Flood Frequency Analysis at Different Rivers in Bangladesh: A Comparison Study on Probability Distribution Functions

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Abstract

This paper is directed to compare probability distribution functions for the study on flood frequency analysis at different rivers in Bangladesh. To analyze this issue we use 5 sets of data of annual maximum runoff of different main rivers in Bangladesh. We compare three widely used distributions. These are: (1) Log Normal (Two parameters, LN2 and three parameters, LN3); (2) Extreme value Type-1 (EV1) or Grumbel and (3) Log-person type-3 (LP3) distributions. The parameters of the distributions have been estimated by using the method of moments and method of maximum likelihood.

Keywords: Flood frequency, Probability distribution, Method of moments, Method of maximum likelihood.

1. Introduction

To evaluate flood (Discharge or water level) as well as rainfall frequency of given returns periods, it is essential that one probability distribution function be used as a standard. In many countries, one distribution function is used as a standard, but in Bangladesh various frequency distribution functions are in use. Many researchers in the USA, studied several distribution functions for flood flows [1,2].

Gumbel [3] published a large number of papers on the application of Fisher - Tippet theory of extreme values to flood frequency analysis. Later, many other hydrologists worked on the extreme value theory for flood frequency analysis [4,5,6,7].

The frequency analysis of discharge data of West Bengal rivers by means of graphical procedures using flow duration curves has been studied by Roy [8]. Cunnane [9] discussed various issues related to probability distribution functions for flood frequency analysis. In the various rivers discharge data in Bangladesh, a Flood Hydrology study [10] and Chowdhary and Karim [11] have used different probability distributions and suggested that the LP3 distributions are suitable for Bangladesh for the frequency analysis of discharge data. The length of data record varies from 16 to 24 years.

The objective of this study is to compare the probability distribution functions for the application of flood frequency analysis considering annul maximum runoff at different rivers in Bangladesh. This study, using 5 sets of annual maximum runoff data in different main rivers in Bangladesh, has compared three widely used distributions.

2. Methodology

2.1 Probability distribution functions used:

Probability distribution functions of discrete and continuos random variables are used to fit distributions in hydrology. There are many distributions that are found useful for hydrological frequency analysis. The Bangladesh water development board which designs and implements all large-scale flood control projects, uses the Gumbel distribution. A few departments and consulting firms use the log normal (LN) distribution. The log-Pearson type-3 (LP3) distribution has been used in the preparation of a national water plan. Three widely used probability functions were compared in this study. These three probability distribution function and the parameters involved in each function are given below.

Log-normal distribution (LN)

The probability density function of this distribution in the case of three parameters (LN3) is

$$f(x)=\frac{1}{(x-\theta)\sigma_{y\sqrt{2\pi}}}\exp\left[\frac{-\{\ln(x-\theta)-\mu_y\}^2}{2\sigma_y^2}\right], x\succ\theta \quad (1)$$

Where μ_y and σ_y are the mean and standard deviation of the natural logarithms of x and θ is a parameter.

The probability density function of this distribution in the case of two parameters (LN2):

$$f(x)=\frac{1}{x\sigma_{y\sqrt{2\pi}}}\exp\left[\frac{-\{\ln x-\mu_y\}^2}{2\sigma_y^2}\right] \quad (2)$$

Where σ_y and μ_y are parameters stated above.

Extreme Value Type-1 or Grumbel Distribution (EV1)

The probability density function of this distribution is:

$$f(x)=\exp\left[-\exp\left(-\frac{x-\zeta}{\alpha}\right)\right] \quad (3)$$

Where $f(x)$ s the non-excedence probability for the value of x, ξ is a location parameter and α is a scale parameter.

Log Pearson Type-3 Distribution (LP3)

The probability density function of Pearson Type-3 distribution is:

$$f(x)=\frac{1}{\alpha\Gamma(\beta)}\left[\frac{x-\upsilon}{\alpha}\right]^{(\beta-1)}\exp\left[-\frac{x-\upsilon}{\alpha}\right] \quad (4)$$

Where α, β and υ are the shape, scale and location parameters to be estimated from the sample and $\Gamma(\beta)$ is the gamma function.

If the logarithm, in of a variable x are distributed as a Pearson Type-3 variable, then the variable x will be distributed as a Log Pearson Type-3 with probability density function:

$$f(x)=\frac{1}{\alpha\Gamma(\beta)}\left[\frac{\ln x-\upsilon}{\alpha}\right]^{(\beta-1)}\exp\left[-\frac{x-\upsilon}{\alpha}\right] \quad (5)$$

Where α, β and Γ are the parameters as before.

2.2 Method of estimating distribution function parameters

The estimation methods techniques are used for estimating various parameters from sample values in such a way that they depart from the population parameters to a minimum. For estimating the parameters from the sample of data, the method of moments (MM) and the method of maximum likelihood (MML) have been used in this study.

2.3 Data Used in this analysis

The annual maximum discharge data have been used in this study. The data have been acquired from the Bangladesh water development board and Institute of flood control and drainage research. For the computation of statistical probability distribution, a total number of 5 sets discharge data and 8 sets of rainfall data have been selected for this study, on various types of rivers and places in Bangladesh. The discharge data record covers up to the year 2000 and rainfall data covers up to 2000 in some cases as given in Table-1. The length of annual maximum discharge data lies between 25 to 63 years. Though there are some breaks in the period of observation, the data are assumed to be continuous in this study.

Table-1: Discharge data (5 sets) of 5 different rivers in Bangladesh.

Serial number	Station and River	Period of Record	Number of years
1	273 Bhairab Bazar (Meghna)	1964 to 2000	29
2	46-9L Bahadurabad (Brahmaputra)	1956 to 1992	36
3	91-9L Baruria (Ganges)	1966 to 1992	25
4	90 Hardinage Bridge (Ganges)	1964 to 2000	63
5	229-5LMyme-nsingh(Old Brahmaputra)	1965 to 1992	26

3. Results and discussion:

For computations of the flood frequency analysis by the distributions LN2, LN3, EV1 and LP3, a computer program in Fortran of these distributions has been developed and then is used for this study. The correctness of these programs have been verified through the examples. The application of the above mentioned distributions have been used for the estimations of T year's events (2, 5, 10, 20, 50 and 100 years) through the method of moments and the method of maximum likelihood. All results are available in the Appendices (in Appendix Al: Annual maximum discharge data of 5 sets in tabular form are given. In Appendix A2: Results of Discharge data are shown).

Conclusion

We have studied the probability distribution functions for discussing the flood frequency at different rivers in Bangladesh. From these distributions it was found that the LP3 distribution gives butter average results than the other distributions. For further investigation, one may use a larger number of data for better understanding.

3. Reference

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Appendix-A1 (I) Detailed discharge data, which have been used in this study, are given in the following tables.

Table No-1 Annual maximum discharges (cumec) with year. Period of record used from 1961 to 2000, Number of years 29, Station Bhairab Bazar (273). River: Meghna. Designation of data set is D1.

Year	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970
Maxi Discharge	13141	7590	9487	12300	12100	14400	12700	13300	11500	16400
Year	1972	1973	1974	1975	1976	1981	1982	1983	1984	1985
Maxi Discharge	11500	12400	19500	12700	16700	11200	13500	16000	13600	14300
Year	1986	1987	1988	1989	1990	1991	1992	1993	2000	
Maxi Discharge	11100	15200	19800	15500	11700	14500	12800	19900	12394	

Table No-2 Annual maximum discharges (cumec) with year. Period of record used from 1956 to 1992, Number of years = 36, Station: Baruria (46-9L). River: Brahmaputra. Designation of data set is D2.

Year	1956	1957	1958	1959	1960	1961	1962	1963	1964	1965
Maxi Discharge	60400	65500	71300	68500	64800	53800	59400	56400	63100	64200
Year	1966	1967	1968	1969	1970	1972	1973	1974	1975	1976
Maxi Discharge	68900	69600	62300	56000	75000	66600	67300	91100	52200	65600
Year	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986
Maxi Discharge	66600	56600	66100	61200	66500	55900	56500	77000	63800	43100
Year	1987	1988	1989	1990	1991	1992				
Maxi Discharge	74000	98600	71100	64400	84100	67500				

Table No-3 Annual maximum discharges (cumec) with year. Period of record used from 1966 to 1992, Number of years= 25, Station Baruria (91-9L). River: Ganges. Designation in data set is D3.

Year	1966	1967	1968	1969	1970	1971	1973	1974	1975	1976
Maxi Discharge	81300	63600	80200	72700	84200	76600	90900	113000	93300	83500
Year	1977	1978	1980	1981	1982	1983	1984	1985	1986	1987
Maxi Discharge	81800	80400	109000	88200	89600	101000	107000	90500	81500	113000
Year	1988	1989	1990	1991	1992					
Maxi Discharge	132000	80000	83800	100000	726000					

Table No-4 Annual maximum discharges (cumec) with year. Period of record: 1934-35 to 1995-96 except 1971-72, 96-2000.Number of years = 63,Station No = 90, Station: Harding Bridge, River = Ganges, Designation of data set is D4

Year	1934-35	1935-36	1936-37	1937-38	1938-39	1939-40	1940-41	1941-42	1942-43
Maxi Discharge	46600	44000	45300	39400	47800	35900	39100	38300	44700
Year	1943-44	1944-45	1945-46	1946-47	1947-48	1948-49	1949-50	1950-51	1951-52
Maxi Discharge	43300	43300	42200	49100	51200	61100	52600	52600	42200
Year	1952-53	1953-54	1954-55	1955-56	1956-57	1957-58	1958-59	1959-60	1960-61
Maxi Discharge	52600	50900	58600	60300	60100	46200	56200	52700	48000
Year	1961-62	1962-63	1963-64	1964-65	1965-66	1966-67	1967-68	1968-69	1969-70
Maxi Discharge	73200	58700	56100	49000	36800	41900	50800	45200	55200
Year	1970-71	1971-73	1973-75	1973-74	1974-76	1977-77	1976-78	1977-79	1978-80
Maxi Discharge	48700	38200	50700	50700	51100	65400	51100	67900	36900
Year	1980-81	1981-82	1982-83	1983-84	1984-85	1985-86	1986-87	1987-88	1988-89
Maxi Discharge	57600	47900	61600	60000	56500	50600	53500	76000	72300
Year	1989-90	1990-91	1991-92	1992-93	1993-94	1994-95	1995-96	1999	2000
Maxi Discharge	31600	51300	56000	41900	44800	46100	49100	55019	60952

Table No-5 Annual maximum discharges (cumec) with year. Period of record used from 1964 to1992, Number of years 26, Station Baruria (228-5) River: Old Brahmaputra. Designation of data set is D5.

Year	1964	1965	1966	1967	1968	1969	1970	1974	1975	1976
Maxi Discharge	2830	3230	3490	3000	2900	2770	3250	3820	3060	3210
Year	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986
Maxi Discharge	3550	2770	2630	3340	2690	2470	2370	4780	3070	1930
Year	1987	1988	1989	1990	1991	1992				
Maxi Discharge	3230	4910	2180	2060	2900	1490				

Appendix-A2 (I) Results and figures of discharge data

Table-1 Comparison of flood frequency results using data D1 obtained by different methods in various return periods.

Return periods (Years Distribution)	2	5	10	20	50	100
EV1 (cumec)						
MM: X_T	13584	16101	17767	19366	21435	22985
MML: X_T	13519	15456	16739	17970	19563	20757
LN2 (cumec)						
MM and MML X_T	13742	15878	17125	18227	19553	20490
LN3 (cumec)						
MM: X_T	13584	15741	17120	18411	20053	21271
MML: X_T	8751	10776	11921	12914	14086	14901
LP3 (cumec)						
MM (Direct): X_T	13599	16598	17039	18300	19914	21121
MML (Indirect) X_T	13487	15641	17111	18556	21985	21985

Table-2 Comparison of flood frequency results using data D2 obtained by different methods in various return periods.

Return periods (Years Distribution)	2	5	10	20	50	100
EV1 (cumec)						
MM: X_T	64555	75023	81954	88603	97208	103657
MML: X_T	64760	75150	82029	88627	97168	103569
LN2 (cumec)						
MM and MML X_T	65357	74585	79918	84607	90214	94158
LN3 (cumec)						
MM: X_T	64715	740064	79939	85388	92247	97293
MML: X_T	60607	69873	75416	80397	86480	90837
LP3 (cumec)						
MM (Direct): X_T	64762	73929	79694	85059	91852	96882
MML (Indirect) X_T	64791	74158	80024	85469	92344	97423

Table-3 Comparison of flood frequency results using data D3 obtained by different methods in various return periods.

Return periods (Years Distribution)	2	5	10	20	50	100
EV1 (cumec)						
MM: X_T	87646	103796	114489	124746	138022	147971
MML: X_T	87477	101229	110334	119068	130373	138844
LN2 (cumec)						
MM and MML X_T	88674	102446	110479	117583	126128	132166
LN3 (cumec)						
MM: X_T	87933	101846	110505	118486	128473	135783
MML: X_T	85637	99731	108856	117481	128543	136813
LP3 (cumec)						
MM (Direct): X_T	88014	101568	110012	117823	127655	134897
MML (Indirect) X_T	87512	101477	110589	119282	130561	139104

Table-4 Comparison of flood frequency results using data D4 obtained by different methods in various return periods.

Return periods (Years Distribution)	2	5	10	20	50	100
EV1 (cumec)						
MM: X_T	49350	58513	64581	70401	77934	83579
MML: X_T	49380	58728	64916	70852	78536	84294
LN2 (cumec)						
MM and MML X_T	49984	58395	63342	67741	70358	76833
LN3 (cumec)						
MM: X_T	50081	58462	63324	67611	72751	76373
MML: X_T	45914	54462	59580	64183	69809	73841
LP3 (cumec)	-				-	-
MM (Direct): X_T		Does	Not	Exit		
MML (Indirect) X_T	50146	58484	63275	67467	72451	75935

Table-5 Comparison of flood frequency results using data D5 obtained by different methods in various return periods.

Return periods (Years Distribution)	2	5	10	20	50	100
EV1 (cumec)						
MM: X_T	2883	3668	4188	4686	5332	5815
MML: X_T	2890	3645	4146	4626	5247	5712
LN2 (cumec)						
MM and MML X_T	2906	3584	4000	4379	4849	5191
LN3 (cumec)						
MM: X_T	2918	3594	4000	4364	4810	5130
MML: X_T	2887	3555	3944	4287	4700	4991
LP3 (cumec)	-				-	-
MM (Direct): X_T		Does	Not	Exist		
MML (Indirect) X_T	2965	3619	3975	4272	4608	4831

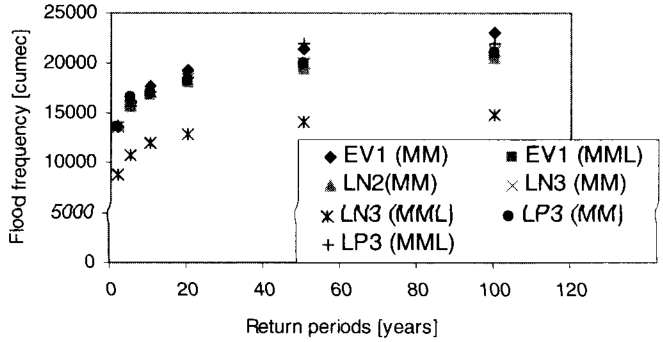

Figure-1 Comparison of flood frequency using the result given in Table-1

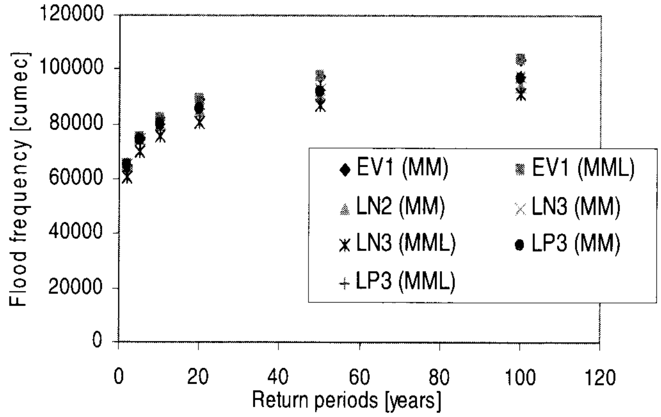

Figure-2 Comparison of flood frequency using the result given in Table-2

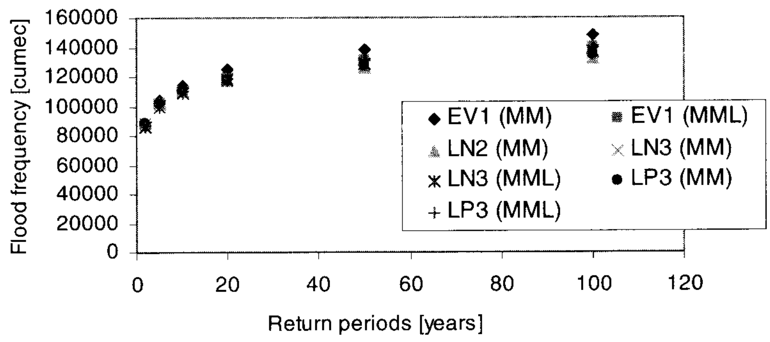

Figure-3 Comparison of flood frequency using the result given in Table-3

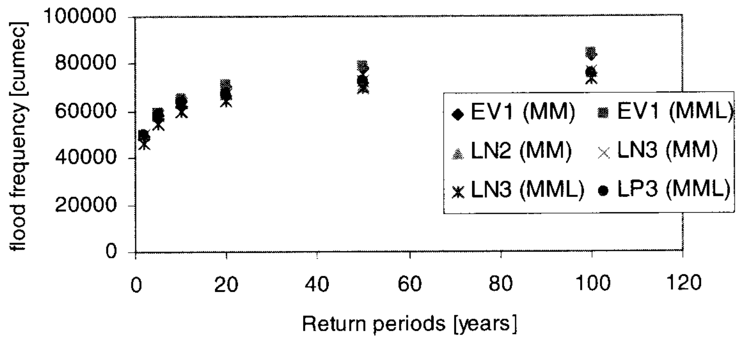

Figure-4 Comparison of flood frequency using the result given in Table-4

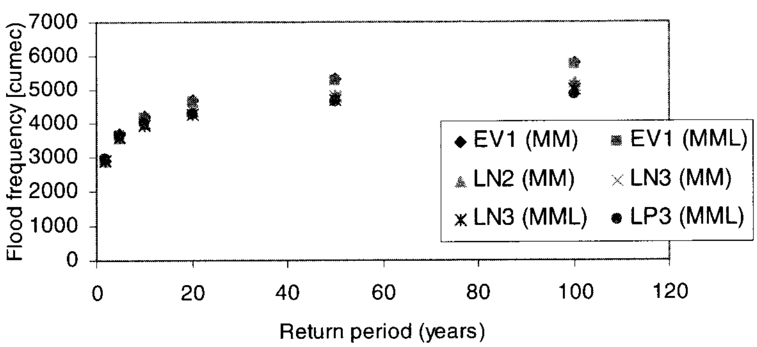

Figure-5 Comparison of flood frequency using the result given in Table-5