Use of Experimental Design to Enhance Performance of an Integrated Approach: Simulated Annealing and Steepest Ascent

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Abstract

An integrated approach between a method of steepest ascent and Simulated Annealing is set up to find optimum settings when the process yield is simulated by the surface of a continuously stirred tank reactor (CSTR) with different levels of random variation (noise) added. The effects of different choices of parameters for the integrated approach, on different performance measures, are investigated. These performance achievements consist of Taguchi 's the larger the better, minimax and mean squared error measures. The approach did not seem sensitive to the parameter choices, within reasonable limits. This approach with the preferable levels of parameters is then compared with the conventional method of steepest ascent. The results suggest that the method of steepest ascent seems to be the most efficient on the CSTR surface at the lower levels of noise. However, the integrated approach with the Simulated Annealing element works well when the standard deviation of the noise is at higher levels. On the average, the standard deviation of the greatest actual concentration of the product and percentage of sequences ended at the optimum from the integrated algorithm. However, it needs more runs, on average, to converge to the optimum.

Key words: Simulated Annealing; Steepest Ascent; Taguchi; Minimax; Mean Squared Error

1. Introduction

The steepest ascent procedure, proposed by Box and Wilson [1], has been widely used in the area of Response Surface Methodology (RSM). The objective of the RSM is to describe how the response of a process varies with changes in k process variables [2]. The process variables determined will depend on the specific field of the application. Most industrial processes have some process variables. For example, a response in a chemical reactor might be *concentration of product* and the process variables affecting this concentration might be *temperature* and *pressure* of a chemical plant [3]. The process variables such as *speed of lathe* and *advance of cutting tool* in machining can be adjusted by plant operators or by automatic control mechanisms to enhance the *efficiency* of the machine. Care must be taken to operate industrial processes within safe limits, but optimal conditions are rarely attained and increased international competition means that deviations from the optimum can have serious financial consequences. In many cases the optimum changes with time and there is a need for a routine mode of operation to ensure that the process always operates at optimal or near-optimal conditions.

On the theory and practice of RSM, it is assumed that the mean response (η) is related to values of the process variables (ξ_1, ξ_2, ..., ξ_k) by an unknown function f. The functional relationship between the mean response and k process variables can be written as $\eta = f(\xi)$, if ξ denotes a column vector with elements ξ_1, ξ_2, ..., ξ_k. Estimation of such surfaces, and hence identification of near optimal settings for process variables is an important practical issue with interesting theoretical aspects. The procedure begins with a factorial experiment around the prevailing operating conditions. A sequence of first order models and line searches are justified on the basis that such a plane would be fitted well as a local approximation to the true response [4]. The estimated coefficients for the first order model are determined using the principles of least squares. A sequence of runs is carried out by moving in the direction of steepest ascent. When curvature is detected, another factorial experiment is conducted. This is used either to estimate the position of the optimum or to specify a new direction of steepest ascent.

There is much current interest in optimisation methods with a stochastic element, such as Genetic Algorithms (GA) and Simulated Annealing (SA). Holland [5] introduced the genetic algorithm for finding the global maximum on a hypersurface [6]. The genetic algorithm (GA) is a set of rules for searching large solution spaces in a manner similar to natural selection in biological evolution. Solutions with desirable characteristics are given a higher probability of being parents for the next generation and will cross their components to offspring, with a possible chance of mutation [7]. The essential parameters are: the number of design points tried initially (population size); length of chromosome, used to code the coordinates of the point, which corresponds to the resolution; probability of crossover; probability of mutation.

A recent study by Luangpaiboon et al. [8] compared a modified simplex method (MSM) by Nelder and Mead [9] and a genetic algorithm for a variety of response surfaces and levels of measurement noise. The former is more efficient if the process noise is negligible but the GA is more robust to process noise. The GA appears to work well in the area of the RSM. However, high variability of the GA when applied to on-line optimisation could be a serious disadvantage [10]. This paper proposes a preliminary study of an application of an integrated approach, Simulated Annealing, on a path of steepest ascent. Simulated Annealing has been used in an interesting analogy between problems in statistical mechanics and optimisation. Its properties expose useful information and overcome the large and noisy systems [11].

The aim of this paper is to investigate the performance of the integrated approach for the process optimisation, and how it depends on the parameter choices. A simulation study is based on the function of three process variables with different levels of noise. The functions represent response surfaces of yields of the continuous stirred tank reactor. The objective of using the integrated approach is to find the values of the process variables which give the greatest yield, and to find these values with a minimum number of process runs at sub-optimal conditions. The integrated approach parameters are varied according to a factorial design. The dependent variable is some measure of performance of replicate trials of the integrated approach. The measures considered are: minimum of the maximum yields of all the trials; Taguchi's '*the larger the better*' signal to noise ratio calculated from the maximum yields of all the trials and the mean squared of error (MSE). Conclusions are drawn, and practical recommendations are made.

2. Related Methods

2.1 Method of Steepest Ascent

The procedure of steepest ascent is that a hyperplane is fitted to the results from the initial 2^k designs. The direction of steepest ascent on the hyperplane is then determined by using principles of least squares and experimental designs. The next run is carried out at a point which is some fixed distance in this direction and further runs are carried out by continuing in this direction until no further increase in yield is noted. When the response first decreases another 2^k design is carried out, centred on the preceding design point. A new direction of steepest ascent is estimated from this latest experiment. Provided at least one of the coefficients of the hyperplane is statistically significantly different from zero, the search continues in this direction. More details are referred to in many statistical texts, for example [12] and [2]. Once the first order model is determined to be inadequate, the area of optimum is identified via a finishing strategy [13].

2.2 Simulated Annealing

Simulated Annealing has been derived from an interesting analogy between problems in statistical mechanics and multivariate or combinatorial optimisation [11]. This algorithm is a set of rules for searching large solution spaces in a manner that mimics the annealing process of metals. The algorithm simulates the behaviour of an ensemble of atoms in equilibrium at a given finite temperature [14] and its original framework can be traced to

Metropolis et al. [15]. This algorithm has been regularly used in global function optimisation and statistical applications.

In case of maximisation the procedures of this algorithm start at a corresponding initial value of the objective function, y_0. The new objective value, y_1, will be then determined. The new solution will be unconditionally accepted if its objective value is improved and the process regularly continues. Otherwise the difference or size of increment in objective values, Δy, is calculated and with an auxiliary experiment the new solution (y_1) would be accepted with probability $P(\Delta y)$ given by:

$$P(\Delta y) = 1, \text{ if } \Delta y = y_1 - y_0 \geq 0 \text{ or}$$

$$P(\Delta y) = EXP(c y_0^g \Delta y) \text{ if } \Delta y < 0,$$

where c and g are an arbitrary positive number and a negative number respectively. A random number, x, is generated from the uniform distribution on (0, 1) and is compared to $EXP(c y_0^g \Delta y)$. If $x < EXP(c y_0^g \Delta y)$, then the new solution is accepted. Otherwise it is rejected. This stochastic element is from Monte Carlo sampling. It occasionally allows the algorithm to accept a new solutions to the problems, which deteriorate rather than improve the objective function value. However, Simulated Annealing includes a number of parameters including g and c, which have been claimed to affect the efficiency of the algorithm.

3. Continuous Stirred Tank Reactor

A diagrammatic representation of a single continuous stirred tank reactor (CSTR) is shown in Figure 1. A stream rich in chemical A of feed concentration $C_{A(in)}$ is flowing into a reactor at a feed flow rate of $F_{(in)}$, and a feed temperature of $T_{(in)}$. FIC is a flow indicator controller, TIC is a temperature indicator controller, $T_{C(in)}$ is the temperature of the coolant to the heat exchanger, F_C is the flow of cooling water and T_C is the temperature of coolant. S is a controlled switch. LI and FI are level and flow indicators respectively [10]. The reaction in the CSTR is an irreversible, first order exothermic reaction. The proportion of chemical A is converted to a desired product B, which, in turn, at high temperature undergoes further reaction and is decomposed to form an undesired by-product C: A to B to C. The stated objective is to explore the operating conditions corresponding to higher concentration of product.

It is also assumed that the level is perfectly controlled, so the volume of material in the tank is constant. This implies that the flow out equals the flow in. The temperature in the reactor may be regulated by manipulating the flow rate of the cooling water (F_C) in the heat exchanger. A mechanistic model adequately accounting for the system under study is suggested purely by physical consideration and the dynamics of the system can then be described by the following set of ordinary, non-linear differential equations.

$$V\frac{dC_A}{dt} = -K_{OA}e^{(-E_A/RT)}C_AV + F_{(in)}(C_{A(in)} - C_A)$$

$$V\frac{dC_B}{dt} = K_{OA}e^{(-E_A/RT)}C_AV - K_{OB}e^{(-E_B/RT)}C_BV + F_{(in)}(-C_B)$$

$$\rho c_pV\frac{dT}{dt} = (-\Delta H_{RA})K_{OA}e^{(-E_A/RT)}C_AV + (-\Delta H_{RB})K_{OB}e^{(-E_B/RT)}C_BV + \rho c_pF_{(in)}(T_{(in)} - T) + \rho c_pF_R(T_R - T)$$

$$\rho c_pV_R\frac{dT_R}{dt} = \rho c_pF_R(T - T_R) + UA(T_C - T_R)$$

$$\rho c_pV_C\frac{dT_C}{dt} = \rho c_pF_C(T_{c(in)} - T_C) + UA(T_R - T_C)$$

The flow of cooling water, F_c, is manipulated by the following control algorithm: $F_c = T_{BIAS} - K_c\ (T_r - T)$. The five process state variables, which depend on time t, are the concentration of reactant C_A, the concentration of product C_B, the reactor temperature T, the temperature of the recycled flow T_R and the temperature of the coolant leaving the heat exchanger (T_C). Initial conditions of process state variables, values of the parameters of the process and parameters of controller are given in Tables 1-3 respectively. The time constant of the system is such that equilibrium is attained after approximately ten minutes.

The integrated approach was determined for the case of the response surface of the simulated continuous stirred tank reactor. There are three process variables which can be set to any chosen values within safe limits. These process variables related to the feed flow are shown in Table 4. This choice of relative ranges for the process variables was based on the chemist's fundamental investigations. The response variable of the process is defined to be the concentration of the desired product B, C_B. The typical three-dimensional response surfaces, with $C_{A(in)}$ fixed at 1 and 15, are shown in Figures 2(a), (b).

4. Details of the Integrated Approach

Parameters of Method of Steepest Ascent: 8 $unit^3$ of the volume of the factorial design; 1 unit of the step length; 10% of the significance level for tests of significance of slopes

Parameters of Simulated Annealing: g; c

Step 1: Perform a 2^3 design at a random centre point.

Step 2: Fit a regression plane to the data so that the fitted model has the form

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 T_{(in)} + \hat{\beta}_2 F_{(in)} + \hat{\beta}_3 C_{A(in)}.$$

Step 3: Test whether there is evidence that either β_1, β_2 or β_3 is different from zero at the 10% level of significance, i.e. does $\left| \frac{\hat{\beta}_i}{\sqrt{C_{ii} MSE}} \right|$ exceed $t_{v,\,0.05}$?, where v is the number of degree of freedom, one for the first experiment increasing by eight for each replicated experiment.

Step 4a: If the result is significant, move one step along the path of steepest ascent, that is along the line whose formula in parametric form is

$$(f\hat{\beta}_1,\ f\hat{\beta}_2,\ f\hat{\beta}_3),\ (-\infty<f<\infty),$$

and determine the yield. The step length is

$$\left[\frac{\hat{\beta}_1}{\sqrt{\hat{\beta}_1^2+\hat{\beta}_2^2+\hat{\beta}_3^2}}, \frac{\hat{\beta}_2}{\sqrt{\hat{\beta}_1^2+\hat{\beta}_2^2+\hat{\beta}_3^2}}, \frac{\hat{\beta}_3}{\sqrt{\hat{\beta}_1^2+\hat{\beta}_2^2+\hat{\beta}_3^2}} \right].$$

Otherwise go to *Step 4b*.

Step 4b: Test whether there is evidence that the interaction or curvature check is significant. If the check is significant, go to *Step 6*. Otherwise, replicate the design and return to *Step 2*.

Step 5a: If the current yield (y_1) is greater than the previous yield (y_0) or the stochastic element meets the requirement of acceptance, continue by moving another step in the same direction.

Step 5b: If the yield is not greater than the previous one, test the element as follows:

Randomly generate a random variable, x, ~ Uniform (0, 1).

If $x < P(\Delta y) \equiv EXP(c y_0^g \Delta y)$, where $\Delta y = y_1 - y_0$, then go to *Step 5a*. Otherwise return to the preceding point then carry out another 2^3 design and return to *Step 2*. If the first step leads to a yield less than the yields obtained in the preceding 2^3 designs then replicate the design and go to *Step 2*.

Step 6: Implement the finishing strategy (see below).

Finishing Strategy

If there is no justification for any assumptions about the shape of the response surface, the use of the finishing strategy based on a hexagon design is recommended [13]. However, if a response surface can reasonably be assumed to be a curved ridge (from the earlier phase of study), the finishing strategy based on a hexagon design is preferable. The central composite design (CCD) is centred on the point ($T_{(in)p}$, $F_{(in)p}$, $C_{A(in)p}$), and consists of 14 design points at (±1, ±1, ±1), ($\pm\sqrt[4]{2^3}$, 0, 0), (0, $\pm\sqrt[4]{2^3}$, 0), (0, 0, $\pm\sqrt[4]{2^3}$) plus six replicates at (0, 0, 0); where (0, 0, 0) now corresponds to the point ($T_{(in)p}$, $F_{(in)p}$, $C_{A(in)p}$). Fit a quadratic surface

$$\begin{aligned} y = {} & \beta_0 + \beta_1 T_{(in)} + \beta_2 F_{(in)} + \beta_3 C_{A(in)} + \beta_4 T_{(in)}^2 \\ & + \beta_5 F_{(in)}^2 + \beta_6 C_{A(in)}^2 + \beta_7 T_{(in)} F_{(in)} \\ & + \beta_8 T_{(in)} C_{A(in)} + \beta_9 F_{(in)} C_{A(in)} \end{aligned} \qquad (1)$$

Find the maximum as the solution of $\partial y/\partial T_{(in)} = 0$, $\partial y/\partial F_{(in)} = 0$, $\partial y/\partial C_{A(in)} = 0$ and call this ($T_{(in)p}$, $F_{(in)p}$, $C_{A(in)p}$). If ($T_{(in)p}$, $F_{(in)p}$, $C_{A(in)p}$) is within the cube with vertices ($\pm\sqrt[4]{2^3}$, $\pm\sqrt[4]{2^3}$, $\pm\sqrt[4]{2^3}$), then ($T_{(in)p}$, $F_{(in)p}$, $C_{A(in)p}$) is taken as the optimum operating condition. If ($T_{(in)p}$, $F_{(in)p}$, $C_{A(in)p}$) is not within this volume, another CCD is carried out, centred on the point from the first CCD with greatest yield. A quadratic surface is now fitted to all the data. If the maximum is outside the volume of the union of the two containing cubes, the ridge is searched for the greatest value of the function, using a step length of 0.05 (from additional experiments by using fewer runs).

4.1 Statistical Experimental Design

The two parameters, g and c, of the integrated approach are varied in a factorial design. Bohachevsky et al. discussed the proper levels of g and c on various functions [14]. In this paper we selected the levels based on these tested functions and they also covered the range of values commonly found in the literature: [-0.5, -1, -1.5] and [4.5, 6.5] for g and c

respectively. Two replicates were performed for each of the 6 sets of parameter values. Each trial used the random initial design points, evenly distributed about edges, furthest from the optimum, of the safe region of operation. The comparisons were made for four different levels of measurement noise added to the response: independent and normally distributed with mean of zero and standard deviations (S.D.) of 0.5, 1.0, 2.0 and 3.0 respectively. The following performance measures were considered.

4.2 Performance Measures

The three performance measures depend only on the yield at the end of each trial.

4.2.1 Taguchi's Measure of Performance (Y_{F1})

Taguchi [16] proposed *'the larger the better'* measure:

$$Y_{F1} = -10 \log \left(\sum_{i=1}^{n} (1/y_i^2)/n\right),$$

in which y_i represents the highest yield at the end of trial i, and n is the number of trials.

4.2.2 Minimax Performance Measure (Y_{F2})

Another measure of the performance of the integrated approach is the minimum of the highest yields at the end of the trials. In the case of ten trials, for example,

$$Y_{F2} = \text{Min}\,(y_1, y_2, \ldots, y_{10}).$$

Y_{F2}. is to be maximized.

4.2.3 Mean Squared Error Performance Measure (Y_{F3})

It is natural to consider combining bias and variance through the mean squared error (MSE) criterion [17]. In this case, for example,

$$Y_{F3} \quad = [(\omega_\mu - \text{T})^2 + \omega_\sigma^2]$$

in which ω_μ represents the average of actual responses, T represents the target value of response and ω_σ is the standard deviation of actual responses. Y_{F3} to be maximized.

4.3 Preferred Levels of Parameters of the Integrated Approach

A typical table of results is given in Table 5 and the analysis of variance and the main effect plots with the error standard deviation of 1.0 for Y_{F1} is shown in Table 6 and Figure 3, respectively.

On the early phase of the parameter study, the main finding was that the probability of g for Y_{F1} and Y_{F3} should be high (-0.5). This leads to higher average and lower level of variance of actual responses. No other statistically significant results were found. The preferred levels of g and c could be high (-0.5 or -1.0) and low (4.5), respectively. Results are included for all cases in which the ANOVA p-values, for main effects and interaction, are less than 0.1 in Table 7.

5. Results and Discussions of the Integrated Approach and the Conventional Method

The comparison between the integrated approach, with the preferred levels of the parameters, and the conventional method of steepest ascent is made with the measurement noise on the concentration of the desired product B (normal and independent with zero mean and standard deviation of 0.5, 1, 2 and 3). There are four performance measures over 100 runs in this study. The first and second measures are an average and a standard deviation of greatest actual concentration of the desired product B from the finishing strategy respectively. The third is an average number of runs until the algorithms converge. Finally, the percentage of sequences that ended at the optimum is shown.

The process settings for all the scenarios are given in Table 8. The performance of the method of steepest ascent and the integrated approach can be explained by the box plots in Figure 4 when the error standard deviation was 2.0 and 3.0. The values of average actual concentrations for both noise levels of the integrated approach seem to be better when compared in this manner. Note that since the efficiency of these algorithms is related to their initial points, it would be helpful to set random starting points for all algorithms. These results show that the performance of the integrated approach under the stochastic element of Simulated Annealing seems superior to the algorithm based on the method of steepest ascent at the higher levels of error standard deviations [18].

Moreover, the percentage of sequences ended at the optimum or near optimum of radius equalling two from the integrated approach is better at higher levels of error standard deviation although a greater number of runs were required to converge to the optimum. As stated earlier, the function of this research was restricted to three process variables. Consequently, comparisons and conclusions between the two algorithms may not be valid for other families of functions. Other stochastic approaches could be extended to the method based on conventional factorial designs to increase its performance, especially in terms of speed of convergence, when the error standard deviation is at higher levels. Moreover, further research will look at the effect of the ranges of parameters of Simulated Annealing. This may enhance the performance of the proposed integrated algorithm.

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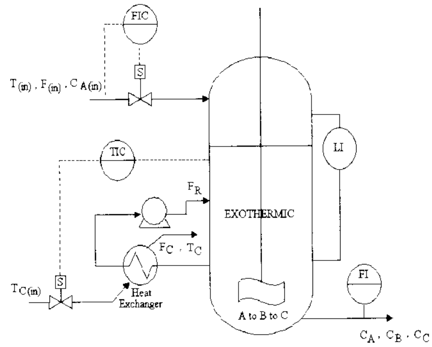

Figure 1 The Continuous Stirred Tank Reactor

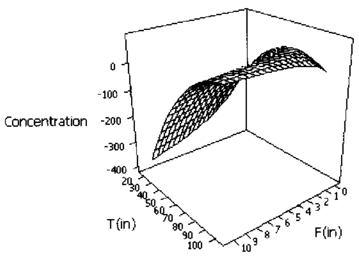

Figure 2(a) The Surface Plot with $C_{A(in)}$ Fixed at 1

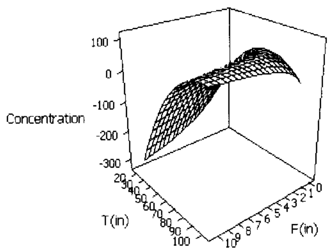

Figure 2(b) The Surface Plot with $C_{A(in)}$ Fixed at 15

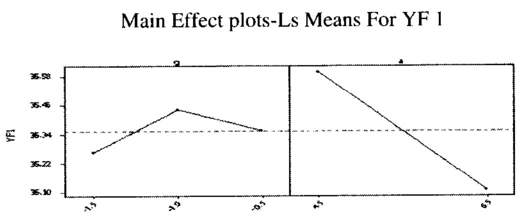

Figure 3 Main Effect Plots for Y_{F1} with the Error Standard Deviation of 1.0

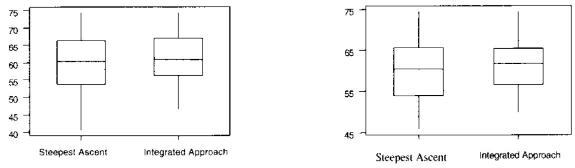

Figure 4 Two Independent Box Plot Comparisons Showing the Performance (Product Concentration) of the Method of Steepest Ascent and the Integrated Approach when the Error Standard Deviation was 2.0 and 3.0 Respectively.

Table 1 Process State Variables and their Initial Conditions

Variables	Description	Unit	Value
C_A	Concentration of reactant A	mole/m^3	747.9
C_B	Concentration of product B	mole/m^3	1609
T	Reactor temperature	K	341.4
T_R	Temperature of the recycled flow	K	333.3
T_C	Temperature of the coolant	K	330.5

Table 2 Parameters of the Process

Parameters	Description	Unit	Value
V	Volume of the CSTR	m^3	3
K_{OA}	Rate coefficient (A to B)	mole/s	7.10^{11}
E_A	Activation energy (A to B)	J	90000
R	Gas constant	J/mole/K	8.314
K_{OB}	Rate coefficient (B to C)	mole/s	9.10^{11}
E_B	Activation energy (B to C)	J	100000
ρ	Process fluid density	kg/m^3	1000
c_p	Process fluid heat capacity	J/kg/K	4180
$-\Delta H_{RA}$	Heat of reaction (A to B)	J/mole	80000
$-\Delta H_{RB}$	Heat of reaction (B to C)	J/mole	40000
F_R	Feed flow rate of recycled stream	m^3/s	0.025
U	Heat transfer coefficient	W/m^{2}K	3000
A	Area, heat exchanger(HX)	m^2	100
V_C	Volume, cooling water in HX	m^3	0.2
V_R	Volume, process stream in HX	m^3	0.2
$T_{C(in)}$	Feed temperature of cooling stream	K	293

Table 3 Parameters of Controller

Parameters	Description	Unit
T_r	Required temperature	K
K_C	Controller gain	m^3/sK
T_{BIAS}	Offset	m^3/s

Table 4 Process Variables of Feed Flow and their Safe Limits

Process Variables	Description	Unit	Feasible Region
$T_{(in)}$	Feed temperature of reactant A	celsius	60-100
$F_{(in)}$	Feed flow rate of reactant A	litre/minute	1-10
$C_{A(in)}$	Concentration of reactant A	mole/litre	1-15

Table 5 Results for the Integrated Approach with the Error Standard Deviation of 1.0

g	c	Highest actual yields calculated by performance measures (Replicate 1) / Highest actual yields calculated by performance measures (Replicate 2)										Y_{F1}	Y_{F2}	Y_{F3}
-1.5	4.5	63.7	53.2	67.6	59.4	67.9	52.6	64.5	45.1	55.9	61.1	35.2	45.1	261
		60.6	58.8	56.3	60.5	56.1	59.1	57.5	62.6	58.3	66.2	35.5	56.1	240
-1.5	6.5	47	50	56.8	55.7	68.7	53.1	54.1	60	57.2	56.8	34.8	47	370
		50.9	65.1	58.7	67.1	74.2	55.5	69.1	53.4	57.9	58.7	35.5	50.9	202
-1	4.5	60.6	71.1	70.7	63.6	65.1	59.5	63.2	60.9	65.7	53.1	35.9	53.1	141
		62	58.8	60	63.3	66.5	58.1	62.2	72.6	61.5	62.6	35.9	58.1	154
-1	6.5	56	71.6	60	45.7	70.2	56.4	55.6	55.5	54.2	58.7	35.1	45.7	283
		65.8	55.8	44.4	57.4	55.4	51.4	63.6	50.4	55.2	58	34.8	44.4	377
-0.5	4.5	59.1	73.6	57.9	68.4	62.1	60	56	59.4	56.8	55.9	35.6	55.9	204
		59	64.7	63.4	62.5	55.6	59.3	62.1	57.5	61.3	50.5	35.4	50.5	242
-0.5	6.5	72.6	60.2	56.2	53.6	57.9	59.8	61.6	69.7	49.7	52.7	35.3	49.7	251
		55.5	55.4	62.8	55	71.7	53.1	42.4	62.2	66.4	59.2	35.1	42.4	285

Table 6 ANOVA for Y_{F1} of the Integrated Approach with the Error Standard Deviation of 1.0

Source	Seq SS	DF	Adj SS	Adj MS	F	p-value
g	0.05786	2	0.05786	0.02893	0.44	0.665
c	0.71612	1	0.71612	0.71612	10.81	0.017
$g*c$	0.36672	2	0.36672	0.18336	0.06628	0.141
Error	0.39766	6	0.39766	0.06628		
Total	1.53836	11				

Table 7 The Preferred Levels of the Parameters of the Integrated Approach

S.D.	Preferred Level		Over all F-significant and p-Value		
	g	c	Y_{F1}	Y_{F2}	Y_{F3}
0.5	-0.5	-	0.081	-	0.077
1.0	-1.0	4.5	0.017	-	-
	-	4.5	-	0.048	-
	-1, -0.5	4.5	-	-	0.040
2.0	-	-	-	-	-
3.0	-	-	-	-	-

Table 8 Four The measurements over 100 runs

S.D. of Noise	Algorithm	Average Greatest Actual Concentration	S.D. of Greatest Actual Concentration	Average Number of Runs	Percentage (ending at optimum)
0.5	Steepest Ascent	59.2011	7.6823	33.3	0.85
	Integrated	56.7053	10.2178	38.17	0.85
1.0	Steepest Ascent	57.4528	7.3680	33.75	0.90
	Integrated	57.0683	6.8899	34.9	0.90
2.0	Steepest Ascent	59.3067	9.5568	32.6	0.85
	Integrated	61.1069	7.7166	34.75	0.90
3.0	Steepest Ascent	60.0803	7.7723	31.35	0.80
	Integrated	61.2676	6.3431	33.3	0.95