A Stress-Based Material Distribution Method for Optimum Shape Design of Mechanical Parts

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Abstract

This paper presents a numerical method for minimum-weight shape design of a mechanical part with a stress constraint. The design method aims to produce a fully-stressed design by distributing material according to the local von Mises stress. Material distribution is represented as element thickness instead of density or elastic modulus in a two-dimensional finite element model, which makes the optimum design practical. During the iterative design process, a trimming process may be conducted, where thin elements are removed to reveal the new boundary shape. Thickness adjustment and trimming lead to the optimum result of a three-dimensional shape. The method is demonstrated with a short cantilever beam design problem. The result is found to be encouraging.

Keywords: Structural optimization, Shape optimization, Topology design.

1. Introduction

Shape optimization of mechanical parts under external load can be performed by several approaches. Among them, free form boundary shape adjustment presented by [1], which utilizes control points on the boundary as design parameters and topology optimization pioneered by [2], which takes local material properties as design parameters. The later is more flexible, hence, tends to provide superior designs.

To date, several methods for topology optimization has been proposed. First is a homogenization method by [2]. The method takes elastic modulus and orientation of micro structure of each element in a finite element model as design parameters. The optimum designs from this method have variable material properties, which are considered impractical. Moreover, the relationship between the elastic modulus and the density of most materials, which plays a significant role in the optimization process, is obscure, although there has been a proof by [3] that the relationship exists for certain materials.

Another method called "evolutionary structural optimization", made well known by [4], is considered an intuitive method where a design evolves mainly through the removal of elements of a finite element model. The elements are removed (or added) according to their stresses or compliance sensitivities. This method is simpler to implement but the optimum designs are discrete and dependent on the scheme used to determine the removal step.

Objectives of shape optimization, studied in the previously mentioned research, are usually one of the following; (i) minimization of material usage subjected to a stress constraint, (ii) maximization of stiffness for a given amount of material. Although [5] proved that the two objectives are equivalent, the first objective is more related to structural applications, where stress is a major design criterion. Hence, this research focused on the first objective.

The shape optimization method presented in this paper utilizes an intuitive scheme to distribute material in the form of element thickness. Thickness of each element is adjusted inversely proportional to its von Mises stress. When certain elements become practically too thin, they can be removed. This leads to creation of new boundary shapes. The scheme is extended from a simple stress-thickness relationship in the beam theory, with a supporting assumption of biological growth [6]. The next section provides a brief discussion of the scheme and its numerical implementation. The design example, presented in section 3, shows the evolution of a rectangular beam into an optimum I-beam shape which is fully-stressed and has significant weight reduction.

2. Material Distribution Method

The topology optimization program, based on the method presented in this paper, runs iteratively in the steps described in Fig. 1. First, a finite element model is formed for the initial shape. The finite element solver then solves for strain and stress distribution. The local von Mises stress distribution is used as information to adjust the elements' thickness. After the adjustment, a new finite element model is created and the process repeated. The optional trimming process (where elements thinner than t_{min} are removed) may be performed at specified iterations. The optimum design is obtained when a fully-stressed condition is achieved. The thickness adjustment scheme is described next.

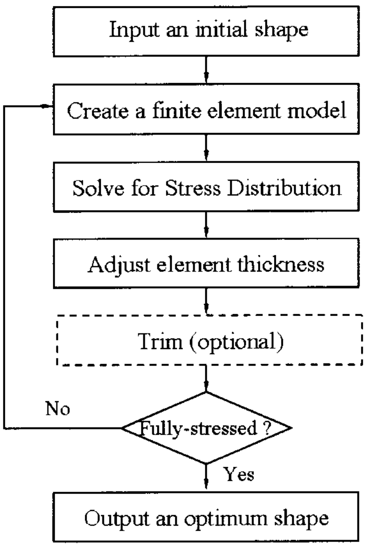


Figure. 1 Topology Optimization Procedure.

From beam theory, major contribution to the element's von Mises stress comes from bending stress, although transverse shear stress exists. The bending stress at any element i, shown in Fig. 2, is a function of bending moment, M, the transverse location, y, and the beam's sectional moment of inertia, I as shown in equation (1a):

$$\sigma_i = -\frac{My}{I} \tag{1a}$$

Substitute an expression of I into equation (1a) to obtain equation (1b).

$$\sigma_i = -\frac{12My}{th^3} \tag{1b}$$

Above, t is the beam's thickness and h is the beam's height.

The bending moment at the position x of element i can be computed for a given load distribution and is considered constant in space. Position y of element i is constant. If h is kept constant, then the thickness of element i can be written in terms of bending stress as:

$$t_i = \frac{k}{\sigma_i} \tag{2}$$

where k is a constant.

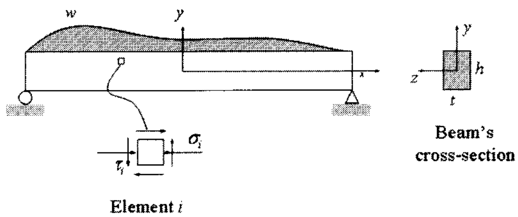


Figure. 2 A simply supported beam under distributed load.

If the load is applied in the x direction, the relationship in equation (2) is still valid. In order to form a fully-stressed beam, one can vary the thickness of the beam's rectangular cross-section along the x-axis inversely proportion to the local maximum stress at a corresponding x-position.

Although there is no proof of validity of equation (2), when the thickness is also varied along the y-axis, the natural trend of biological growth supports the equation. More material should be distributed to the elements with high stress, which is inagreement with the trend of the equation. Hence, the relationship in equation

(2) is assumed as a rough approximation to the sensitivity information of local thickness to local stress. As any numerical approximation, the solution process requires some form of relaxation. Thus, replace σ_i in equation (2) with local von Mises stress, apply relaxation factor and the scheme takes the following form.

$$t_i^{j^*} = t_i^j\left(\frac{\sigma_i}{\sigma_{set}}\right)$$
$$t_i^{j+1} = \min[(1-r)t_i^j + rt_i^{j^*}, t_{\max}] \qquad (3)$$

where t_i^j is the original thickness of element i, t_i^{j+1} is the adjusted thickness of element i for the next iteration, $t_{\max}$ is the maximum allowable thickness, σ_i is the von Mises stress of element i, σ_{set} is the maximum allowable stress and r is a relaxation factor, $0 < r \le 1$. The relaxation factor, r, controls the pace of the design iteration. If r is set to one, the scheme fully follows equation (2), which may result in an overshoot (i.e. element stresses exceed σ_{set}.)

3. Design Example

A short cantilever beam in Fig. 3 is used to show the design method. The computation is conducted in dimensionless manner with the initial shape being l = 2, h = 1 and t = 0.1. A generic finite element program using iso-parametric quadrilateral elements is utilized as a solver. The finite element model of this beam consists of 40x20 elements. Other parameters are: point load, F = 1; Youngs modulus, E = 1; Poisson ratio ν = 0.3 and relaxation factor, r = 0.5. ; The maximum allowable von Mises stress, σ_{set} is set to the maximum value of the initial beam.

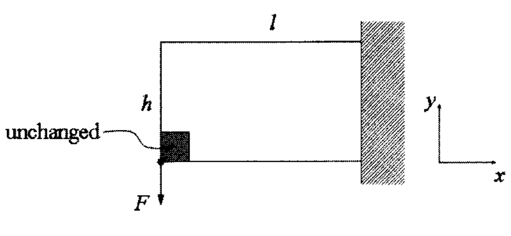


Figure. 3 Short cantilever beam.

The shaded portion at the point load in Fig. 3 represents the unchanged area. The elements containing the supported edge are also left unchanged. With this treatment, the effect of stress concentration is avoided.

To ensure a smooth convergence, there is no thickness constraint during the iterations, and the trimming process is conducted after the fully stressed beam is achieved. t_{min} is set to 5 percent of the initial thickness. Since the density is a constant, the program tracks the volume reduction of the beam instead of weight. Numerically, the fully-stressed condition is satisfied when the maximum and the minimum elements' von Mises stresses lie within one percent of σ_{set}.

Observe from the design history in Fig. 4, the beam evolves rapidly at the beginning causing volume reduction at a fast rate, down to about 25 percents of the original volume. However, element stresses diverge. Then, after the twentieth iterations, there is almost no volume reduction but the shapes keep evolving. The maximum and minimum von Mises stresses gradually converge to σ_{set}. At the 199^{th} iteration, a fully-stressed condition is achieved. Trimming is conducted at the 200^{th} design, where 75 elements are removed simultaneously, which causes the maximum stress to rise beyond σ_{set}. Fifty more iterations are performed to bring the beam back to the fully-stressed condition. Only minor thickness changes occur to some elements after trimming. This demonstrates the robustness of the optimum design. The volume of the optimum beam is 24 percent of the original volume.

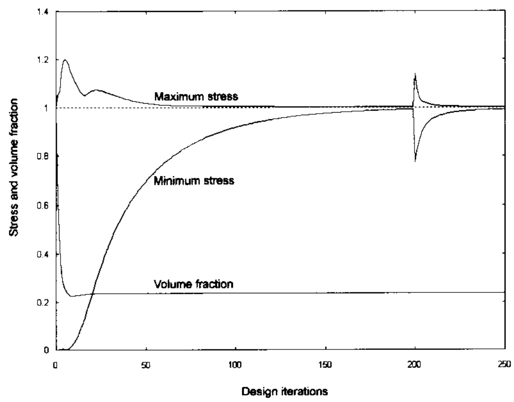
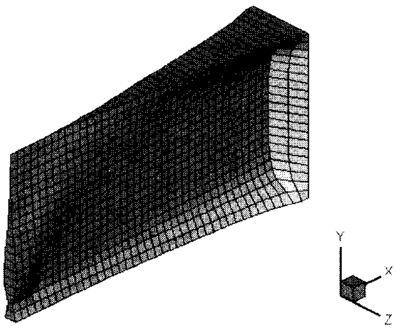
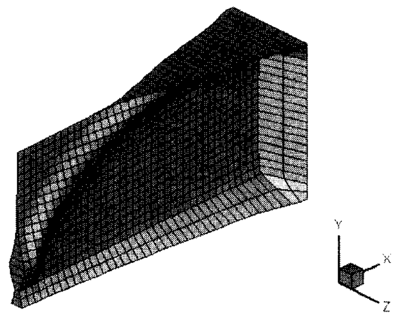


Figure. 4 Design history.

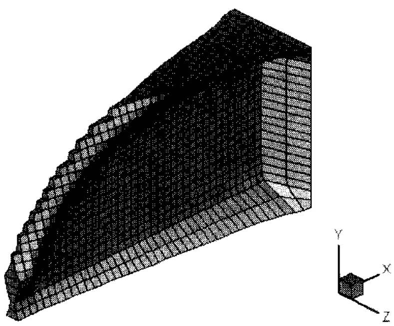
Figures 5 show physical shapes of the design from iteration 10, the fully-stressed design from iteration 199 and the optimum design obtained after trimming from iteration 250. Note that the thickness (z-axis) is scaled up three times for visibility. Also, for a presentation purpose, the elements' thickness is interpolated to nodal thickness and thickness of the unchanged elements is shown to be equal to the thickness of the nearby elements.



(a) The 10th iteration.



(b) Fully-stressed design. (the 199^{th} iteration)



(c) Optimum design after trimming.
(the 250^{th} iteration)

Figure. 5 Shapes of the short cantilever beams.
(z-scale is magnified three time)

Figure 6 shows cross-sections of the fully-stressed beam, which resemble an I-shape, which is known to be an optimum engineering solution. This demonstrates the effectiveness of the method.

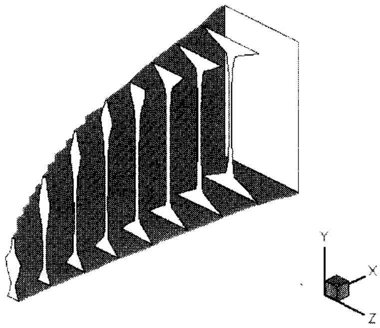


Figure. 6 Cross-sections of the final design.
(z-scale is magnified three time)

The proposed method is also implemented for other design problems involving different types of mechanical elements such as design of a simply-supported beam subjected to a horizontally-translated vertical load and design of a center-supported wheel subjected to a torque on the rim. The final designs for both design problems are shown in Fig. 7 and 8 respectively. Both designs shown are found to be fully-stressed and have significant weight reduction. Details of both cases can be found in [7].

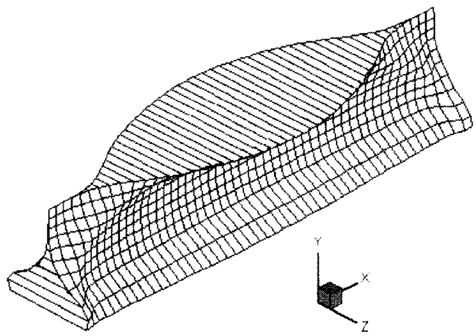


Figure. 7 Optimum design of a simply-supported beam subjected to a horizontally-translated vertical load.

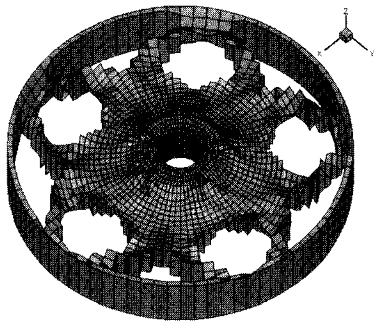


Figure. 8 Optimum design of a center-supported wheel subjected to a torque on the rim.

4. Conclusion

The presented method relies on an intuitive scheme that adjusts the thickness of each element inversely proportional to its von Mises stress. Although there is no theoretical proof of the thickness-stress relationship that forms the basis of the presented design method, the short cantilever beam example does validate the method.

The advantages of this design method over other mathematical based topology design methods are its simplicity and manufacturability of the optimum design. The method can be applied to any finite element source codes that are available. The method can also be extended to multiple load cases by employing virtual stress distributions (union product of von Mises stress distribution from all load cases).

The limitation of the method is that it is currently applicable only to in-plane load cases. Stresses in the optimum design are based on the plane stress assumption, so, fine tuning is required upon constructing a three-dimensional model of the optimum design. Further study is required to extend the method to cover fully three-dimensional load cases.

5. References

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