Inventory/Distribution Plan in a One-Warehouse/Multi-Retailer Supply Chain

Chumpol Monthatipkul and Pisal Yenradee
Industrial Engineering Program, Sirindhorn International Institute of Technology
Thammasat University, Pathum Thani 12121, Thailand
Tel. 662-986-9009 Ext.2107, Fax 662-986-9112, E-mail: pisal@siit.tu.ac.th

Abstract

This paper proposes a new inventory control system called the optimal inventory/distribution plan (IDP) control system for a one-warehouse/multi-retailer supply chain. The IDP control system includes three major components, namely, a linear programming model, an adjustment rule, and a rationing rule. Implementing the IDP control system begins with solving the proposed linear programming model and then following the obtained optimal inventory/distribution plan by adopting the adjustment and rationing rules. The efficiency of the IDP control system is compared to that of the traditional installation-stock s,Q system under two uncertain demand patterns. The experimental results show that the IDP control system gives lower total cost with higher fill rates than the traditional installation-stock s,Q system for the two demand patterns.

Keywords: One-Warehouse/Multi-Retailer, Linear programming, Installation-stock, and Optimal inventory/distribution plan

1. Introduction

Recently, inventory management in a supply chain has received much more attention. In this case, inventory control systems, supply chain structures, lead times, coordination, and information sharing processes are important factors, especially when the supply chain faces stochastic parameters. The Bullwhip Effect [1] makes inventory control problems in a supply chain more complicated. Most traditional inventory control systems (s,Q; s,S; R,S; R,Q; and R,s,S, etc. where the parameters s, Q, S, R-denotes reorder points, reorder quantities, order-up-to levels, and periodic review periods, respectively—are constant) are still used in the supply chain environment. However, determining the control parameters for the supply chain environment is more complicated than determining those for a single firm. Considering all members of the supply chain, rather than a single facility, is an approach to obtain better control parameters and lower inventory costs. Besides the traditional inventory control systems, new systems which can be used as inventory control systems are studied by some researchers. Examples of the new systems are the DRP system of Ganeshan et al. [2] and a SMART s,S system of Giannoccaro and Pontrandolfo [3].

Inventory management systems of supply chains can be categorized into two broad classes, namely, installation-stock and echelon-stock inventory control systems. The former refers to a situation, in which each entity makes a decision based on its own current inventory status. In contrast, the latter considers its own inventory as well as all downstream and pipeline inventories. The echelon-stock inventory control system can be implemented only if each member shares its own information to upstream members. Details of the two classes can be seen in Axsäter and Rosling [4]. The structure of supply chains can be classified into two types, namely, a serial supply chain and a non-serial supply chain. The serial supply chain is a network, which consists of several stages and each stage contains only one entity. The non-serial supply chain refers to a multi-stage/multi-entity network. If a non-serial supply chain is considered, rationing rules must be taken into account when the upstream members have insufficient inventories to replenish all demands of several downstream members.

A number of research works try to determine optimal parameters of traditional inventory control systems that minimize total related costs using different solution techniques. The related costs include ordering costs, holding costs, and shortage costs of all supply chain members as well as some additional important costs, for example, transportation and in-transit holding costs. Ganeshan [5] proposed a non-linear programming approach to determine parameters of the installation-stock s,Q system. The author considered total logistic costs and adopted the *Newton* or the conjugate gradient method to search reorder points and reorder quantities of a warehouse and retailers simultaneously. Yoo et al. [6] proposed two effective order planning methods based on the installation-stock s,Q and R,Q systems for a one-central distribution center/N-regional distribution center network. They considered backlogging, holding, and ordering costs of all supply chain members.

Abdul-Jalbar et al. [7] studied installation-stock inventory control systems of a one-warehouse/N-retailer network under both centralized and decentralized policies. The authors obtain inventory control parameters (replenishment times and reorder quantities of a warehouse and retailers) by solving various types of mathematical models concerning holding and ordering costs. Axsäter [8] studied the installation-stock s,Q system of a one-warehouse/N-retailer network and then proposed a simple technique for approximate optimization of reorder points. The model considers holding costs at all locations and backorder costs at retailers.

Ryu and Lee [9] considered the installation-stock s,Q system in a dual-sourcing model with stochastic lead times and constant demands. The authors determine order quantities and reorder points by solving a mathematical model concerning ordering costs, holding costs, and shortage costs. Yokoyama [10] addressed a Multi-DC/Multi-Retailer model controlled by the installation-stock R,S system. The target inventory and the transportation amount are determined so as to minimize the sum of transportation, holding, and shortage costs.

Tagaras [11] studied the installation-stock R,S system in a one-warehouse/N-retailer network. Order-up-to quantities are obtained by solving a mathematical model concerning holding, shortage, and transshipment costs if transshipment between retailers is allowed. Heijden [12] dealt with the optimization of stock levels in general divergent networks under the echelon-stock R,S system. The goal is to attain target fill rates and minimize total holding costs of the entire networks.

In addition to determine optimal inventory parameters, some research works study additional policies extended from traditional inventory control policies. Banerjee et al. [13] studied partial shipment policies in a one-manufacturer/multi-buyer supply chain. The buyers replenish their inventories according to the installation-stock s,Q system and the manufacturer produces products at every fixed interval with predetermined production quantities. The authors show that partial shipment policies increase customer service levels significantly. Solving for parameters of the installation-stock R,S system, Tagaras [11] also identified benefits of lateral transshipments. Tagaras [11] demonstrated that customer service is increased and total system costs are decreased, if transportation between retailers is allowed in cases of unexpected shortages.

Some research works focus on improving traditional inventory control systems, and some of them also compare their improved systems to a traditional inventory control system. Yoo et al. [6] compared improved installation-stock s,Q and R,Q systems to the traditional Distribution Resource Planning (DRP). In the improved systems, regional distribution centers can make a decision to reduce the related costs-order only the amount available or postpone the ordering. The authors performed numerical simulations and concluded that both improved systems yield total costs lower than that of the traditional DRP method. Ganeshan et al. [2] studied two inventory control systems, namely, DRP and Reorder Point systems in a four-echelon network. The DRP system refers to a calculation of upstream reorder quantities and reorder intervals by aggregating all downstream demands and offsetting them by related lead times. The Reorder Point system refers to a situation that manufacturers forecast needs at the distribution centers. The authors concluded that the DRP system gives higher service levels and lower cycle times.

Giannoccaro and Pontrandolfo [3] proposed an s,S system where s and S vary with states (SMART s,S system) for a three-stage serial supply chain. Their problem is formulated as a SMDP (Semi-Markov decision process) model and then solved by SMART (Semi-Markov average reward technique) algorithm. The authors compared the SMART s,S system to the echelon-stock R,S system and then concluded that the SMART s,S system gives lower total costs and is more robust as long as demand undergoes only slight changes. Wang et al. [14] proposed just-in-time distribution requirements planning (JIT-DRP) which aims to pull material through a multi-warehouse/multi-retailer supply chain effectively. The JIT-DRP gives optimal solutions within deterministic conditions.

In this paper, a new inventory control system called the optimal inventory/distribution plan (IDP) control system is proposed. The proposed system is to manage inventories in a one-warehouse/multi-retailer supply chain under demand uncertainty. An important aspect that distinguishes this paper from most research found in the literature is that the IDP control system controls each supply chain member by the optimal inventory/distribution plan (IDP) rather than using or improving traditional inventory control systems. The IDP is obtained by using linear programming. The objective function is to minimize the sum of ordering, holding, in-transit holding, transportation, and lost-sale costs. The performance of the proposed IDP control system is compared to that of a traditional control system, namely, the installation-stock s,Q system.

2. Problem description

A supply chain under consideration, which is modified from interesting supply chains in Thailand, for example, dried food, beverage, consumable product, and paint supply chains, consists of one warehouse and multiple retailers. The retailers face uncertain demand of a single type of product and need some safety stocks to protect against a shortage problem. Unsatisfied demand is considered as a lost sale. The retailers replenish their inventories from the warehouse, which, in turn, replenishes its inventory from a vendor/manufacturer outside the concerned supply chain. The warehouse might face uncertain orders from the retailers and also needs some safety stocks. It is assumed that all storage and transportation capacities are unlimited. Lateral transshipments between retailers are not allowed. The related costs are ordering, holding, in-transit holding, and transportation costs.

The supply chain is controlled by the inventory/distribution plan or the installation-stock s,Q system. In this paper, the control parameters of the installation-stock s,Q system (reorder points s and reorder quantities Q) are determined based on a model of Ganeshan [5]. For a non-serial supply chain in this paper, material rationing in case of shortages is an important issue. If the warehouse has insufficient inventory to fulfill all retailers' orders, all order quantities should be adjusted according to a rationing rule. In this paper, the rationing rule tries to equalize the fill rates of the supply chain members that demand the products or materials. After adopting the rationing rule, the actual amount to be shipped is told to each retailer immediately so that each retailer can calculate its actual inventory position which is the sum of on-hand inventory plus the actual amount to be received.

3. The installation-stock s,Q system

The installation-stock s,Q system operates as follows. When the inventory position (on-hand inventory + outstanding orders) is at or below reorder point s, a reorder quantity Q is placed, in order to increase the inventory position. Since the supply chain consists of one-warehouse and multiple identical retailers, there are four important control parameters, namely, warehouse's reorder point s_w, retailer's reorder point s_r, warehouse's reorder quantity Q_w, and retailer's reorder quantity Q_r. The control parameters s_w, s_r, Q_w, and Q_r in this paper are determined by solving the model of Ganeshan [5]. The model of Ganeshan [5] with few modifications is briefly described below:

Notation:

Decision variables

s_w reorder point at the warehouse
s_r reorder point at the retailers
Q_w order quantity at the warehouse
Q_r order quantity at the retailers

Parameters

N_r number of retailers
n number of vendors

A_w ordering cost at the warehouse
A_r ordering cost at the retailers
μ_w mean daily demand at warehouse
μ_r mean daily demand at each retailer
v product price
h holding cost fraction for a period
Y_w mean demand during lead time at warehouse
Y_r mean demand during lead time at a retailer
L_w constant lead time for the warehouse
L_r constant lead time for each retailer
g_w unit transportation cost of the warehouse
g_r unit transportation cost of each retailer
ρ_w order fill rate at the warehouse
ρ_r order fill rate at each retailer
k a positive integer number.
ES_r expected shortage units per replenishment cycle at each retailer
ES_w expected retailer's orders with shortage per replenishment cycle at the warehouse.

The s,Q model:

Objective function

$$\text{Min EADC} = A_w \mu_w / Q_w + \mu_r N_r A_r / Q_r + (L_w \mu_w + Q_w/2 + s_w - Y_w) vh + (L_r \mu_r + Q_r/2 + s_r - Y_r) vh N_r + g_w \mu_w + g_r N_r \mu_r \quad (1)$$

Constraints

$$ES_w \leq Q_w (1 - \rho_w) / Q_r \quad (2)$$
$$ES_r \leq Q_r (1 - \rho_r) \quad (3)$$
$$Q_w = k Q_r \quad (4)$$
$$Q_w \geq \mu_w \quad (5)$$
$$s_r, s_w \geq 1 \quad (6)$$

The objective function is to minimize the expected average daily costs (EADC), which comprise the total ordering cost, the total holding costs, and the total transportation costs. The 1st and 2nd terms in equation 1 are ordering costs at the warehouse and at the retailers, respectively. The terms $L_l \mu_l$, $Q_l/2$, and $s_l - Y_l$, where $l = w, r$ in equation 1 represent the in-transit, cycle stock, and safety stock, at the warehouse and each of the retailers, respectively. The 5th and 6th terms in equation 1 are transportation costs at the warehouse and at the retailers, respectively. Formulas 2 and 3 define relationships between the expected shortage measures and the order fill rates. Formula 2 states that ES_w is not greater than the target number of retailer's orders that are subject to shortage per replenishment cycle. Also, formula 3 means that ES_r is not greater than the target number of shortage units per replenishment cycle. Equation 4 assumes that Q_w is an integer multiple of Q_r to reduce computational time for solving the problem. Formula 5 assures that the warehouse does not place more than one order (on average) in each day, which is common in practice.

A few modifications to the model of Ganeshan [5] are summarized as follows. An experimental case in this paper has one vendor (n =1) rather than multiple vendors. All lead times are deterministic rather than stochastic and the transportation cost parameters g_w and g_r are constants rather than functions.

As shown in the model of Ganeshan [5], daily demand at each retailer is assumed to follow a Poisson distribution with a mean λ_r. Thus, ES_r can be determined by:

$$ES_r = \sum_{i=s_r}^{\infty} (i - s_r) e^{-\lambda_r L_r} (\lambda_r L_r)^i / i! \quad (7)$$

Similarly, ES_w can be determined by:

$$ES_w = \sum_{i=s_w}^{\infty} (i - s_w) e^{-\lambda_w L_w} (\lambda_w L_w)^i / i! \quad (8)$$

where λ_w is estimated by $N_r \lambda_r / Q_r$ when $N_r \geq 20$ (see Ganeshan [5] page 346). Note that λ_w represents retailers' orders with order size Q_r arriving according to a Poisson process.

In this paper, two types of stochastic demand, namely, Demand Type I and Demand Type II, are considered. Demand Type I involves a situation in which the demand at the retailers, which is slightly uncertain, has a constant mean during the year. In this case, there is a single value of μ_r. Demand Type II involves a situation in which the mean immediately changes from a low level to a high level for a certain interval and immediately returns to the low level again. This demand behavior is not a rare case because some products, alcoholic beverages for instance, may have high demands on Fridays and Saturdays, but low demands on the other days. In this case, there is a high number for μ_r, denoted by μh_r, and a low number for μ_r, denoted by μl_r. This change can occur several times during the year. In the former situation, the decision variables s_w, s_r, Q_w, and Q_r are obtained by solving the model and used throughout the year. Unfortunately, the model of Ganeshan [5] does not consider Demand Type II. Therefore, in the latter situation, an adaptation of the model of Ganeshan [5] must be considered.

The adaptation of the model of Ganeshan [5] to the Demand Type II is based on the change of reorder point and reorder quantity from a low level to a high level according to the change of mean daily demand. This change must be done at an appropriate time so that the right amount of product is shipped to the demand points at the right time. The appropriate time to change the reorder point and reorder quantity from the low level to the high level is easily determined by lead-time offsetting as shown in Fig. 1. The s,Q model is solved for the low level demand and the obtained decision variables are used during the low level demand interval, denoted by $IV[t_1,t_2\text{-}L]$. Note that t_1 and t_2 represent the times at which the low level demand occurs and ends, respectively, and L represents the related lead time (L equals L_r for determining decision variables for retailers but equals L_w+ L_r for determining decision variables for the warehouse). For the high level demand interval, the decision variables are recalculated and used during the interval $IV[t_2\text{-}L,t_3\text{-}L]$, where t_3 represents the time at which the high level demand ends.

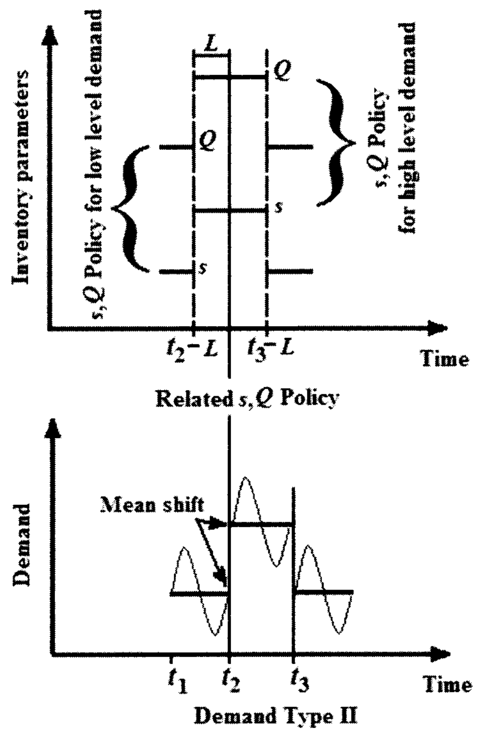

Fig 1. s,Q policies under Demand Type II

Considering the point at which the demand is changed from the low level to the high level (t_2), all retailers will have inventory positions below their reorder points. Thus, they will place the orders to the warehouse. The first order which the retailers place to the warehouse should not be equal to their high-level reorder quantities, because the retailers are placing orders while their inventory positions are not at high-level reorder points, but at low-level reorder points. Thus, the first order size should be equal to the high-level reorder quantity plus the high-level reorder point minus the current inventory positions as expressed in equation 9.

$$FOR(t_2) = HQ_r + HS_r - IPR(t_2) \qquad (9)$$

where $FOR(t_2)$ is the first order size that retailers place to the warehouse at time t_2, HQ_r is the high-level reorder quantity of retailers, HS_r is the high-level reorder point of retailers, and $IPR(t_2)$ is the current inventory positions of retailers at time t_2.

Similarly, the first order which the warehouse places to its vendor should not be equal to its own high-level reorder quantity, but should be equal to the retailers' high-level reorder quantity plus the retailers' high-level reorder point, multiplied by the number of the retailers in the chain, and finally, minus the warehouse current inventory position as shown in equation 10.

$$FOW(t_2) = (HQ_r + HS_r)N_r - IPW(t_2) \qquad (10)$$

where $FOW(t_2)$ is the first order size the warehouse places to a vendor at time t_2, and $IPW(t_2)$ is the current inventory positions of the warehouse at time t_2.

For example, suppose that a warehouse supplies a product to 20 identical retailers and the control parameters of the installation-stock s,Q system are shown in Table 1. The first order size which the retailers place to the warehouse $FOR(t_2)$ should be equal to 60+20-10=70. Similarly, the first order size which the warehouse places to the vendor $FOW(t_2)$ should be equal to (60+20) 20 -100 = 1,500.

Table 1. Example of parameters in the s,Q mode

Node	Level	Reorder Point	Reorder Quantity
Warehouse	High	200	600
	Low	100	300
Retailer	High	20	60
	Low	10	30

4. Inventory/Distribution Plan System

The inventory/distribution plan (IDP) system is composed of three components, namely, an LP model, a safety stock policy, and rules for adjusting the transportation quantities.

4.1. LP model

The inventory/distribution planning model aims to minimize the total costs including ordering, inventory holding, lost sales, and transportation costs, subject to the safety stock policy, and material balance constraints. The following additional notations are used in the model:

Indices

t period index (1 .. T)

i retailer index (1 .. N_r)

Decision variables

T_{twi} amount of product sent from the warehouse at the beginning of period t to retailer i

A_{tw} size of order that the warehouse places to its vendor at the beginning of period t

IB_{tw} beginning inventory at the warehouse at t

IE_{tw} ending inventory at the warehouse at t

IB_{ti} beginning inventory at retailer i at period t

IE_{ti} ending inventory at retailer i at period t

L_{ti} lost sales at retailer i at the end of period t

B_{tw} binary variable for placing an order by the warehouse at period t (B_{tw}= 1, if A_{tw} >0, otherwise B_{tw} = 0)

B_{ti} binary variable for placing an order by the retailer i at period t (B_{ti} = 1, if T_{twi} >0, otherwise B_{ti} = 0)

Parameters

M a large positive number

sl_{ti} unit lost-sale cost at retailer i at period t

k_1 safety stock parameter at the warehouse

k_2 safety stock parameter at retailers

σ_{tw} standard deviation of demand at the warehouse at period t

σ_{ti} standard deviation of demand at retailer i at period t

d_{ti} demand of product at retailer i in period t

Objective function

The objective function is to minimize the total costs including ordering, holding, holding in transit, transportation, and lost sale costs.

$$\text{Min } TC = \sum_t A_w B_{tw} + \sum_{i,t} A_r B_{ti} + vh(\sum_t IE_{tw}) + vh\sum_{i,t}\{(IB_{ti} + IE_{ti})/2\} + vh\sum_t A_{tw} + vh\sum_{i,t} T_{twi} + \sum_t g_w A_{tw} + \sum_{i,t} g_r T_{twi} + \sum_{i,t} sl_{ti} L_{ti} \quad (11)$$

In equation 11, the 1st term represents the ordering costs at the warehouse. The 2nd term represents the ordering costs at retailers. Note that the ordering costs at retailers are determined from the multiplication of ordering cost A_r and binary variable B_{ti}, which is a function of transportation amount T_{twi}. This is because the transportation from the warehouse to each retailer occurs after the retailer has placed orders to the warehouse. Thus, it can be considered that the ordering cost has been incurred when transportation from the warehouse to each retailer occurs. The 3rd and 4th terms are the holding costs at the warehouse and retailers, respectively. Note that the holding cost at the warehouse is determined based on ending inventory at each period. This is because inventory at the warehouse reduces at the beginning of a period and remains constant during the period. However, the holding cost at the retailers is determined from the average of beginning and ending inventory. This is because inventory at the retailers reduces according to the arrival of customers which may occur any time during the period. The 5th and 6th term is the holding cost in transit of product transported from an outside vendor and the warehouse. The 7th term represents the cost for transporting the product from the vendor to the warehouse. Note that the transportation amount is equal to the amount that the warehouse orders from the vendor. The 8th term represents the cost for transporting the product from the warehouse to retailers. The 9th term represents the lost sales cost.

Constraints

1. Safety stock policy constraints

Based on a basic inventory model [15], the safety stock level is set proportional to the standard deviation of demand during lead time. In this paper, the amount of inventory at each entity cannot be lower than the target safety stock level. The safety stock policies are expressed as:

$$IE_{tw} \geq k_1\sigma_{tw} \qquad \forall t > L_w \qquad (12)$$

$$IE_{ti} \geq k_2\sigma_{ti} \qquad \forall t > L_w + L_r \qquad (13)$$

The time index t in constraint 12 is greater than lead time L_w because it takes L_w for each shipment from the vendor to get the warehouse. Considering the beginning period (t=1) of each planning horizon, if the initial inventory level of the warehouse is lower than the level of policy, $k_1\sigma_{tw}$, it must take L_w to raise the warehouse inventory level to above $k_1\sigma_{tw}$. This means that the inventory level of the warehouse must be allowed to be lower than the policy $k_1\sigma_{tw}$ during the first L_w period of each planning horizon. Similarly, the inventory level of each retailer must also be allowed to be lower than the policy $k_2\sigma_{ti}$ during the first L_w+ L_r period of each planning horizon. The time index t in constraint 13 is L_w+ L_r because it must take L_w+ L_r to raise the inventory level of each retailer to above $k_2\sigma_{ti}$ if the warehouse has insufficient inventory to ship to retailers immediately.

2. Inventory balance constraints

The LP model assumes that the transportation starts at the beginning of period t and ends at the beginning of the period t plus a transportation lead time, which is an integer number of periods. Thus, the beginning inventory level of each stocking point must be equal to the end of the last period inventory level plus the total incoming quantity. The end of period inventory level must also be equal to the beginning inventory level minus the total outgoing quantity. Expressions are shown in equations 14 to 17.

$$IB_{tw} = IE_{(t-1)w} + A_{(t-L_w)w} \qquad \forall t \qquad (14)$$

$$IE_{tw} = IB_{tw} - \sum_i T_{twi} \qquad \forall t \qquad (15)$$

$$IB_{ti} = IE_{(t-1)i} + T_{(t-L_r)wi} \qquad \forall t, i \qquad (16)$$

$$IE_{ti} = IB_{ti} - (d_{ti} - L_{ti}) \qquad \forall t, i \qquad (17)$$

3. Fixed-charge constraints

Constraints 18 and 19 represent that the fixed ordering costs are always incurred when ordering or transportation occurs. The binary variables B_{tw} and B_{ti} must be equal to 1 if the ordering amount A_{tw} or transportation amount T_{twi} is not zero, respectively.

$$M\,B_{tw} \geq A_{tw} \qquad \forall t \qquad (18)$$

$$M\,B_{ti} \geq T_{twi} \qquad \forall t, i \qquad (19)$$

4. Non-Negativity and binary variables

Constraints 20 and 21 are to ensure that the decision variables must not be negative.

$$T_{twi}, IB_{ti}, IE_{ti}, L_{ti} \geq 0 \qquad \forall t, i \qquad (20)$$

$$A_{tw}, IB_{tw}, IE_{tw} \geq 0 \qquad \forall t \qquad (21)$$

B_{tw}, B_{ti} are binary.

The proposed model is a linear programming model and can be solved optimally. Solving the proposed model, the planner obtains the optimal inventory/distribution plan containing the optimal acquisition quantity, optimal transportation quantity, and optimal inventory level at each entity.

Since mean values of the stochastic demands are used to generate IDP (entering the parameter d_{ti} with the mean daily retailer demand μ_r for all retailers i and all periods t), one cannot conclude that following IDP results in the minimum total cost in practice. Practically, the real demand might be more or less than the mean value. This may result in a shortage or an excess amount of inventories. In this paper, an approach to solve this problem is to keep an appropriate amount of safety stock in the supply chain. The method to find the appropriate amount of safety stock is described in section 4.2. Also, the actual total cost is not equal to the total cost obtained from IDP. In this paper, the actual total cost is determined by a simulation experiment. Some rules for adjusting the transportation quantities presented in section 4.3 are needed to make the real inventory level close to the optimal inventory level (based on IDP) when the supply chain is subject to demand uncertainty.

4.2. Safety stock policy

The assumption for this study is that only two locations are available to keep the safety stock. Thus, the safety stock policy is denoted by (k_1, k_2), where k_1 and k_2 represent the safety stock parameters at the warehouse and retailers, respectively. Note that the values of k_1 and k_2 affect fill rates at both the warehouse and the retailers. Thus, the desired fill rates can be obtained by adjusting the values of k_1 and k_2. The standard deviations σ_{ti} and σ_{tw} must also be determined before calculating the required safety stock level according to the safety stock policy. In the case of Poisson-distributed demand, the standard deviations σ_{ti} and σ_{tw} are equal to $\sqrt{\lambda_r}$ and $\sqrt{N_r\lambda_r}$ at the associated time t, respectively.

4.3. Adjustment Rules

IDP is regenerated once every fixed-period re-planning time. Therefore, each supply chain member must adopt the latest plan until a new plan is generated. During each re-planning time, each member might face demand which is more or less than the mean value. Thus, the actual inventory may not conform to the optimal plan. In this case, the members of the supply chain must try to follow the plan as much as possible by adopting an adjustment rule.

A primary rule is needed to develop the adjustment rule. This assumes that the products can be shipped only if IDP allows us to do so. The product cannot be shipped in any period if IDP does not show positive transportation quantities. The adjustment rule is defined by formulas 22 and 23.

$$A\_T_{twi} = T_{twi} + IE_{(t\text{-}1)i} - A\_IE_{(t\text{-}1)i} \qquad (22)$$

$$A\_A_{tw} = A_{tw} + IE_{tw} - P\_IE_{tw} \qquad (23)$$

where $P\_IE_{tw} = A\_IE_{(t\text{-}1)w} + A\_A_{(t-L_w)w} - \sum_i A\_T_{twi}$

The notation $A\_$ represents the actual quantity of the related transportation, inventory, and acquisition. The P_IE_{tw} is the projected inventory level at the end of period t at the warehouse.

Based on the adjustment rule 22, the actual transportation quantity will be increased from the transportation quantity recommended by IDP if the actual inventory level ($A\_IE_{(t\text{-}1)i}$) is lower than the optimal inventory level ($IE_{(t\text{-}1)i}$), and vice versa. The inventory parameters in equation 22 have a time index of t-1 because they are the latest values that are available.

The adjustment rule 23 is similar to rule 22 except for the time index of inventory parameters, which is t, not t-1. The P_IE_{tw} is calculated at the beginning of period t when the A_T_{twi} is known-informed from the retailers. Then A_A_{tw} is calculated using the adjustment rule 23.

Based on the adjustment rule, the actual amount to be shipped or acquired in each period is determined. However, the actual amount to be shipped from the warehouse may be re-adjusted by the rationing rule (described in section 2) if the warehouse has insufficient inventory to supply the retailers.

5. Experimental case

The comparison between the IDP control system and the installation-stock s,Q system is illustrated through an experimental case. A supply chain under consideration comprises one warehouse and twenty identical retailers. All given data are shown in Table 2. Note that a daily period is used and there are seven days in a week. For each simulation run, the first week is a warm-up period. Note that based on a pilot run, a warm-up period of one week is sufficient. The related costs and fill rates are recorded from the second to fourth week. The number of replications is 100 times, since at this level the 95 % confidence interval of the means of fill rates and total costs do not exceed 5% of their means.

Table 2. Parameters setting for the experiment

Data	value	Demand	value
v	5	Type I	
h	0.14%	μ_r	λ_r =10
A_w	3	Type II	
A_r	1	μh_r	λ_r =50
ρ_w, ρ_r	99%	μl_r	λ_r =10
L_w, L_r	1	Low interval	1^{st}-15^{th},
M	100,000		26^{th}-28^{th}
g_w, g_r	0.01	High interval	16^{th}-25^{th}
sl_{ti}	50% of v		

When the installation-stock s,Q system is used, the control parameters s_w, s_r, Q_w, and Q_r are determined based on the demand types as described in section 3. For Demand Type I and the low-demand period of Demand Type II, (s,Q) at the retailers and warehouse are (13.8, 21.74) and (304.3, 217.4), respectively. For the high-demand period of Demand Type II, (s,Q) at the retailers and warehouse are (58.1, 41.67) and (1,301.6, 1,000.0), respectively.

For the IDP control system, the inventory/distribution plan covers a 9-period planning horizon, and it is regenerated once every seven periods (the plan is rolled every 7 periods). The planning horizon (9 periods) is longer than the re-planning period (7 periods) to smoothly determine the new plan based on the real execution of the existing plan.

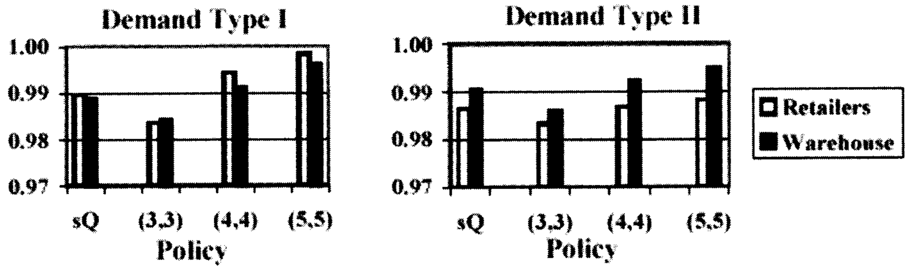

Fig 2. Fill rates obtained from the simulation

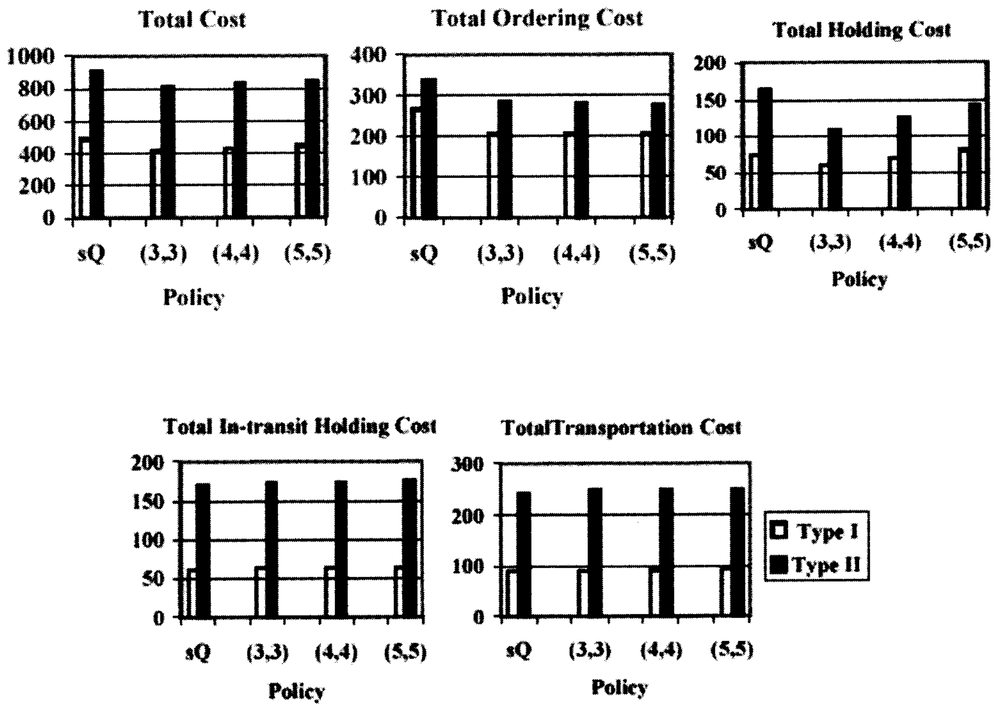

Fig. 3. Average costs obtained from the simulation under Demand Types I and II

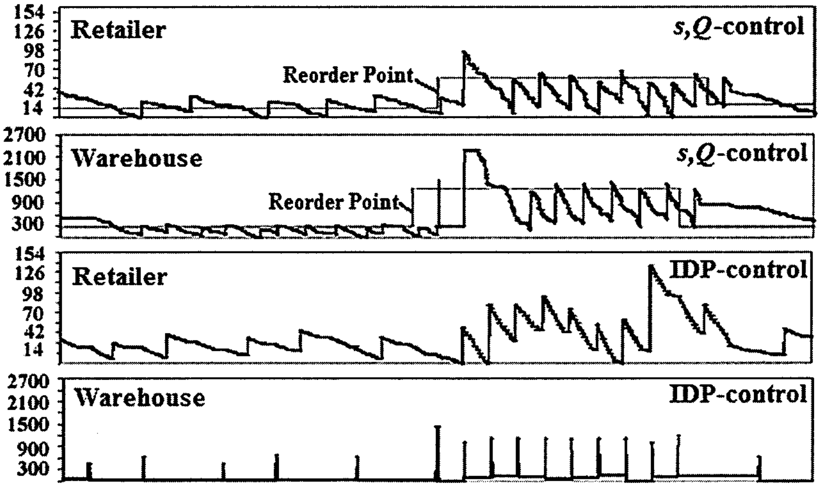

Fig. 4. On-hand inventory movement under the demand type II

6. Results and discussion

Fig. 2 shows the fill rates obtained from the simulation and Fig. 3 compares the average costs of ordering, holding, holding in transit, transportation, and total cost. Note that lost sales cost is not included in the total costs. However, the lost sales are considered implicitly because they are inversely proportional to the fill rates at the retailers.

From Fig. 2, the results of Demand Type I and Demand Type II are similar. Thus, the following analysis is valid for both demand patterns. It is observed that the installation-stock s,Q system gives fill rates near 99% as expected and can be compared to the IDP control system when (k_1, k_2) equals (4,4), denoted by IDP(4,4). Despite giving the lowest total cost, the IDP(3,3) should not be used to compare because it gives lower fill rates than the installation-stock s,Q system.

From Fig. 3, it can be seen for both demand patterns that IDP(4,4) gives a total cost lower than the installation-stock s,Q system. It is because IDP(4,4) gives the ordering costs at the warehouse and at the retailers lower than those of the installation-stock s,Q system (lower total ordering costs). Also, IDP(4,4) gives lower total holding cost. Even though the transportation and holding in transit costs of IDP(4,4) are slightly higher than those of the installation-stock s,Q system, they are dominated by the savings from the total ordering and holding costs.

Interestingly, the IDP(5,5), which has a higher fill rate, also gives a total cost lower than the installation-stock s,Q system (see Fig. 2, and 3). This means that the IDP(5,5) can serve the customer better with lower inventory cost. Similarly, this is because the IDP(5,5) gives much lower total ordering costs and lower total holding costs.

For more-in-depth analysis, considering an example of movement of the on-hand inventory presented in Fig. 4, one can see that, at the warehouse, the average on-hand inventory of the IDP(4,4) is much less than that of the installation-stock s,Q system. This is because the IDP control system always suggests the warehouse ship their products to destinations simultaneously. This results in an instantaneous decrease of inventory on-hand at the warehouse and a very low holding cost. On the contrary, the installation-stock s,Q system allows the warehouse to ship the product to the retailers at any time when its inventory position reaches its reorder point, resulting in a gradual decrease of inventory on-hand at the warehouse. It can be seen that the number of orders at the warehouse of IDP(4,4) is also less than that of the installation-stock s,Q system. This is because IDP(4,4) will place an order only if the optimal plan suggests, but the installation-stock s,Q system will place an order if the inventory position reaches the reorder point. This rule results in an uncontrollable number of orders, depending on demand uncertainty.

One can see that the on-hand inventory movements at the retailers of both control systems are quite similar. They deplete inventory according to customers' demand. Fig. 4 shows that the holding cost at the retailers of the IDP(4,4) is slightly higher than that of the installation-stock s,Q system. Nevertheless, the higher holding cost at the retailers is dominated by the saving of holding cost at the warehouse.

In summary, the IDP control system is more efficient and is also robust for both demand patterns. The higher efficiency of the IDP control system is mainly due to the fact that the decision of basic actions (i.e. ordering time and amount) is more sophisticated. In fact, the inventory and transportation quantities of the IDP control system are not constant. Thus, replenishment orders are placed as close to the optimal plan as possible. Especially at the warehouse, the appropriate amount of product will be acquired and sent to the next stage. As a result, an appropriate quantity of desired safety stock is kept and low holding cost is incurred. In contrast, the installation-stock s,Q system has a inflexible reorder point and reorder quantity (for the Demand Type I). This results in a relatively high inventory level under stochastic demands.

7. Conclusions

The inventory control system named the IDP control system for a one-warehouse/multi-retailer is developed and then evaluated against the installation-stock s,Q system. The supply chain faces two types of demand patterns. The total logistic costs include ordering, holding, holding in transit, and transportation costs. The IDP control system tries to follow the optimal inventory/distribution plan as much as possible by adopting an adjustment rule and a rationing rule. The optimal inventory/distribution is obtained by solving the proposed linear programming model. The control parameters of the installation-stock s,Q system of Ganeshan [5] is obtained by solving a non-linear model.

Based on the experimental results, the IDP control system can give better results, compared to the installation-stock s,Q system. It can give both higher fill rates and lower total logistic costs; two major costs in inventory management (holding and ordering costs) are lower.

The main contribution of this paper is the development of a new and effective control system for a one-warehouse/multi-retailer supply chain. The main idea of the proposed inventory control system is the adaptation of a simple linear programming model with the proposed adjustment rule and rationing rule. However, implementing an IDP control system needs some coordination among supply chain members. Each member should transfer some information, at least, its on-hand inventory to the planner to generate a new optimal plan. This is quite a difficult task if each member belongs to different owners. Therefore, future research should study a coordination scheme or a reward system (e.g. incentive mechanisms) to fairly allocate the benefits obtained from an IDP control system among the members.

8. Acknowledgement

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