

Extraction of Static Hysteresis Model Parameters Using an Arbitrarily-Dimensioned Toroid

Paiboon Nakmahachalasint

Department of Electrical Engineering, Faculty of Engineering
Thammasat University, Klong Luang, Pathum Thani 12121, Thailand
E-mail: npaiboon@engr.tu.ac.th

Khai D. T. Ngo

University of Florida
Department of Electrical and Computer Engineering
Gainesville, FL 32611-6130 USA
E-mail: kdtan@ufl.edu

Abstract

A method for extracting the parameters of a static hysteresis (B - H) model is presented that effectively removes the requirement that the sample toroidal core be “thin” (so that the field is uniform radially). In the method, the field-based B - H model is replaced by its circuit-based version, the flux-mmF (magnetomotive force) model. The flux-mmF model explicitly depends on the applied voltage and current, as well as the B - H model parameters, making field uniformity or core dimensions irrelevant in parameter extraction. Numerical optimization is employed to extract the hysteresis model parameters from measured voltages and currents for three commercial power ferrites. Calculations suggest that the material parameters extracted with and without assuming uniform field distribution differ by less than 10% for cores with inner-radius/outer-radius ratios greater than 0.5.

Keywords: Ferrites, magnetic cores, magnetic hysteresis, soft magnetic materials.

1. Introduction

Static hysteresis models [1]-[3] are used in circuit simulators [4] and magnetic design software to characterize core loss, permeability, and other magnetic phenomena. Such models typically contain a set of parameters, referred to herein as “model parameters” or “material parameters,” that need to be extracted from experimental data. While the hysteresis models are usually in terms of field variables, such as magnetic flux density (B) and magnetic field intensity (H), experimental data are usually in terms of circuit variables, such as voltages (v) and currents (i) measured from a sample toroidal core. Thus, to extract the material parameters using the hysteresis model directly, B and H need to be solved from v and i . This is generally nontrivial analytically since B is nonlinear with respect to H , which varies inversely with the toroidal radius. In order to obtain the

approximate solution of B and H from v and i , the standard practice has been to assume that the field distribution is uniform radially [5]-[6]. This means that the sample toroid has to be “sufficiently thin.” In fact, particular toroidal dimensions have been suggested [7]-[8] or used by core producers [9]-[11] for material characterization.

The objective of the paper is to re-formulate the parameter extraction problem so that static hysteresis model parameters can be extracted using any toroid that might be available in one’s laboratory. Toward this goal, Section 2 first reviews the static hysteresis model described in [2]-[3] for soft power ferrites, then formulates a flux-mmF (magnetomotive force) model that contains the material parameters to be extracted, but not the field variables B and H . Since field variables are not involved, field uniformity/non-

uniformity and toroidal sizes are no longer an issue.

Section 3 is dedicated to the experimental and numerical aspects of parameter extraction, and gives results for three commercial power ferrites. The optimization capability of Matlab [12] is used to extract the hysteresis model parameters to best-fit the measured flux and mmf, calculated from the measured voltages and currents.

Thanks to its simplicity, the assumption of “uniform field distribution” is frequently invoked in practice, e.g., via the use of “effective length” and “effective area” [13]. Thus, another objective of the paper is to assess the validity of this assumption. This is done in Section 4 by comparing model parameters extracted with and without the uniform-field assumption. The main results are summarized and future directions suggested in the concluding section.

2. Static Hysteresis (B - H) and Flux-mmF Models

The static hysteresis (B - H) model introduced by Basso-Bertotti [2], and refined by the authors to characterize static hysteresis in soft power ferrites [3], is reviewed in this section. The flux-mmF model for a toroidal core with arbitrary dimensions is next derived, parametric with respect to the hysteresis model parameters.

2.1 Review of Basso-Bertotti's Static Hysteresis Model

The Basso-Bertotti's static hysteresis model comprises a set of equations that describe the initial magnetization curve and all minor and major B - H loops. Each point on a B - H trajectory is associated with a domain wall position x , or $x_{initial}$ if the trajectory happens to be the initial magnetization curve. The first point of each B - H trajectory is called the “turning point,” and is characterized by (B_0, H_0, x_0) ; the first point of the initial magnetization curve has $B_0 = H_0 = x_0 = 0$. As H is varied from H_0 by $\Delta H = H - H_0$, x varies from x_0 both reversibly and irreversibly according to the following respective functions:

$$P_{rev}(\Delta H) = \Delta H = H - H_0 \quad (1)$$

$$P_{irr}(\Delta H) = (\Delta H - H_{c,irr}) + H_{c,irr} \times \exp\left(\frac{-n\Delta H}{H_{c,irr}}\right) \sum_{k=1}^n \frac{k}{n \cdot (n-k)!} \left(\frac{n\Delta H}{H_{c,irr}}\right)^{n-k} \quad (2)$$

where n is the positive integer of the irreversible process; and $H_{c,irr}$ is related to the coercive force H_c and the weighting factor c of the reversible process (see (4) and (5) below) according to:

$$H_c = (1-c)H_{c,irr} \quad (3)$$

The domain wall position is the weighted average of P_{rev} and P_{irr} :

$$x_{initial}(H) = \chi \operatorname{sgn}(H) \times \left[(1-c)P_{irr}(|H|) + cP_{rev}(|H|) \right] \quad (4)$$

$$x(H) = x_0 + 2\chi \operatorname{sgn}(\Delta H) \times \left[(1-c)P_{irr}\left(\left|\frac{\Delta H}{2}\right|\right) + cP_{rev}\left(\left|\frac{\Delta H}{2}\right|\right) \right] \quad (5)$$

where χ is the maximum differential susceptibility. In [3] the following function was suggested to determine B from x :

$$\frac{B[x(H)]}{B_s} = \begin{cases} \sqrt{m_t} \tanh\left(\frac{x}{\sqrt{m_t}}\right) & \text{for } |x| \leq x_t \\ \operatorname{sgn}(x) \left(1 - \frac{1-m_t}{1-x_t+|x|}\right) & \text{for } |x| > x_t \end{cases} \quad (6)$$

$$\text{with } x_t = \sqrt{m_t} \tanh^{-1}(\sqrt{m_t}) \quad (7)$$

where B_s is the saturation flux density; and m_t is the parameter defining the transition between the linear and saturation regions. The static hysteresis model embodied in (1) - (7) was used to generate the scaled B - H loops in Fig. 1(a).

The six material-dependent parameters for the B - H model reviewed above are χ , c , H_c , B_s , m_t , and n . The most obvious way to extract these parameters is to measure B versus H . Unfortunately, B and H are difficult to measure in a bulk toroidal core, and are difficult to infer from voltage and current measurements unless

the field distribution is uniform. Thus, it is desirable to synthesize a circuit-oriented model that also contains the six parameters χ , c , H_c , B_s , m_t , and n so that the parameters can be extracted

from the measurements of circuit variables. The next sub-section describes one such model, which relates flux to mmf.

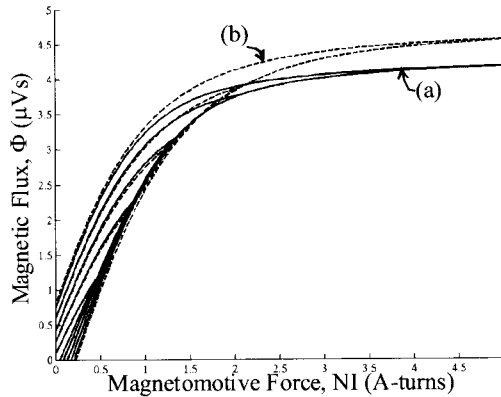


Fig. 1 One quadrant of (a) scaled static hysteresis loops for MN8CX ferrite [10] with $\chi = 0.014079$ m/A, $c = 0.568183$, $H_c = 12.420370$ A/m, $B_s = 0.476905$ T, $m_t = 0.849555$, and $n = 1$; (b) the corresponding flux-mmfm loops for a toroid with $2r_i = 3.175$ mm, $2r_o = 9.525$ mm, and $h = 3.175$ mm.

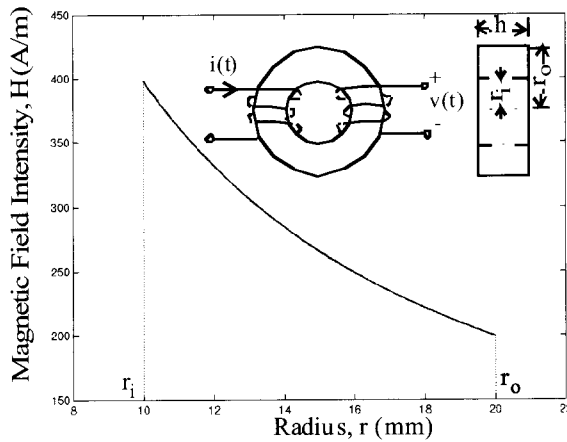


Fig. 2 Non-uniform field distribution in a toroid with radii r_i and r_o , and height h .

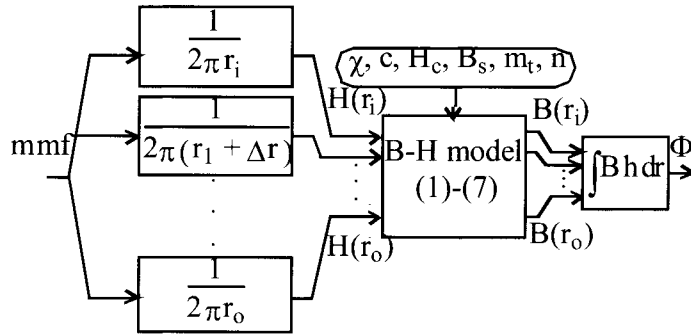


Fig. 3 Discretized relationship between the flux-mmF model and the B - H model for the toroidal core shape.

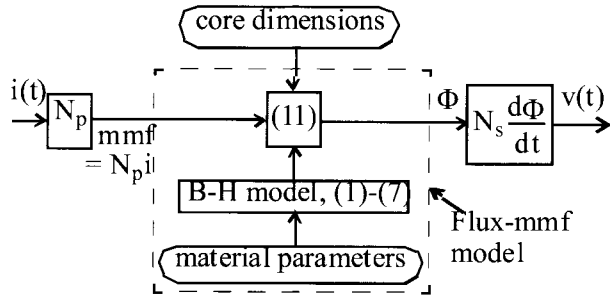


Fig. 4 Relationships among the voltage, current, flux-mmF model, core dimensions, and B - H model.

2.2 Flux-mmF Model for Toroids

Most producers of soft power ferrites often select the toroid with uniform square cross-section shown in Fig. 2 as the test core for hysteresis measurement. The closed magnetic path of the ring-shaped toroid generates zero demagnetizing effects, making the toroid a perfect shape for material characterization. Distributed uniformly around the toroid are bifilar windings with $N_p = N_s = N$ turns.

As illustrated in Fig. 2 and Fig. 3, an mmf is generated by injecting a current i into the excitation winding:

$$\text{mmf}(t) = N_p i(t) \quad (8)$$

The voltage v across the open-circuited sense winding is integrated to obtain the “measured” flux:

$$\Phi_{\text{measured}}(t) = \frac{1}{N_s} \int v(t) dt \quad (9)$$

Thus, mmf and Φ can be readily obtained by measurements of the current and voltage on

the windings. What remains to be done is to relate Φ to mmf by a set of equations that contain the material model parameters. This is illustrated in Fig. 3, which shows that the flux-mmF model comprises the B - H model, and thus the six material parameters, and the core geometry parameters. The interplay among the material parameters, geometry parameters, and hysteresis model to synthesize the flux-mmF model is further delineated in Fig. 4 for the toroid in Fig. 2.

Given an mmf, H can be found by Ampere’s law:

$$\oint H \cdot dl = Ni \rightarrow H(r) = \frac{\text{mmf}}{2\pi r} \quad (10)$$

where dl is the differential length along the integration contour. Note that the core height h does not enter the preceding equation. The nonuniform distribution of H along the toroidal radius is plotted in Fig. 2, which shows that H peaks at the inner radius r_i and drops as r increases toward the outer radius r_o . The

discretized form of (10) is illustrated in the left half of Fig. 4.

With H known, $B(H)$ can be found via the hysteresis model in (1) - (7). The magnetic flux is then parametric with respect to the material parameters as follows:

$$\Phi = \int_{r_i}^{r_o} B(H[r]) h dr \quad (11)$$

$$= h \int_{r_i}^{r_o} B\left(\frac{mmf}{2\pi r}, \chi, c, H_c, B_s, m_i, n\right) dr$$

The discretized form of the preceding equation is illustrated in the right half of Fig. 3. Equations (11) and (1) - (7) constitute the flux-mmfm model. One quadrant of the flux-mmfm loops is shown in Fig. 1(b). The flux-mmfm loops are not simply scaled B - H loops because of field non-uniformity. The next section explains how the material parameters are extracted from the flux-mmfm model.

3. Parameter Extraction Using the Flux-mmfm Model

This section describes the procedure to extract the material parameters for a given ferrite, using the flux-mmfm model discussed in the previous section

3.1 Parameter Extraction

The procedure for extracting the material parameters χ , c , H_c , B_s , m_i , and n is summarized in Fig. 6. It has been implemented in the Optimization Toolbox of Matlab [12]. The mmf, toroidal dimensions, and the trial values for the material parameters are used in (11), which calls (1) - (7), to compute Φ_{modeled} . The error $\Delta\Phi$ between Φ_{measured} and Φ_{modeled} drives a nonlinear least-square optimization algorithm that searches the set of material parameters that minimize $\Delta\Phi$.

Since n is a positive integer, it could not be extracted using the nonlinear least-squares optimization function in a straightforward manner. Thus, n was swept between 1 and 5. For each n , the five real parameters χ , c , H_c , B_s , and m_i were extracted and the minimum squared 2-norm error ($\text{resnorm} = \sum (\Delta\Phi_j)^2$, $j = 1, 2, \dots$, total number of points) recorded. The set of parameters with the lowest resnorm was then selected. The function was called with its default options and the following initial guesses: $B_s \approx 0.5$ T, $\chi \approx 0.01$ (A/m) $^{-1}$, $c \approx 0.5$, $H_c \approx 10$ A/m, and $m_i \approx 0.8$. All the flux-mmfm branches were compiled into a matrix input to the function.

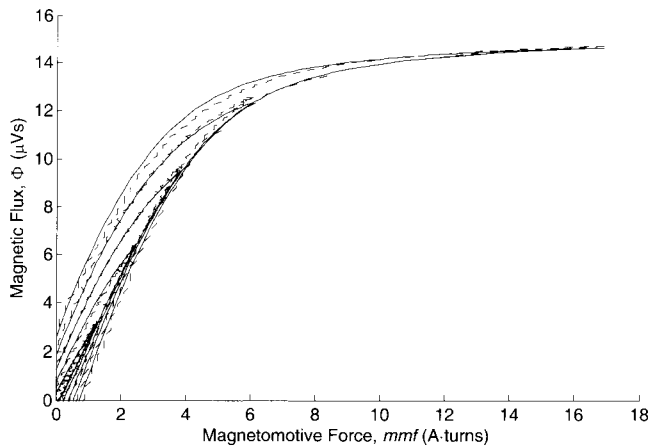


Fig. 5 Measured and modeled flux-mmfm loops at room temperature and 10 kHz for the MN8CX toroid described in Table 1.

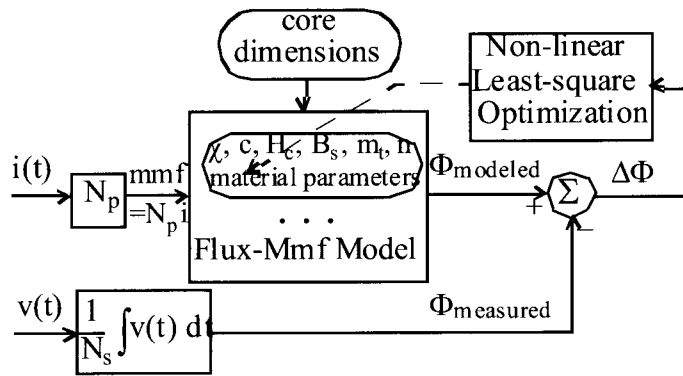


Fig. 6 Extraction procedure for B - H model parameters using the flux-mmF model.

3.2 Sample Results

For the ferrites and cores tabulated in Table 1, the material parameters extracted using the method outlined above are listed in Table 2.

Fig. 5 compares the first quadrants of the modeled and measured flux-mmF loops of the MN8CX toroid described in Table 1. The error per point, computed by $\text{resnorm}^{0.5}/\Phi_{\max}/(\text{number of points})$, based on 500 data points per loop is less than 0.1% for the major loop, and is less than 0.05% for each minor loop.

Table 1 Materials and dimensions of the measured toroids

Ferrite	MN8CX	K	PC40
r_o (mm)	11.645	11.050	6.000
r_i (mm)	7.470	6.860	3.000
h (mm)	7.700	6.350	3.000

Table 2 Extracted material parameters

Ferrite	MN8CX	K	PC40
χ	0.014079	0.010088	0.023551
c	0.568183	0.534172	0.290071
H_c	12.420370	23.968955	18.858222
B_s	0.476905	0.453032	0.464389
m_t	0.849555	0.605447	0.573578
n	1	3	2

4. Comparison of Parameters Extracted with and without Uniform-Field Assumption

In this section the “effective” material parameters are extracted assuming uniform field distribution, and are compared with the “actual” material parameters extracted in the previous section, taking field non-uniformity into account.

The uniform magnetic field intensity $H_{\text{effective}}$ is customarily computed using the effective length $l_{e,IEC}$, and the corresponding flux $\Phi_{\text{effective}}$ is computed using the effective area $A_{e,IEC}$:

$$H_{\text{effective}} = \frac{Ni}{l_{e,IEC}} = \frac{mmf}{l_{e,IEC}} \quad (12)$$

$$\Phi_{\text{effective}} = B \left(\frac{mmf}{l_{e,IEC}} \right) A_{e,IEC} \quad (13)$$

where, for toroidal cores with sharp corners, the IEC standards [13] give the following relationships, derived in [14]:

$$\frac{l_{e,IEC}}{2\pi r_o} = \frac{k_r}{1-k_r} \ln \left(\frac{1}{k_r} \right) \quad (14)$$

$$\frac{A_{e,IEC}}{hr_o} = \frac{k_r}{1-k_r} \ln^2 \left(\frac{1}{k_r} \right) \quad (15)$$

where the radius ratio k_r is defined as:

$$k_r = \frac{r_i}{r_o} \quad (16)$$

Equation (13) constitutes the “effective” flux-mmF model for the case of uniform field. When the procedure in Fig. 7 is executed using this model, a set of “effective” material parameters is extracted, that depends on the radius ratio k_r , and that converges to the “actual” material parameters as k_r approaches unity. To establish the range of k_r over which the assumption of uniform field distribution is

acceptable, the relative differences between the effective and the actual material parameters are defined:

$$\varepsilon_{H_c}(k_r) = \frac{|H_{c, \text{effective}}(k_r) - H_c|}{H_c}, \text{ and so on.} \quad (17)$$

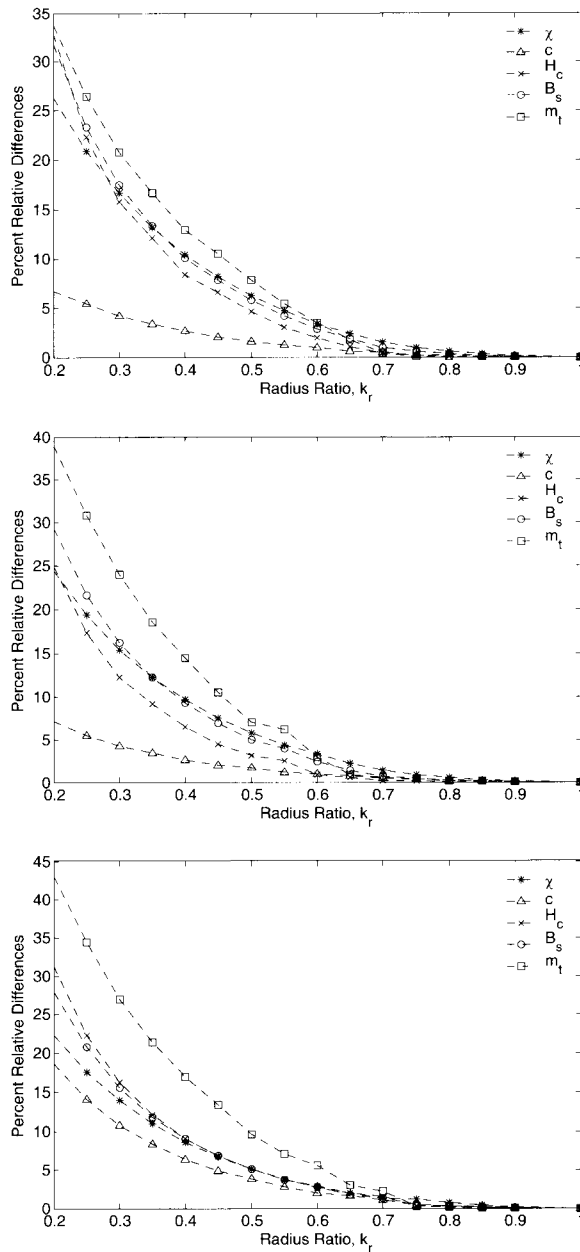


Fig. 7 Relative differences between the effective material parameters for the ferrites in Table 1 and the actual material parameters listed in Table 2 for those ferrites.

Fig. 7 plots the relative differences ε_{H_c} , ε_{B_s} , ε_{m_i} , ε_χ , and ε_c for the ferrites in Table 1 versus k_r ($\varepsilon_n = 0$). From these relative differences, the root-mean-squared (rms) relative difference between the two sets of parameters can be defined:

$$\varepsilon_{rms}(k_r) = \sqrt{\frac{\varepsilon_{H_c}^2 + \varepsilon_{B_s}^2 + \varepsilon_{m_i}^2 + \varepsilon_\chi^2 + \varepsilon_c^2}{5}} \quad (18)$$

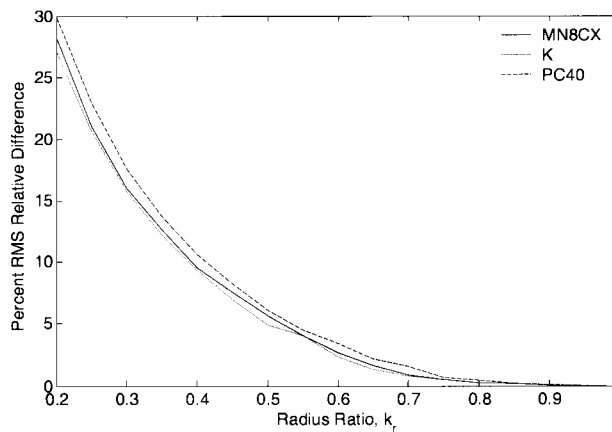


Fig. 8 Rms relative difference computed from Fig. 7.

Fig. 8 is for the ferrites in Table 1. The effective material parameters differ from the actual material parameters by less than 10% as long as k_r is above 0.5.

5. Conclusions

The flux-mmF model has been found to be an effective formulation for the extraction of static hysteresis model parameters. The model has been incorporated into a procedure to extract model parameters that involves measurements of winding voltage and current, followed by numerical optimization. The procedure has been applied to extract the parameters of a static hysteresis model (introduced by Basso-Bertotti and refined by the authors to model minor and major hystereses in soft power ferrites) for three commercial power ferrites. The frequent assumption of uniform radial field distribution has been found to be valid as long as the inner radius is greater than 0.5 times the outer radius.

6. Acknowledgment

This work was supported by the National Science Foundation under Grant #ECS-9906254, by the State of Florida Integrated

Electronics Center, and by the Thailand Research Fund under Contract #MRG4780186. The authors wish to thank Mr. George E. Schaller (the Executive Vice President of Ceramic Magnetics), Rob Coit (of Magnetics Inc.), and MH&W International Corp. (sales agent of TDK ferrites) for providing the sample toroids.

7. References

- [1] F. Liorzou, B. Phelps, and D.L. Atherton, Macroscopic Models of Magnetization, IEEE Transactions on Magnetics, Vol. 36, No. 2, pp. 418-428, March 2000.
- [2] V. Basso and G. Bertotti, Hysteresis Models for the Description of Domain wall Motion," IEEE Transactions on Magnetics, Vol. 32, No. 5, pp. 4210-4212, September 1996.
- [3] P. Nakmahachalasint and K. D. T. Ngo, Generalized Formulation for the Description of Hysteresis in Soft Magnetic Materials," IEEE Transactions on Magnetics, Vol. 38, No. 1, pp. 200-204, January 2002.

- [4] Analyzing Designs Using SaberDesigner, Release 5.0, Analogy, Inc., 1999.
- [5] P. Nakmahachalasint, K. D. T. Ngo, and L. Vu-Quoc, Effective Magnetic Parameters in the Presence of Hysteresis, IEEE Transactions on Aerospace and Electronic Systems, Vol. 40, No. 3, pp. 1100 -1105, July 2004.
- [6] P. Nakmahachalasint, K. D. T. Ngo, and L. Vu-Quoc, Effective Parameters for Toroidal Cores Based on Nonlinear Magnetization, IEEE Transactions on Magnetics, Vol. 41, No. 9, pp. 2432 -2435, September 2005.
- [7] IEEE, IEEE Standard for Test Procedures for Magnetic Cores, IEEE Std. 393-1991, 1991.
- [8] Magnetic Materials Producers Association, Soft Ferrites, a user Guides, MMPA SFG-98, 1998.
- [9] Ceramic Magnetics, Engineered Ferrites Catalog, Fairfield, NJ: Ceramic Magnetics, Inc., 1999.
- [10] Ferrite Cores Design Manual, Magnetics Inc., 1999.
- [11] TDK Ferrite Cores for Power Supply and EMI/RFI Filter, TDK, 1999.
- [12] Optimization Toolbox User's Guide, version 2, Math Works, MA, 1999.
- [13] International Electrotechnical Commission, Calculation of the Effective Parameters of Magnetic Piece Parts, IEC 60205, 2nd ed., 2001.
- [14] E. C. Snelling, Soft Ferrites: Properties and Applications, 2nd ed. London, UK: Butterworth, 1988.

Paiboon Nakmahachalasint is currently an Assistant Professor in the Department of Electrical Engineering, Thammasat University, Thailand. His current research interests include magnetic materials and components.

Khai D. T. Ngo is currently a Professor of Electrical and Computer Engineering at the University of Florida. His current research interests include magnetic materials and components, power converters, and power integrated circuits.