model of I-class sub-networks, are then recalculated. The number of pumps would be changed under these circumstances. If the optimal solution has to be fixed, a flooding problem can occur as a result.

5. Conclusions and discussions

This study aims to reduce the pressure of urban drainage and efficiently combine flood control with the utilisation of pumping resources. When the study is applied, it will increase the capacity of drainage systems and protect people's lives and property of the capital, Bangkok. It can improve the city environment, with the preparation of alternative pumps, when the conditions change. Modeling of drainage systems in the study area of Inner Bangkok was conducted using an adapted model of mine ventilation. The results of the network analysis with nodes included in the model, or I-class, with a combination of nodes excluded, O-class, show the proper numbers of pumps in each station on all sub-networks. Moreover, further investigation was carried out to measure various parameters affecting the yield of the mathematical model. It is necessary for further improvement to introduce sensitivity analysis so that significant parameters of the model are predetermined to find the increment of pumping capacity, or to acquire adequate storage of pumps, in order to cope with future circumstances.

6. Acknowledgments

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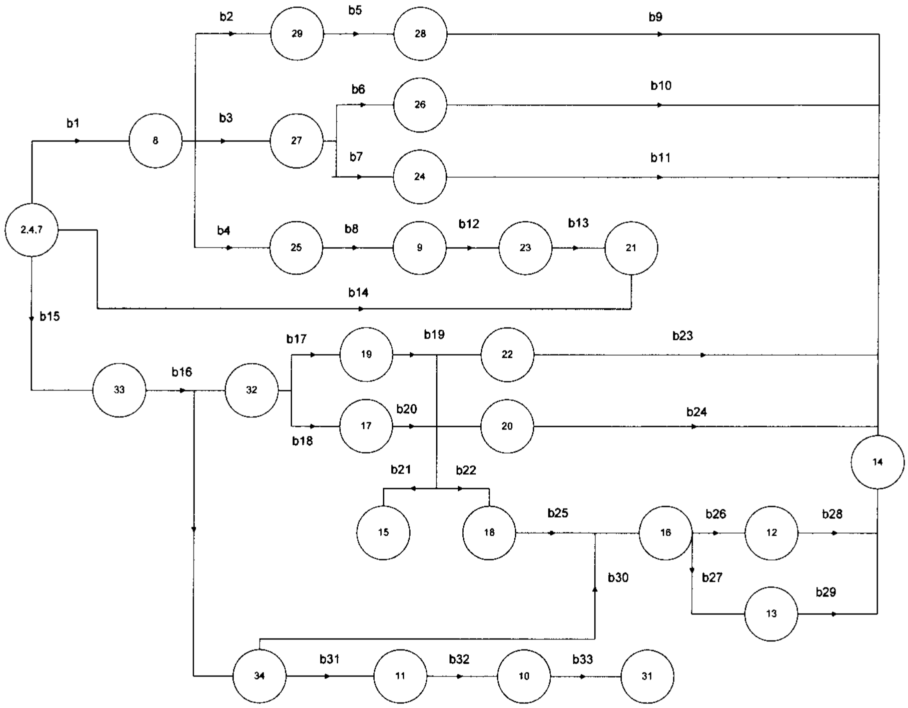

Figure 1 Sub-network: A

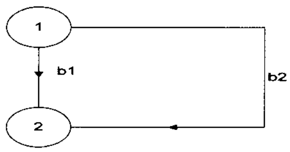

Figure 2 Sub-network: B

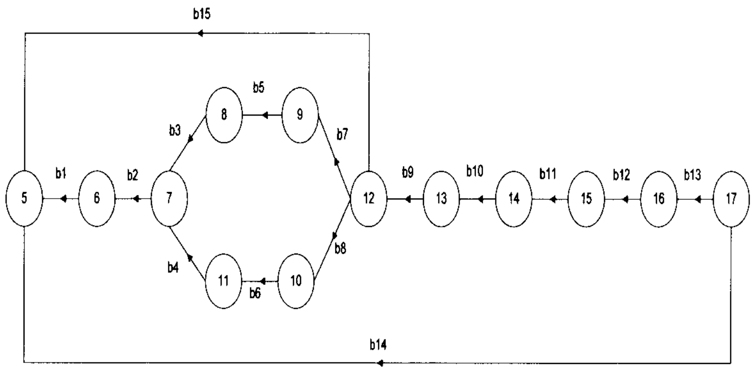

Figure 3 Sub-network: C

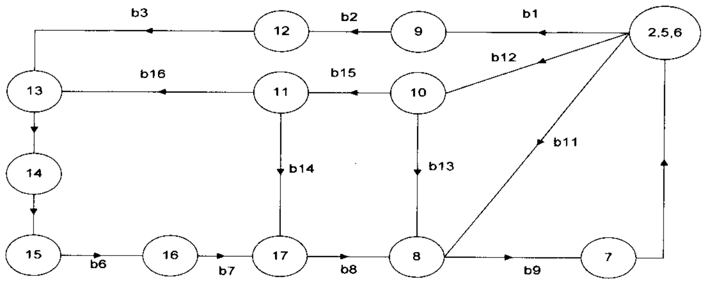

Figure 4 Sub-network: D

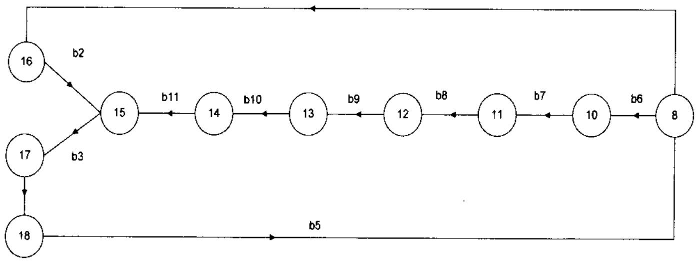

Figure 5 Sub-network: E

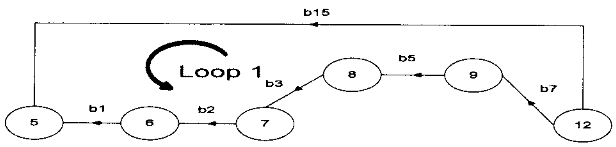

Figure 6 Kirchhoff's flow for sub-network: C, Loop 1

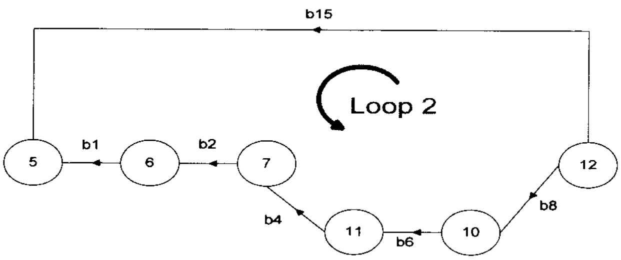

Figure 7 Kirchhoff's flow for sub-network: C, Loop 2

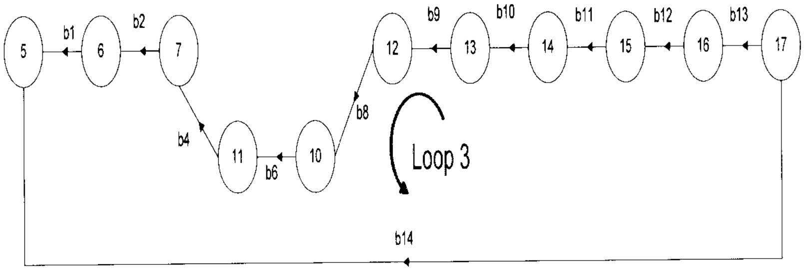

Figure 8 Kirchhoff's flow for sub-network: C, Loop 3

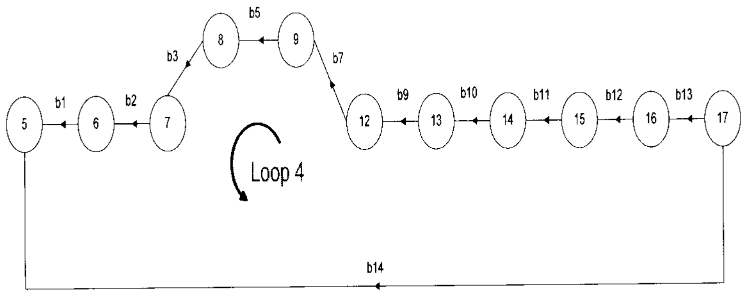

Figure 9 Kirchhoff's flow for sub-network: C, Loop 4

	-1	0	0	-1	0	0	0	-1	0	0	0	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	-1	-1	0	0	-1	0	0	0	-1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	1	0	1	0	-1	0	0	0
	-1	-1	0	0	-1	0	0	0	-1	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	-1	0	-1	-1	0	0	0
	-1	-1	0	0	-1	0	0	0	-1	0	0	0	0	0	1	1	1	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0
	-1	0	0	0	0	-1	0	0	0	-1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	1	0	1	0	-1	0	0	0
	-1	-1	0	0	-1	0	0	0	-1	0	0	0	0	0	1	1	0	1	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0
	-1	-1	0	0	-1	0	0	0	-1	0	0	0	0	0	1	1	0	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0
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	-1	-1	0	0	-1	0	0	0	-1	0	0	0	0	0	1	1	0	1	1	1	0	1	0	0	1	1	0	1	0	0	0	0	0
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	-1	-1	0	0	-1	0	0	0	-1	0	0	0	0	0	1	1	0	1	0	1	0	1	0	0	1	1	0	1	0	0	0	0	0
	-1	-1	0	0	-1	0	0	0	-1	0	0	0	0	0	1	1	0	1	0	1	0	1	0	0	1	0	1	0	1	0	0	0	0
	-1	0	-1	0	-1	0	0	0	-1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	0	1	-1	0	0	0
[B] =	-1	0	-1	0	-1	0	0	0	-1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	1	0	1	0	-1	0	0	0
	-1	0	-1	0	-1	0	0	0	-1	0	0	0	0	0	1	1	-1	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0
	-1	0	-1	0	-1	0	0	0	-1	0	0	0	0	0	1	1	0	-1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0
	-1	0	-1	0	-1	0	0	0	0	-1	0	0	0	0	1	1	0	0	-1	1	0	0	0	1	0	0	0	0	0	0	0	0	0
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	-1	0	-1	0	0	0	-1	0	0	0	-1	0	0	0	1	1	-1	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0
	-1	0	-1	0	0	0	-1	0	0	0	-1	0	0	0	1	1	0	-1	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0
	-1	0	-1	0	0	0	-1	0	0	0	-1	0	0	0	1	1	-1	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0
	-1	0	-1	0	0	0	-1	0	0	0	-1	0	0	0	1	1	0	-1	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0
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	-1	0	-1	0	0	-1	0	0	0	-1	0	0	0	0	1	1	0	-1	0	1	0	1	0	0	1	1	0	1	0	0	0	0	0
	-1	0	-1	0	0	-1	0	0	0	-1	0	0	0	0	1	1	-1	0	1	0	0	1	0	0	1	0	1	0	1	0	0	0	0
	-1	0	-1	0	0	-1	0	0	0	-1	0	0	0	0	1	1	0	-1	0	1	0	1	0	0	1	0	1	0	1	0	0	0	0
	-1	0	-1	0	0	-1	0	0	0	-1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	0	1	-1	0	0	0

Figure 10 Fundamental mesh matrix for sub-network: A (k = A)

$$[B] = \begin{vmatrix} 1 & -1 \end{vmatrix}$$

Figure 11 Fundamental mesh matrix for sub-network: B (*k* = B)

$$[B] = \begin{vmatrix} -1 & -1 & -1 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & -1 & 0 & -1 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & -1 & 0 \end{vmatrix}$$

Figure 12 Fundamental mesh matrix for sub-network: C (*k* = C)

$$[B] = \begin{vmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & -1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & -1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

Figure 13 Fundamental mesh matrix for sub-network: D (*k* = D)

$$[B] = \begin{vmatrix} 1 & 1 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

Figure 14 Fundamental mesh matrix for sub-network: E (*k* = E)

Table 1 A Comparison of motor power of pumps for sub-network C.

Node	Class	Before		After	
		Motor Power of Pump (Kilowatts)	Total Cost (Baht)	Motor Power of Pump (Kilowatts)	Total Cost (Baht)
1	O	300	3200000	420.97	4470185
2	O	150	1625000	84.79	940295
3	O	55	627500	45.47	527435
4	O	132	1436000	64.89	731345
5	I	300	3200000	0	0
6	I	150	1625000	0	0
7	I	264	2822000	0	0
8	I	150	1625000	0	0
9	I	60	630000	32	386000
10	I	110	1155000	0	0
11	I	110	1155000	47	543500
12	I	132	1386000	195	2097500
13	I	22	231000	0	0
14	I	60	630000	0	0
15	I	132	1386000	0	0
16	I	22	231000	0	0
17	I	30	315000	67	753500
			23279500		10449760

Table 2 Number of submersible motor pumps for sub-network C

No.	Name	Specifications			Standard		Model
		Head (Meter)	Pumping Rate (Cub. Meter/Sec)	Motor Power (Kilowatts)	Kilo Watt	Number of Pumps	
1	C1	2.30	19.37	420.97	30.00	14	400 JSS 30 12T
2	C2	0.80	1.60	84.79	22.00	4	350 JSS 22 8T
3	C3	0.70	1.04	45.47	22.00	2	350 JSS 22 8T
4	C4	0.60	1.13	64.89	22.00	3	350 JSS 22 8T
5	C5	1.50	5.42	32.00	22.00	1	350 JSS 22 8T
6	C6	0.50	1.63	47.00	22.00	2	350 JSS 22 8T
7	C7	1.00	2.00	195.00	22.00	9	350 JSS 22 8T
8	C8	0.50	0.54	67.00	22.00	3	350 JSS 22 8T