A Heuristic for Solving a Stochastic Knapsack Problem with Discrete Random Capacity

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Abstract

In this paper, a set of n items with a_j and c_j as the weight coefficient and cost coefficient of item j, $j = 1,2,\ldots,n$, must be decided, to allocate within a discrete random capacity b_i, in order to minimize the total cost plus the cost of not meeting the capacity, with g and h the per unit penalty costs. The problem can be formulated as the stochastic knapsack problem with discrete random capacity (SKPDRC). A heuristic for solving SKPDRC is proposed and computationally experimented.

Keywords: stochastic knapsack problem, discrete random capacity

1. Introduction

Consider a class of stochastic knapsack problem with discrete random capacity (SKPDRC) as follows:

(1) Minimize $f = \sum_{j=1}^{n} c_j x_j + \sum_{i=1}^{m} (p_i g u_i + p_i h v_i)$

Subject to $\sum_{j=1}^{n} a_j x_j + u_i - v_i = b_i$

$0 \le x_j \le t_j$ and integer

$\sum_{i=1}^{m} p_i = 1$

where x_j is a decision variable, for $j = 1,2,\ldots,n$,

t_j is an upper bound of x_j, for $j = 1,2,\ldots,n$,

u_i is the slack variable and $u_i \ge 0$, for $i = 1,2,\ldots,m$,

v_i is the surplus variable and $v_i \ge 0$, for $i = 1,2,\ldots,m$,

a_j is the weight coefficient of item j and $a_j \ge 0$, for $j = 1,2,\ldots,n$,

c_j is the cost coefficient of item j and $c_j \ge 0$, for $j = 1,2,\ldots,n$,

b_i is the capacity of alternative i, $b_i \ge 0$ for $i = 1,2,\ldots,m$,

p_i is the probability of having capacity b_i, $p_i \ge 0$ for $i = 1,2,\ldots,m$,

g is the per unit cost of having u_i, $g \ge 0$,

h is the per unit cost of having v_i, $h \ge 0$.

Assume that $b_i \le b_{i+1}$, for $i = 1,2,\ldots,m-1$,

and $\frac{c_j}{a_j} \le \frac{c_{j+1}}{a_{j+1}}$, for $j = 1,2,\ldots,n-1$.

The basic problem statement of the above model (1) can be described as follows. Given c_j, a_j, t_j, for $j = 1,2,\ldots,n$, p_i, b_i, for $i = 1,2,\ldots,m$, g and h, the decision problem is to search for x_j, for $j = 1,2,\ldots,n$, and u_i, v_i, for

$i = 1, 2, ..., m$, so that all constraints are satisfied in order to minimize the objective function.

2. Literature Reviews

Dantzig [2] founded the general concept of linear programming (LP) including introducing LP with uncertain parameters referred to as stochastic linear programming (SLP). SLP was described by Birge and Louveaux [1], and Kall and Wallace [4].

In many realistic situations, the objective coefficients c_j are not known with certainty. This case mostly arises in resource allocation problems, capital budgeting problems, and project selection problems. Parks and Steinberg [7] and Henig [3] described dynamic programming to solve this type of stochastic knapsack problem. Moreover, the weight coefficients a_j may not be known with certainty. Pisinger [8] presented a knapsack problem with stochastic weight. Furthermore, Kleywegt et al. [6] and Kleywegt and Papastavrou [5] studied a knapsack problem when both objective coefficients c_j and weight coefficients a_j are stochastic. However, studying uncertainty in a capacity b_i is also important. For example, in a capital budgeting problem, we may be faced with the problem that there is a probability of how much budget we will receive. In a cutting stock problem, it may have a variation in standard length. In a cargo loading problem, when the truck has many customers to load the items. And we do not know how many items that the previous customers loaded into that truck. Hence, the remaining capacity of the truck is uncertain. In a carrying problem, there are many persons that can carry the different weights. And we do not know exactly which one will carry the items at that time.

There are two types of random capacity: discrete random capacity and continuous random capacity. The studied problem is a class of stochastic knapsack problem with discrete random capacity.

3. Methodology

First, we relaxed the integer constraint of SKPDRC and solve the relaxed problem (SKPDRCR). Then we proposed a heuristic for solving SKPDRC by using the optimal solution of SKPDRCR as an initial solution.

SKPDRCR is shown as follows.

(2) Minimize $f = \sum_{j=1}^{n} c_j x_j + \sum_{i=1}^{m} (p_i g u_i + p_i h v_i)$

Subject to $\sum_{j=1}^{n} a_j x_j + u_i - v_i = b_i$

$0 \le x_j \le t_j$

$\sum_{i=1}^{m} p_i = 1$

where

x_j is a decision variable, for $j = 1, 2, ..., n$,

t_j is an upper bound of x_j, for $j = 1, 2, ..., n$,

u_i is the slack variable and $u_i \ge 0$, for $i = 1, 2, ..., m$,

v_i is the surplus variable and $v_i \ge 0$, for $i = 1, 2, ..., m$,

a_j is the weight coefficient of item j and $a_j \ge 0$, for $j = 1,2,\ldots,n$,

c_j is the cost coefficient of item j and $c_j \ge 0$, for $j = 1,2,\ldots,n$,

b_i is the capacity of alternative i, $b_i \ge 0$ for $i = 1,2,\ldots,m$,

p_i is the probability of having capacity b_i, $p_i \ge 0$ for $i = 1,2,\ldots,m$,

g is the per unit cost of having u_i, $g \ge 0$,

h is the per unit cost of having v_i. $h \ge 0$.

Assume that $b_i \le b_{i+1}$, for $i = 1, 2, ..., m-1$,

and $\frac{c_j}{a_j} \le \frac{c_{j+1}}{a_{j+1}}$, for $j = 1, 2, ..., n-1$.

The following theorem proves optimality of the results obtained by the proposed algorithm for SKPDRCR.

There exist four cases that are the following:

Case I: the optimal solution is as follows.

$x_j = 0, \forall j$

$v_i = 0, \forall i$

$u_i = b_i, \forall i$

where $\frac{c_1}{a_1} \ge g$

Proof

$$u_i = b_i - \sum_{j=1}^{n} a_j x_j + v_i \text{ , } i = 1,2,...,m$$

Next is to substitute basic variables into the objective function as follows.

$$f = \sum_{j=1}^{n} c_j x_j + g \sum_{i=1}^{m} p_i (b_i - \sum_{j=1}^{n} a_j x_j + v_i)$$

$$+ h \sum_{i=1}^{m} p_i v_i$$

$$= g \sum_{i=1}^{m} p_i b_i + \sum_{i=1}^{m} (g+h) p_i v_i$$

$$+ \sum_{j=1}^{n} (c_j - g a_j \sum_{i=1}^{m} p_i) x_j$$

$$= g \sum_{i=1}^{m} p_i b_i + \sum_{i=1}^{m} (g+h) p_i v_i$$

$$+ \sum_{j=1}^{n} (c_j - g a_j) x_j$$

The minimum value of f can be found when all reduced costs of nonbasic variables are greater than or equal to zero. In this case, nonbasic variables are x_j for all j and v_i for all i. In order to obtain the minimum value of f, the reduced costs of these nonbasic variables must be greater than or equal to zero as follows.

1. $(g+h) p_i \geq 0, \forall i$
2. $(c_j - g a_j) \geq 0, \forall j$

Since $g, h \geq 0$ and $p_i \geq 0$ for all i, $(g+h) p_i \geq 0$, for all i. To minimize f, $(c_j - g a_j) \geq 0$, for all j must be greater than or equal to zero, that is equivalent to $\frac{c_j}{a_j} \geq g$ for all j. According to the assumption $\frac{c_j}{a_j} \leq \frac{c_{j+1}}{a_{j+1}}$, for $j = 1,2,...,n-1$, the condition for this case is $\frac{c_1}{a_1} \geq g$. In this case, the minimum value of f is $g \sum_{i=1}^{m} p_i b_i$.

Case II: the optimal solution is as follows.

$$x_j = t_j \text{ , } \forall j$$

$$v_i = \sum_{j=1}^{n} a_j t_j - b_i \text{ and } u_i = 0, \quad i \in B_2$$

$$u_i = b_i - \sum_{j=1}^{n} a_j t_j \text{ and } v_i = 0, \quad i \in B_1$$

where $B_1 = \{i = 1:m : b_i \geq \sum_{j=1}^{n} a_j t_j\}$,

$$B_2 = \{i = 1:m : b_i < \sum_{j=1}^{n} a_j t_j\}$$

$$\frac{c_n}{a_n} \leq g \sum_{i \in B_1} p_i - h \sum_{i \in B_2} p_i$$

Proof

$$x_j = t_j - r_j \text{ , } \forall j$$

$$r_j \geq 0, \forall j$$

$$u_i = b_i - \sum_{j=1}^{n} a_j t_j + \sum_{j=1}^{n} a_j r_j + v_i \text{ , } i \in B_1$$

$$v_i = -b_i + \sum_{j=1}^{n} a_j t_j - \sum_{j=1}^{n} a_j r_j + u_i \text{ , } i \in B_2$$

Next is to substitute basic variables into the objective function as follows.

$$f = \sum_{j=1}^{n} c_j (t_j - r_j) + g \sum_{i \in B_1}^{m} p_i (b_i - \sum_{j=1}^{n} a_j t_j + \sum_{j=1}^{n} a_j r_j + v_i)$$

$$+ h \sum_{i \in B_1}^{m} p_i v_i + h \sum_{i \in B_2}^{m} p_i (-b_i + \sum_{j=1}^{n} a_j t_j - \sum_{j=1}^{n} a_j r_j + u_i)$$

$$+ g \sum_{i \in B_2}^{m} p_i u_i$$

$$= \sum_{j=1}^{n} c_j t_j + g \sum_{i \in B_1}^{m} p_i (b_i - \sum_{j=1}^{n} a_j t_j)$$

$$+ h \sum_{i \in B_2}^{m} p_i (-b_i + \sum_{j=1}^{n} a_j t_j)$$

$$+ \sum_{i \in B_1}^{m} (g+h) p_i v_i + \sum_{i \in B_2}^{m} (g+h) p_i u_i$$

$$+ \sum_{j=1}^{n} (-c_j + g a_j \sum_{i \in B_1}^{m} p_i - h a_j \sum_{i \in B_2}^{m} p_i) r_j$$

In this case, nonbasic variables are v_i for $i \in B_1$, u_i for $i \in B_2$ and r_j for all j. In order to obtain the minimum value of f, the reduced costs of these nonbasic variables must be greater than or equal to zero as follows.

1. $(g+h)p_i \geq 0,\ \forall i$
2. $(-c_j + ga_j\sum_{i\in B_1}^{m} p_i - ha_j\sum_{i\in B_2}^{m} p_i) \geq 0,\ \forall j$

Since $g,h \geq 0$ and $p_i \geq 0$ for all i, condition 1 has been met. To minimize f, $(-c_j + ga_j\sum_{i\in B_1}^{m} p_i - ha_j\sum_{i\in B_2}^{m} p_i)$ for all j must be greater than or equal to zero, that is equivalent to $\frac{c_j}{a_j} \leq g\sum_{i\in B_1}^{m} p_i - h\sum_{i\in B_2}^{m} p_i$ for all j. According to the assumption $\frac{c_j}{a_j} \leq \frac{c_{j+1}}{a_{j+1}}$, for $j = 1,2,\ldots,n-1$, the condition for this case is $\frac{c_n}{a_n} \leq g\sum_{i\in B_1}^{m} p_i - h\sum_{i\in B_2}^{m} p_i$.
In this case, the minimum value of f is $\sum_{j=1}^{n} c_j t_j + g\sum_{i\in B_1}^{m} p_i(b_i - \sum_{j=1}^{n} a_j t_j) + h\sum_{i\in B_2}^{m} p_i(-b_i + \sum_{j=1}^{n} a_j t_j)$.

Case III: the optimal solution is as follows.

$$x_j = t_j,\ j = 1,\ldots,k-1$$
$$x_k = \frac{b_q - \sum_{j=1}^{k-1} a_j t_j}{a_k}$$
$$x_j = 0,\ j = k+1,\ldots,n$$
$$v_i = b_q - b_i \text{ and } u_i = 0,\quad i = 1,2,\ldots,q-1$$
$$u_q = 0 \text{ and } v_q = 0$$
$$u_i = b_i - b_q \text{ and } v_i = 0,\quad i = q+1,q+2,\ldots,m$$

where

$q \in \{1,\ldots,m\}$ such that:

$$(-\frac{c_k}{a_k} + g\sum_{i=q}^{m} p_i - h\sum_{i=1}^{q-1} p_i) \geq 0 \text{ and}$$
$$(\frac{c_k}{a_k} + h\sum_{i=1}^{q} p_i - g\sum_{i=q+1}^{m} p_i) \geq 0$$

$k \in \{1,\ldots,n\}$ such that $\sum_{j=1}^{k-1} a_j t_j \leq b_q$ and $\sum_{j=1}^{k} a_j t_j \geq b_q$. If $\sum_{j=1}^{n} a_j t_j < b_q$, then $k = n$.

Proof

Suppose k and q have been specified:

$$f(q) = \sum_{j=1}^{k-1} c_j x_j + c_k x_k + \sum_{j=k+1}^{n} c_j x_j + g\sum_{i=1}^{q-1} p_i u_i + h\sum_{i=1}^{q-1} p_i v_i + g\sum_{i=q+1}^{m} p_i u_i + h\sum_{i=q+1}^{m} p_i v_i + gp_q u_q + hp_q v_q \qquad (1)$$

Let,

$$x_j = t_j,\ j = 1,\ldots,k-1$$
$$x_k = \frac{(b_q - u_q + v_q - \sum_{j=1}^{k-1} a_j x_j - \sum_{j=k+1}^{n} a_j x_j)}{a_k} = \frac{(b_q - u_q + v_q - \sum_{j=1}^{k-1} a_j t_j - \sum_{j=k+1}^{n} a_j x_j)}{a_k}$$
$$v_i = \sum_{j=1}^{k-1} a_j x_j + a_k x_k + \sum_{j=k+1}^{n} a_j x_j + u_i - b_i$$
$$= \sum_{j=1}^{k-1} a_j x_j + a_k\frac{(b_q - u_q + v_q - \sum_{j=1}^{k-1} a_j x_j - \sum_{j=k+1}^{n} a_j x_j)}{a_k} + \sum_{j=k+1}^{n} a_j x_j + u_i - b_i$$
$$= b_q - b_i + u_i - u_q + v_q,\quad i = 1,2,\ldots,q-1$$
$$u_i = b_i + v_i - \sum_{j=1}^{k-1} a_j x_j - a_k x_k - \sum_{j=k+1}^{n} a_j x_j$$

$$= b_i + v_i - \sum_{j=1}^{k-1} a_j x_j$$

$$- a_k \frac{(b_q - u_q + v_q - \sum_{j=1}^{k-1} a_j x_j - \sum_{j=k+1}^{n} a_j x_j)}{a_k}$$

$$- \sum_{j=k+1}^{n} a_j x_j$$

$$= b_i - b_q + v_i + u_q - v_q,$$
$$i = q+1, q+2, \ldots, m$$

The next part is to substitute basic variables into equation 1. Thus equation 1 becomes:

$$f(q) = \sum_{j=1}^{k-1} c_j t_j$$

$$+ c_k \frac{(b_q - u_q + v_q - \sum_{j=1}^{k-1} a_j t_j - \sum_{j=k+1}^{n} a_j x_j)}{a_k}$$

$$+ \sum_{j=k+1}^{n} c_j x_j + g \sum_{i=1}^{q-1} p_i u_i$$

$$+ h \sum_{i=1}^{q-1} p_i (b_q - b_i + u_i - u_q + v_q)$$

$$+ g \sum_{i=q+1}^{m} p_i (b_i - b_q + v_i + u_q - v_q)$$

$$+ h \sum_{i=q+1}^{m} p_i v_i + g p_q u_q + h p_q v_q$$

$$= \sum_{j=1}^{k-1} c_j t_j + c_k \frac{(b_q - \sum_{j=1}^{k-1} a_j t_j)}{a_k}$$

$$+ h \sum_{i=1}^{q-1} p_i (b_q - b_i) + g \sum_{i=q+1}^{m} p_i (b_i - b_q)$$

$$+ \sum_{j=k+1}^{n} (c_j - \frac{c_k a_j}{a_k}) x_j + \sum_{i=1}^{q-1} (g+h) p_i u_i$$

$$+ \sum_{i=q+1}^{m} (g+h) p_i v_i + (-\frac{c_k}{a_k} + g \sum_{i=q}^{m} p_i - h \sum_{i=1}^{q-1} p_i) u_q$$

$$+ (\frac{c_k}{a_k} + h \sum_{i=1}^{q} p_i - g \sum_{i=q+1}^{m} p_i) v_q$$

In this case, nonbasic variables are x_j for $j = k+1, k+2, \ldots, n$, u_i for $i = 1, 2, \ldots, q-1$, v_i for $i = q+1, q+2, \ldots m$, u_q and v_q. In order to obtain the minimum value of f, the reduced costs of these nonbasic variables must be greater than or equal to zero as follows.

1. $c_j - \frac{c_k a_j}{a_k} \geq 0, \; j = k+1, k+2, \ldots, n$
2. $(g+h) p_i \geq 0, \; \forall i$
3. $(-\frac{c_k}{a_k} + g \sum_{i=q}^{m} p_i - h \sum_{i=1}^{q-1} p_i) \geq 0$
4. $(\frac{c_k}{a_k} + h \sum_{i=1}^{q} p_i - g \sum_{i=q+1}^{m} p_i) \geq 0$

From assumption, $\frac{c_j}{a_j} \leq \frac{c_{j+1}}{a_{j+1}}$, for $j = 1, 2, \ldots, n-1$ then $c_j - \frac{c_k a_j}{a_k} \geq 0$ for $j = k+1, k+2, \ldots, n$. Since $g, h \geq 0$ and $p_i \geq 0$ for all i, condition 2 has been met.

To minimize f, $(-\frac{c_k}{a_k} + g \sum_{i=q}^{m} p_i - h \sum_{i=1}^{q-1} p_i)$ and $(\frac{c_k}{a_k} + h \sum_{i=1}^{q} p_i - g \sum_{i=q+1}^{m} p_i)$ must be greater than or equal to zero. Therefore, we select q such that :

$$(-\frac{c_k}{a_k} + g \sum_{i=q}^{m} p_i - h \sum_{i=1}^{q-1} p_i) \geq 0 \text{ and}$$
$$(\frac{c_k}{a_k} + h \sum_{i=1}^{q} p_i - g \sum_{i=q+1}^{m} p_i) \geq 0.$$

We also select k. If $\sum_{j=1}^{n} a_j t_j < b_q$, then: $k = n$. Otherwise, select k from $\sum_{j=1}^{k-1} a_j t_j \leq b_q$ and $\sum_{j=1}^{k} a_j t_j \geq b_q$. In this case, the minimum value of f

is $\sum_{j=1}^{k-1} c_j t_j + c_k \dfrac{(b_q - \sum_{j=1}^{k-1} a_j t_j)}{a_k} + h\sum_{i=1}^{q-1} p_i (b_q - b_i)$

$+g \sum_{i=q+1}^{m} p_i (b_i - b_q)$. Thus, the theorem is proven.

Case IV: the optimal solution is as follows.

$$x_j = t_j,\ \ j = 1,\ldots,k$$
$$x_j = 0,\ \ j = k+1,\ldots,n$$
$$v_i = \sum_{j=1}^{k} a_j t_j - b_i \text{ and } u_i = 0,$$

$i = 1,2,\ldots,q$

$$u_i = b_i - \sum_{j=1}^{k} a_j t_j \text{ and } v_i = 0,$$
$$i = q+1, q+2,\ldots,m$$

where

$q \in \{1,\ldots,m\}$ such that

$$\left(\frac{c_{k+1}}{a_{k+1}} + h\sum_{i=1}^{q} p_i - g\sum_{i=q+1}^{m} p_i\right) \geq 0$$

$k \in \{1,\ldots,n\}$ such that $\sum_{j=1}^{k} a_j t_j > b_q$ and

$\sum_{j=1}^{k} a_j t_j < b_{q+1}$.

Proof

Suppose k and q have been specified:

$$f(q) = \sum_{j=1}^{k} c_j x_j + \sum_{j=k+1}^{n} c_j x_j + g\sum_{i=1}^{q} p_i u_i + h\sum_{i=1}^{q} p_i v_i + g\sum_{i=q+1}^{m} p_i u_i + h\sum_{i=q+1}^{m} p_i v_i \tag{2}$$

Let,

$$x_j = t_j,\ \ j = 1,\ldots,k$$
$$v_i = \sum_{j=1}^{k} a_j x_j + \sum_{j=k+1}^{n} a_j x_j + u_i - b_i$$
$$= \sum_{j=1}^{k} a_j t_j + \sum_{j=k+1}^{n} a_j x_j + u_i - b_i,$$
$$i = 1,2,\ldots,q$$
$$u_i = b_i + v_i - \sum_{j=1}^{k} a_j x_j - \sum_{j=k+1}^{n} a_j x_j$$
$$= b_i + v_i - \sum_{j=1}^{k} a_j t_j - \sum_{j=k+1}^{n} a_j x_j,$$
$$i = q+1, q+2,\ldots,m$$

The next part is to substitute basic variables into equation 2. Thus equation 2 becomes:

$$\begin{aligned} f(q) &= \sum_{j=1}^{k} c_j t_j + \sum_{j=k+1}^{n} c_j x_j + g\sum_{i=1}^{q} p_i u_i \\ &+ h\sum_{i=1}^{q} p_i \Big(\sum_{j=1}^{k} a_j t_j - b_i + u_i + \sum_{j=k+1}^{n} a_j x_j\Big) \\ &+ g\sum_{i=q+1}^{m} p_i \Big(b_i - \sum_{j=1}^{k} a_j t_j + v_i - \sum_{j=k+1}^{n} a_j x_j\Big) \\ &+ h\sum_{i=q+1}^{m} p_i v_i \\ &= \sum_{j=1}^{k} c_j t_j + h\sum_{i=1}^{q} p_i \Big(\sum_{j=1}^{k} a_j t_j - b_i\Big) \\ &+ g\sum_{i=q+1}^{m} p_i \Big(b_i - \sum_{j=1}^{k} a_j t_j\Big) + \sum_{i=1}^{q} (g+h) p_i u_i \\ &+ \sum_{i=q+1}^{m} (g+h) p_i v_i \\ &+ \sum_{j=k+1}^{n} \Big(c_j + h a_j \sum_{i=1}^{q} p_i - g a_j \sum_{i=q+1}^{m} p_i\Big) x_j \end{aligned}$$

In this case, nonbasic variables are x_j for $j = k+1, k+2,\ldots,n$, u_i for $i = 1,2,\ldots,q$, v_i for $i = q+1, q+2, \ldots m$. In order to obtain the minimum value of *f*, the reduced costs of these nonbasic variables must be greater than or equal to zero as follows.

1. $(g+h) p_i \geq 0,\ \forall i$
2. $(c_j + h a_j \sum_{i=1}^{q} p_i - g a_j \sum_{i=q+1}^{m} p_i) \geq 0,$

   $j = k+1, k+2,\ldots,n$

Since $g,h \geq 0$ and $p_i \geq 0$ for all i, condition 1 has been met. To minimize f, $(c_j + ha_j\sum_{i=1}^{q} p_i - ga_j\sum_{i=q+1}^{m} p_i)$ for $j = k+1, k+2, ..., n$ must be greater than or equal to zero that is equivalent to $(\frac{c_j}{a_j} + h\sum_{i=1}^{q} p_i - g\sum_{i=q+1}^{m} p_i) \geq 0$ for $j = k+1, k+2, ..., n$. According to the assumption $\frac{c_j}{a_j} \leq \frac{c_{j+1}}{a_{j+1}}$, for $j = 1,2,...,n-1$, the condition for this case is $(\frac{c_{k+1}}{a_{k+1}} + h\sum_{i=1}^{q} p_i - g\sum_{i=q+1}^{m} p_i) \geq 0$.

Therefore, we select k and q such that : $(\frac{c_{k+1}}{a_{k+1}} + h\sum_{i=1}^{q} p_i - g\sum_{i=q+1}^{m} p_i) \geq 0$, $\sum_{j=1}^{k} a_j t_j > b_q$ and $\sum_{j=1}^{k} a_j t_j < b_{q+1}$. In this case, the minimum value of f is $\sum_{j=1}^{k} c_j t_j + h\sum_{i=1}^{q} p_i(\sum_{j=1}^{k} a_j t_j - b_i)$ $+g\sum_{i=q+1}^{m} p_i(b_i - \sum_{j=1}^{k} a_j t_j)$. Thus, the theorem is proven.

At this point, the optimal solution for SKPDRCR is reached. Next is to propose a heuristic for solving SKPDRC.

The flow chart of a heuristic for SKPDRC is shown in Figure 1.

In this study, the proposed method is developed for SKPDRC and written in MATLAB software as an M-file program and compared with a general purpose method using LINGO software. An experiment is conducted by varying m and n and the elapsed time and solutions obtained are collected and compared. All computations are tested on a PC notebook with Pentium M, 1.6 Ghz and 512 MB RAM. c_j, a_j, t_j, for $j = 1,2,...,n$, is generated with uniform[0,10]. b_i for $i = 1,2,...,m$, are generated with uniform $[0, \sum_{j=1}^{n} a_j t_j]$. $p_i = \frac{pp_i}{\sum_{i=1}^{m} pp_i}$ where pp_i, for $i = 1,2,...,m$, is generated with uniform [0,1]. g and h are generated with uniform[0,10].

4. Results and Discussion

Table 1 The average computing time of general purpose method using LINGO when n and m are varied.

n	average computing time (excluding parameter generating) (sec)			
	$m = 100$	$m = 500$	$m = 1000$	$m = 5000$
100	0.0769	0.5385	1.3077	29.1538
500	1.0769	3.4615	9.7692	163.4615
1000	5.4706	10.3077	21.6923	608.4615
5000	122.0714	128.1538	474.7692	N/A

Table 2 The average computing time of the proposed method when n and m are varied.

n	average computing time (excluding parameter generating) (sec)			
	$m = 100$	$m = 500$	$m = 1000$	$m = 5000$
100	0.0115	0.0584	0.1172	0.9117
500	0.0865	0.3816	0.7746	3.9164
1000	0.233	0.8872	2.1071	8.5669
5000	4.591	20.1234	38.8557	198.3927

Table 3 The percent error between general purpose method using LINGO and the proposed method when n and m are varied.

n	average percent error			
	$m = 100$	$m = 500$	$m = 1000$	$m = 5000$
100	0.00087	0.00014	0.00005	0.00003
500	0.00016	0.00004	0	0
1000	0.00004	0.00004	0	0
5000	0	0	0	N/A

Remark: N/A means that LINGO cannot solve the studied problem of size (n, m).

According to Tables 1 and 2, the average elapsed time of the proposed method is shorter than the average elapsed time of the general purpose method using LINGO. Moreover, the percent error between the proposed method and the general purpose method using LINGO is very small as shown in Table 3. However, the general purpose method using LINGO cannot solve the studied problem when m and n are very large, e.g. $n = 5000$ and $m = 5000$.

5. Conclusion

A heuristic method for a stochastic knapsack problem with discrete random capacity has been developed and compared with a general purpose method. The results indicated that the proposed method is faster than the general purpose method. However, the general purpose method using LINGO cannot solve the studied problem when m and n are very large, e.g. $n = 5000$ and $m = 5000$.

6. Referrences

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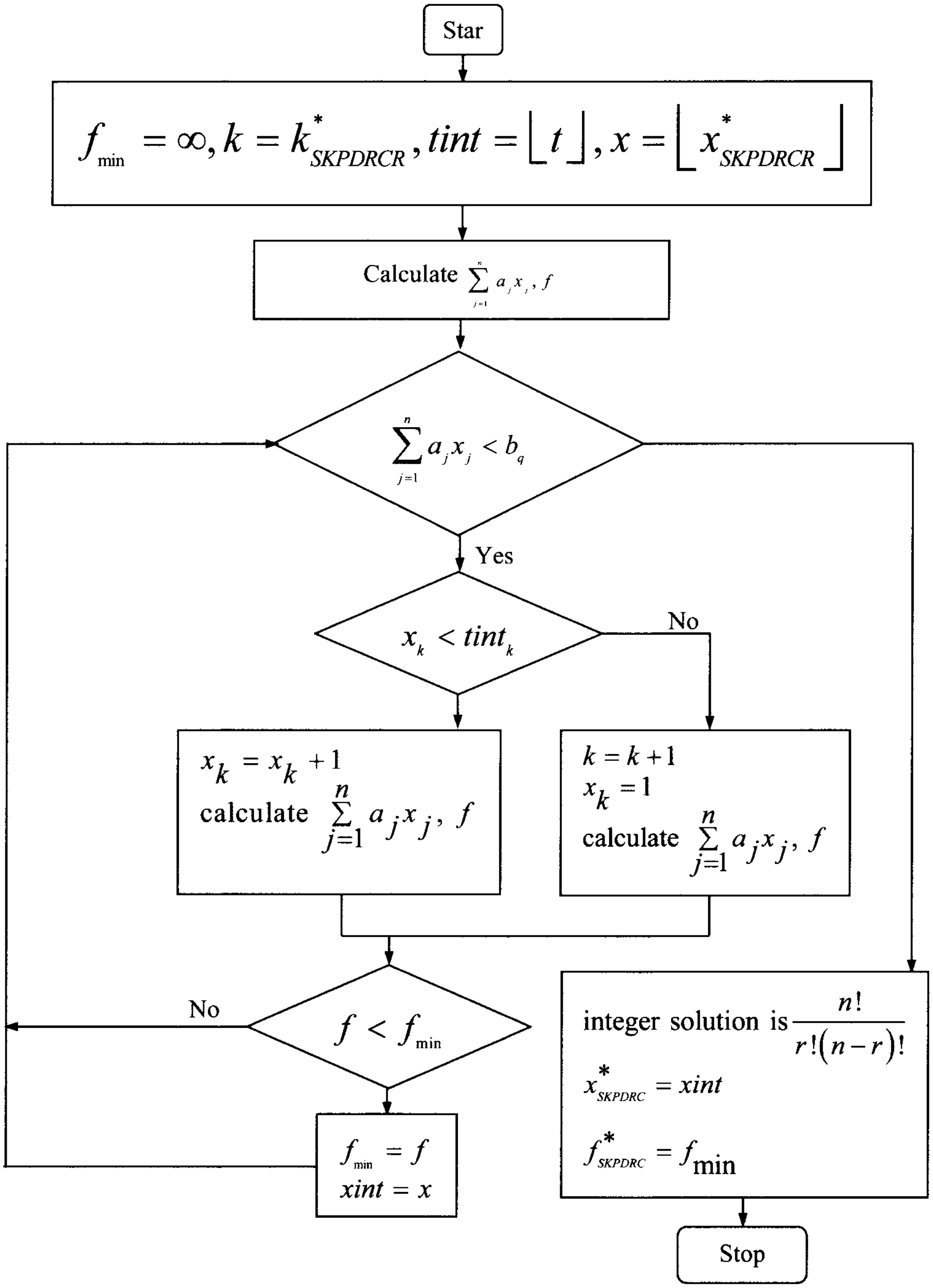

Figure 1 Flow chart of a heuristic for SKPDRC