Local Similarity Solutions for Unsteady MHD free Convection and Mass Transfer Flow Past an Impulsively Started Vertical Porous Plate with Dufour and Soret Effects

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Abstract

An unsteady free convection and mass transfer flow of an electrically conducting, viscous, incompressible fluid, past an infinite vertical porous plate in the presence of a transverse magnetic field is studied, when the plate is moved impulsively with a constant velocity in the direction of the flow. Both the Dufour and Soret effects are considered for a hydrogen-air mixture as the non-chemically reacting fluid pair. The non- linear partial differential equations, governing the problem under consideration, have been transformed by a similarity transformation into a system of ordinary differential equations, which are solved numerically by using the Nachtsheim-Swigert shooting iteration technique together with a sixth order Runge-Kutta integration scheme. The resulting velocity, temperature and concentration distributions are shown graphically for different values of the parameters entering into the problem. Finally, the numerical values of the local skin-friction coefficient, local Nusselt number and local Sherwood number are also presented in a tabular form.

Keywords: MHD, Free convection, Unsteady flow, Vertical plate, Dufour and Soret effects.

1 Introduction

The hydrodynamic flow of a viscous incompressible fluid past an impulsively started infinite horizontal plate was studied by Stokes [1], and because of its practical importance this problem was extended to bodies of different shapes by a number of researchers. Soundalgekar [2] studied the above problem along an infinite vertical plate, when it is cooled or heated by the free convection currents. It is also known that flows arising from differences in concentration have great significance not only for their own interest but also for the application to geophysics, aeronautics and engineering. In light of the above applications, many researchers studied the effects of mass transfer on magnet hydrodynamics (MHD) free convection flow; some of them are, Raptis and Kafoussias [3], Rahman and Sattar [4], Yih [5], Aboeldahab and Elbarbary [6], Megahead et al. [7] and Kim [8]. In the above stated papers, the diffusion-thermo term and thermal-diffusion term were neglected from the energy and concentration equations respectively. But when heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of a more intricate nature. It has been found that an energy flux can be generated not only by temperature gradients but by composition gradients as well. The energy flux caused by a composition gradient is called the Dufour or diffusion-thermo effect. On the other hand, mass fluxes can also be created by temperature gradients and this is the Soret or thermal-diffusion effect. In general, the thermal-diffusion and diffusion-thermo effects are of a smaller order of magnitude than the effects described by Fourier's or Fick's law and are often neglected in heat and mass transfer processes. However, exceptions are observed therein. The thermal-diffusion (Soret) effect, for instance, has been utilized for isotope separation, and in mixtures between gases with very light molecular weight (H_2, He) and of medium molecular weight (N_2, air) the diffusion-thermo (Dufour) effect was found to be of a considerable magnitude such that it cannot be ignored (Eckert and Drake [9]). In view of the importance of this diffusion-thermo effect, Jha and Singh [10] studied the free-convection and mass transfer flow about an infinite vertical flat plate moving impulsively in its own plane, taking into account the Soreteffects. Kafoussias [11] studied the same problem in the case of MHD flow. They madeanalytical studies based on the Laplace transformtechnique. Later, Kafoussias and Williams [12] studied thermal-diffusion and diffusion-thermo effects on mixed free-forced convective and mass transfer boundary layer flow with temperature dependent viscosity, whereas Anghel et al. [13] investigated the Dufour and Soret effects on a free convection boundary layer over a vertical surface embedded in a porous medium. Recently, Takhar et al. [14] studied unsteady free convection flow over an Infimte porous plate due to the combined effects of thermal and mass diffusion, magnetic field and Hall currents. Very recently, Postelnicu [15] studied numerically the influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects.

Therefore, the objective of this paper is to study the Dufour and Soret effects on unsteady free convection and mass transfer flow, past an impulsively started infinite vertical porous flat plate, of a viscous incompressible and electrically conducting fluid, in the presence of a uniform transverse magnetic field.

2. Mathematical Analysis

We consider an unsteady two-dimensional flow of an incompressible and electrically conducting viscous fluid, along an infinite vertical porous flat plate. The x-axis is taken along the plate in the upward direction and the *y-axis* is taken normal to the plate. A magnetic field of uniform strength is applied transversely to the direction of the flow. Initially the plate and the fluid are at the same temperature T_∞> in a stationary condition with concentration level C_∞> at all points. For time t>0, the plate starts moving impulsively in its own plane with a velocity U_0, its temperature is raised to T_w and the concentration level at theplate is raised to C_w. The fluid is assumed to have constant properties except for the influence of the density variations with temperature and concentration, which are considered only in the body force term. Under the above assumptions, the physical variables are functions of y and t only and therefore the basic equations, which govern the problem, are:

$$\frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial v}{\partial y} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)$$

$$+ g\beta^*(C - C_\infty) - \frac{\sigma B_0^2 u}{\rho}, \tag{2}$$

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{D_m k_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2}, \tag{3}$$

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2} + \frac{D_m k_T}{T_m}\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

where the variables and related section quantities are defined in the Nomenclature section. The initial and boundary conditions for the above problem are:

for $t \le 0 : u = v = 0,\ T = T_\infty,\ C = C_\infty$ for ally. for $t > 0$:

$$u = U_0, v = v(t), T = T_w, C = C_w \text{ at } y = 0, \tag{5a}$$

$$u = 0, v = 0, T = T_\infty, C = C_\infty \text{ as } y \to \infty, \tag{5b}$$

The last term on the right-hand side of the energy equation (3) and concentration equation (4) signify the Dufour or diffusion-thermo effect and the Soret or thermal-diffusion effect, respectively.

Now in order to obtain a local similarity solution in the time of the problem under consideration, we introduce a time dependent length scale δ as:

$$\delta = \delta(t) \tag{6}$$

In terms of this length scale, a convenient solution of the equation (1) is considered to be in the following form:

$$v = v(t) = -v_0 \frac{\upsilon}{\delta}, \tag{6}$$

where Vo is the suction parameter.

We now introduce the following dimensionless variables:

$$\left.\begin{aligned} &\eta = \frac{y}{\delta}, \\ &u = U_0 f(\eta), \\ &\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \\ &\phi(\eta) = \frac{C - C_\infty}{T_w - T_\infty}, \end{aligned}\right\} \tag{7}$$

Then, introducing the relations (6)-(8) into the equations (2), (3) and (4), respectively, we then obtain the following ordinary differential equations:

$$f'' + \eta\left(\frac{\delta}{\upsilon}\frac{d\delta}{dt}\right) f' + v_0 f' + Gr\theta + Gm\phi - Mf = 0, \tag{9}$$

$$-\eta\left(\frac{\delta}{\upsilon}\frac{d\delta}{dt}\right)\theta' + v_0\theta' \frac{1}{\Pr}\theta'' + Dr\phi'', \tag{10}$$

$$-\eta\left(\frac{\delta}{\upsilon}\frac{d\delta}{dt}\right)\phi' - v_0\phi' \frac{1}{Sc}\phi'' + Sr\theta'', \tag{11}$$

where

$\Pr = \frac{\upsilon}{\alpha}$ is the Prandtl nuber, $Sc\frac{\upsilon}{D_m}$, is the Schmidt number, $M = \frac{\sigma B_0^2 \delta^2}{\upsilon\rho}$ is the Magnetic field parameter, $Sr = \frac{D_m k_T (T_W - T_\infty)}{T_m \upsilon (C_W - C_\infty)}$, is the Soret number, $Df = \frac{D_m k_T (C_W - C_\infty)}{c_s c_p \upsilon (T_W - T_\infty)}$, is the Dufour number, $Gr = \frac{g\beta(T_W - T_\infty)\delta^2}{\upsilon U_0}$, is the local Grashof number and

$Gm = \frac{g\beta^*(C_W - C_\infty)\delta^2}{\upsilon U_0}$, is the local modified Grashof number.

The corresponding boundary conditions for $t > 0$ are obtained as:

$$f = 1,\ \theta = 1,\ \phi = 1,\ \text{at } \eta = 0, \tag{12a}$$
$$f = 0,\ \theta = 0,\ \phi = 0,\ \text{as } \eta = \infty. \tag{12b}$$

Now the equations (9)-(11) are locally similar except the term $\left(\frac{\delta}{\upsilon}\frac{d\delta}{dt}\right)$, where t appears explicitly. Thus, the local similarity condition requires that the term $\left(\frac{\delta}{\upsilon}\frac{d\delta}{dt}\right)$ in the equations (9)-(11) must be a constant quantity.

Hence, following the works of Hasimoto [16], Sattar and Hossain [17] and Sattar et al. [18], one can try a class of solutions of the equations (9)- (11) by assuming that:

$$\left(\frac{\delta}{\upsilon}\frac{d\delta}{dt}\right) = \lambda \text{ (a constant)} \tag{13}$$

Integrating (13) we have

$$\delta = \sqrt{2\lambda \upsilon t}\,, \tag{14}$$

where the constant of integration is determined through the condition that $\delta = 0$ when $t = 0$. We have considered the problem for small time. In this case the normal velocity in (7) will be large i. e., suction will be large, which can be applied to increase the lift of airfoils. From (14), choosing $\lambda = 2$, the length scale $\delta(t) = 2\sqrt{\upsilon t}$ exactly corresponds to the usual scaling factor for various unsteady boundary layer flows (Schlichting [19]). Since δ is a scaling factor as well as a similarity parameter, any value of λ in (13) would not change the nature of the solutions, except that the scale would be different.

Now introducing (13) (with/ $\lambda = 2$] in the equations (9)-(11) respectively, we obtain the following dimensionless ordinary differential equations which are locally similar in time but not explicitly time dependent.

$$f'' + (2\eta + v_0)f' + Gr\theta + Gm\phi - Mf = 0 \tag{15}$$

$$\theta'' + \Pr(2\eta + v_0)\theta' + \Pr Df\phi'' = 0 \tag{16}$$

$$\phi'' + Sc(2\eta + v_0)\phi' + ScSr\theta'' = 0 \tag{17}$$

where primes denote differentiation with respect to η.

The equations (15)-(17) constitute a set of ordinary differential equations, the solutions of which should unfold the characteristics of the problem under consideration. These equations under the boundary conditions (12) are solved numerically by using the Nachtsheim-Swigert (20] shooting iteration technique together with a sixth- order Runge Kutta integration scheme.

3. Skin-friction, rate of heat and mass transfer

Now it is important to calculate the physical quantities of the primary interest, which are the local wall shear stress, local surface heat flux and the local surface mass flux respectively from the following definitions:

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \tag{18}$$

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}, \tag{19}$$

$$M_w = -D_m\left(\frac{\partial C}{\partial y}\right)_{y=0}. \tag{20}$$

where μ is the viscosity, k is the thermal conductivity and Dm is the mass diffusivity . The dimensionless local wall shear stress, local surface heat flux and the local surface mass flux for an impulsively started plate are respectively obtained as:

$$\frac{\tau_w \delta}{\mu U_0} = f'(0), \tag{21}$$

$$\frac{q_w \delta}{k(T_w - T_\infty)} = -\theta'(0), \tag{22}$$

$$\frac{M_w \delta}{D_m(C_w - C_\infty)} = -\phi'(0), \tag{23}$$

Hence the dimensionless skin-friction coefficient, Nusselt number and Sherwood number for impulsively started plate are given by:

$$C_f = \frac{2\tau_w}{\rho U_0^2} = 2(\mathrm{Re}_\delta)^{-1} f'(0), \tag{24}$$

$$Nu = \frac{q_w \delta}{k(T_w - T_\infty)} = -\theta'(0), \tag{25}$$

$$Sh = \frac{M_w \delta}{D_m(C_w - C_\infty)} = -\phi'(0). \tag{26}$$

where $\mathrm{Re}_\delta = \dfrac{U_0 \delta}{\upsilon}$ is the Reynolds number.

These dimensionless values of the local skin- friction coefficient, local Nusselt number and local Sherwood number for impulsively started plate are obtained from the process of numerical calculations and are sorted in Tables 1-2.

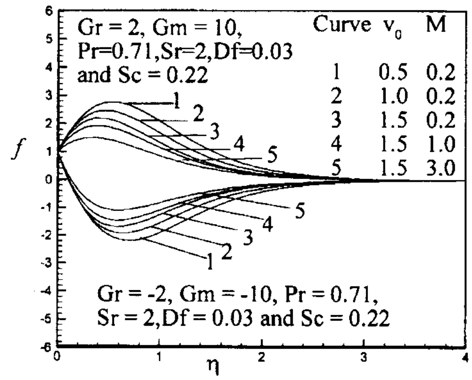

Fig. 1 Velocity profiles for different values of v_0 and M.

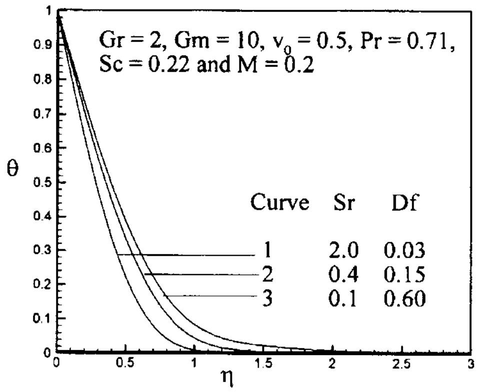

Fig. 4 Temperature profiles for different values of Sr and Df.

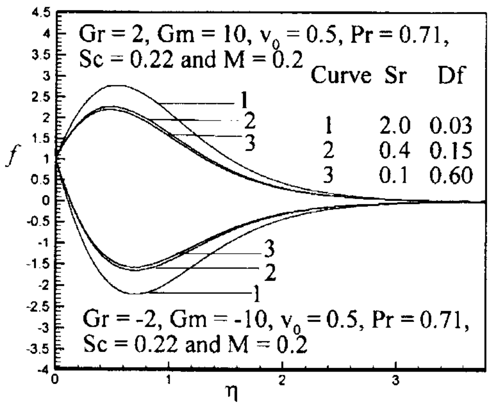

Fig. 2 Velocity profiles for different values of Sr and Df.

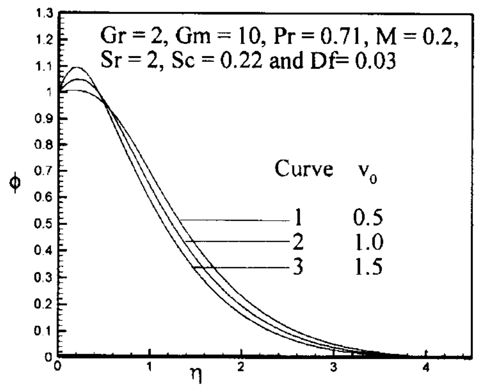

Fig. 5 Concentration profiles for different values of v_0.

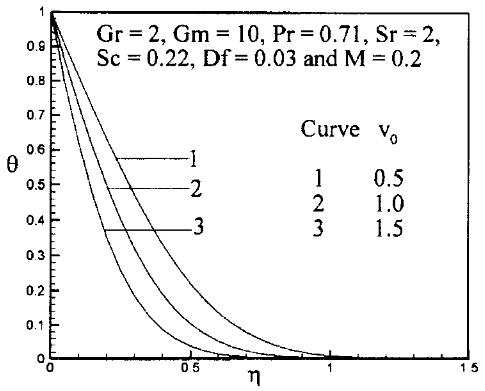

Fig. 3 Temperature profiles for different values of v_0.

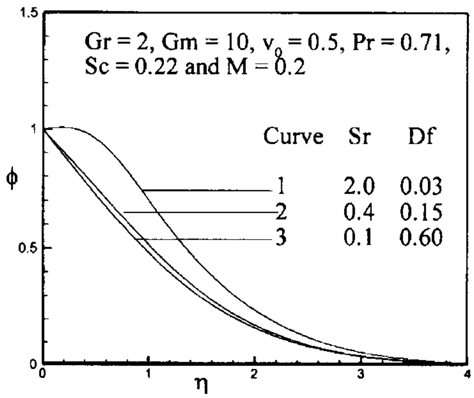

Fig. 6 Concentration profiles for different values of Sr and Df.

4. Results and Discussion

For the purpose of discussing the effects of various parameters on the flow behaviour near the plate, numerical calculations have been carried out for different arbitrary values of suction parameter v_0, magnetic field parameter M and for fixed values of Prandtl number Pr, Schmidt number Sc, Grashof number Gr and modified Grashof number Gm. The value of Prandtl number Pr is taken equal to 0.71 which corresponds physically to air. The value of Schmidt number Sc = 0.22 has been chosen to represent hydrogen at approx $Tm = 25^{\circ}$C and 1 atm. The values of Grashof number Gr and modified Grashof number Gm are taken to be both positive and negative, since these values represent respectively, cooling and heating of the plate. Finally, the values of Soret number Sr and Dufour number Df are chosen in such a way that their product is constant.

Under the above assumptions, the dimensionless velocity, temperature and concentration profiles are shown graphically in Figs. 1-6 for both cooling and heating of impulsively started plate. The effects of suction parameter Vo and magnetic field parameter M on the velocity field are shown in Fig. 1 for both cooling and heating of the plate.

It can be seen that for cooling of the plate ($Gr,Gm>0$), the velocity profiles decrease monotonically with the increase of suction parameter indicating the usual fact that suction stabilizes the boundary layer growth. By sucking the slowed boundary layer material into the inside of the body through narrow slits on the wall boundary layer separation can be prevented. For cooling of the plate and for fixed suction velocity (v_0), velocity is found to increase and reaches a maximum value in a region close to the surface of the plate, then gradually decreases to zero. From this figure we also see that as the magnetic field parameter increases, the velocity decreases. This indicates that the magnetic field retards the fluid motion. The effects of Soret and Dufour numbers on the velocity field for cooling and heating of the plate are shown in Fig. 2. We observe that for cooling of the plate, quantitatively when η = 0.5 and as Sr decreases from 2 to 0.4 (or Df increases from 0.03 to 0.15), there is a 22.47% decrease in the velocity value, whereas the corresponding decrease is 3.25%, when Sr decreases from 0.4 to 0.1. In all the figures mentioned above, compared to the case of the cooling of the plate, opposite effects are observed in the case of the heating of the plate.

Since the energy and concentration equations are independent of all parameters except the suction parameter, Dufour number and Soret number, the temperature and concentration profiles are shown only for v_0, Df and Sr.

The temperature profiles are shown in Figs. 3 and 4 for cooling of the plate. From Fig. 3 we see that the temperature decreases with the increase of suction parameter. From Fig. 4, when 17 = 0.5 and Sr decreases from 2 to 0.4 (or Df increases from 0.03 to 0.15), there is a 50.82% increase in the temperature value, whereas the corresponding increase is 15.42%, when Sr decreases from 0.4 to 0.1.

In Figures 5 and 6, the concentration profiles are shown for cooling of the plate. It is observed from Fig. 5 that the concentration increases with the increase of suction parameter close to the wall (approx. $\eta \leq 0.60$), whereas for $\eta \geq 0.60$, the concentration decreases with increase of suction parameter. In Fig. 6, the effects of Soret and Dufour numbers on the concentration profiles are shown. It is seen from this figure that for 1] = 1 and as Sr decreases from 2 to 0.4 (or Df increases from 0.03 to 0.15), there is a 35.96% decrease in the concentration value, whereas the corresponding decrease is 7.09% when Sr decreases from 0.4 to 0.1.

Table-l Numerical values ofskin-friction coefficient, Nusselt number and Sherwood number for $Pr = 0.71$, $Sr = 2.0$, $Df = 0.03$ and $Sc = 0.22$.

Gr	GM	M	v_0	C_f	Nu	Sh
-2	-10	0.2	0.5	-10.227144	1.940148	-0.082728
-2	-10	0.2	1.0	-10.793929	2.944274	-0.452290
-2	-10	0.2	1.5	-11.256771	4.445256	-1.040100
-2	-10	1.0	1.5	-10.518761	4.445252	-1.040102
-2	-10	3.0	1.5	-9.395594	4.445239	-1.040107
+2	+10	0.2	0.5	7.156127	1.940148	-0.082728
+2	+10	0.2	1.0	6.998597	2.944274	-0.452290
+2	+10	0.2	1.5	6.686176	4.445256	-1.040100
+2	+10	1.0	1.5	5.505187	4.445252	-1.040102

+2	+10	3.0	1.5	3.435226	4.445239	-1.040107

Table-2 Numerical values of skin-friction coefficient, Nusselt number and Sherwood number for Pr = 0.71, v_o = 0.5, M= 0.2 and Sc = 0.22.

Gr	*GM*	*Sr*	*Df*	C_f	*Nu*	*Sh*
+2	+10	2.0	0.03	7.205083	1.934014	0.087042
+2	+10	0.4	0.15	5.775135	1.517723	0.495844
+2	+10	0.1	0.60	5.58190	1.364413	0.575167
-2	-10	2.0	0.03	-10.2761	1.934014	0.087042
-2	-10	0.4	0.15	-8.8461	1.517723	0.495844
-2	-10	0.1	0.60	-8.6529	1.364413	0.575167

Finally, the effects of various parameters on *Cj, Nu* and *Sh* are shown in Tables 1 and 2. The conclusions and discussion regarding the behaviour of the parameters on skin-friction coefficient, local Nusselt number and local Sherwood number are self evident from the tables and hence are not discussed for brevity.

Nomenclature:

B_0 = applied magnetic field
C = concentration
c_p = specific heat at constant pressure
c_s = concentration susceptibility
Df= Dufour number
D_m = mass diffusivity
f = dimensionless velocity
g = acceleration due to gravity
Gr = local Grashof number
Gm = local modified Grashof number
K_T = thermal diffusion ratio
M = magnetic field parameter
M_w = mass flux
Nu = Nusselt number
Pr = Prandtl number
q_w = heat flux
Sc = Schmidt number
Sh = Sherwood number
Sr = Soret number
T = temperature
T_m = mean fluid temperature
U_o = constant plate velocity
u, v = velocity components in the x- and y-direction respectively
x, y = Cartesian coordinates along the plate and normal to it

Greek Symbols:

a = thermal diffusivity
β = coefficient of thermal expansion
β^* = coefficient of concentration expansion
σ = electrical conductivity
ρ = density of the fluid
ν = kinematic viscosity
θ =dimensionless temperature
ϕ = dimensionless concentration
δ = time dependent length scale
τ_w = wall shear stress

Subscripts:

w = condition at wall
∞ = condition at infinity

Superscript:

$'$ differentiation with respect to η

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