

Efficient Electrical Distribution Network Observability Algorithm for State Estimator

Jovitha Jerome

Electrical and Electronics Engineering Department
PSG College of Technology, Coimbatore 641004
Tamilnadu, India

Abstract

A distribution automation system (DAS) aims for better management and control of the distribution networks. An efficient network observability, bad data detection and state estimation solution technique are prerequisites for the success of DAS. This paper presents an efficient and robust three-phase state estimation (SE) algorithm for application to radial distribution networks. The extension of the method to the network observability analysis and bad data processing is discussed in detail. This method exploits the radial nature of the network and uses a forward and backward propagation scheme to estimate the line flows, node voltages and loads at each node, based on the measured quantities. The SE cannot be executed without an adequate number of measurements. The proposed method has been tested to analyze several practical distribution networks of various voltage levels and also having high r/x ratio of lines. The results for a typical network are presented for illustration purposes.

Keywords: Distribution automation system, State estimation, Network observability, Bad data processing

1. Introduction

The power business is moving into new territory with market deregulation. Deregulation of the power industry has made power quality a distinguishing feature of the distribution service. DAS is the key to address all these challenges to improve the operation of the distribution system and the quality of supply. DAS is essential for efficient operation of the distribution networks. In real-time environment the state estimator consists of different modules, such as topology processor, observability analysis, state estimator and bad data processing. Many researches have addressed distribution state estimation, proposing different approaches. A distribution state estimation technique for real-time monitoring was proposed in [1]. When using current measurements, there may be situations where the network observability cannot be determined. This is due to the possibility of multiple solutions for certain network and measurement configurations. Distribution systems [2] that use the numerical observability

analysis based on triangular factorization of the gain matrix can be applied to the three-phase SE without major modifications. A distribution state estimator based on weighted least square (WLS) approach and three-phase modeling techniques was formulated in [3]. The multiphase power flow and state estimation is discussed in [4]. In the distribution system observability analysis, the conventional topological method cannot be used directly because the definition of the system state is not the same as in the traditional sense and not all buses are three phase buses.

An efficient and robust three-phase power flow algorithm for application to radial distribution networks was presented in [5]. It uses the forward and backward propagation technique to calculate branch currents and node voltages. This method has been extended to state estimation algorithms, network observability analysis and bad data processing for application distribution networks. Conventional state estimation methods based on the least square method technique may fail to give a solution to

the distribution state estimation in many cases due to ill-conditioned gain matrix and Jacobian matrices. Also these methods are applicable to systems with lower numbers of nodes and lower r/x ratio of lines and computationally not efficient. In this paper a new formulation and solution algorithm for solving power system state estimation is proposed for three-phase unbalanced distribution systems. In the proposed technique the observability routine decides if the measurement set is sufficient to allow the computation of the SE. Bad measurement data are detected, eliminated and replaced by pseudo or calculated values.

2. Methodology

The figure 1 shows the computational steps in the approach. Node numbers are ordered to generate proper parent-child relations based on the network topology [5].

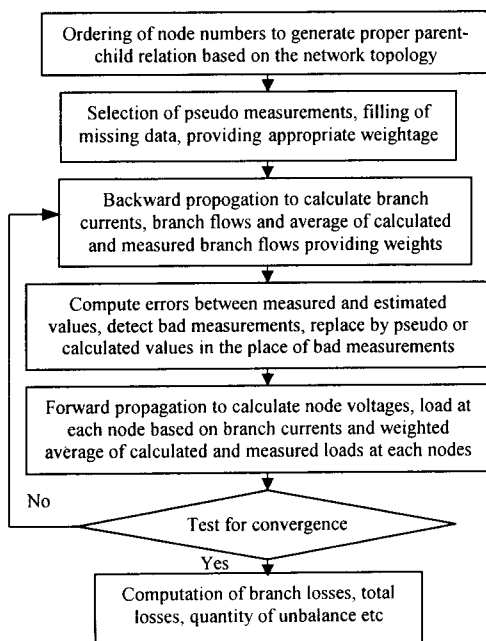


Figure 1: Basic computational blocks in the proposed algorithm

The measurement values are checked. If the measurements are not available then the values are either calculated or pseudo measurements provided. Accordingly appropriate index is provided to the measurement values. Selections of pseudo measurements, filling of missing data,

providing appropriate weightage are the functions of the observability analysis algorithm. Backward propagation is used to calculate branch currents, branch flows and average of calculated and measured branch flows providing weights. During the iterative process the bad data is detected and replaced by pseudo or calculated values. Computation of errors between measured and estimated values, detection of bad measurements, replacement by pseudo or calculated values in the place of bad measurements are the functions of the bad data processing algorithm. Node voltages and load at each node are calculated during forward propagation, based on branch currents and weighted averages of calculated and measured loads at each nodes. Voltage at each node is computed and the test for convergence is performed. The absolute errors of measured and calculated values of real and reactive power flows and injection are also performed. Computation of branch losses, total losses, and quantity of unbalance in current and voltage is done once the program converges.

3. Network Observability

A minimum amount of real-time data is necessary for the state estimator to be effective. In a radial distribution network, the source substation node voltage are assumed to be known (voltage magnitude measured and angle are taken as reference). Then the state estimator has to compute $(n-1)$ voltage magnitudes, $(n-1)$ voltage angles in an n -node radial network. If we have the node measurements of both active and reactive power loads at $(n-1)$ load nodes, the network is observable. In general it may not be possible to have load measurements at all load nodes, instead we may have line flows both active and reactive power in some selected branches of the network. Due to radial nature of the network, from the available branch flows around a node, it may be possible to predict the load at that node. Figure 2 gives the basic steps in the proposed observability analysis. With the help of the sample radial network shown in Figure 3, the following steps explain the observability process. Consider a network having (n) number of nodes, and (m) number of branches. Let $J_{pd}(i)$, $J_{qd}(i)$ be the index for active and reactive load measurements at node i , $J_{pf}(l)$, $J_{qf}(l)$ be the index for active and reactive power flow measurements in branch l ,

Where $i = 1$, Source node; $i = 2, \dots, n$ other nodes; $l = 1, \dots, m$ are branches.

A: Create the parent child table, as shown in Table 1 for the example network. Initialize the indices of real power demand, reactive power demand, real power flow, and reactive power flow to zero. So $Jpd(i) = 0$, $Jqd(i) = 0$, $Jpf(l) = 0$, $Jqf(l) = 0$; Where $i = 2, \dots, n$ and $l = 1, \dots, m$.

B: Scan the available measurements of real power demand, reactive power demand, real power flow and reactive power flow and set the corresponding indices of these measurements as 1. $Jpd(i) = 1$, $Jqd(i) = 1$, $Jpf(l) = 1$, $Jqf(l) = 1$; where i is measurement available node; l is measurement available branch.

C: Check for the indices $Jpd(i)$, $Jqd(i)$, $i = 2, \dots, n$ of real power demand, reactive power demand are equal to 1. If yes the state estimation is performed then go to step 10. If not, then go to step 4.

D: Select the nodes where the index of real power demand $Jpd(i)$, reactive power demand $Jqd(i)$, is zero, search for the branch flow information [5] suitable to be used for load demand calculation. If, such branch flow measurement is available, make the load measurement equal to the flow measurement and make the corresponding index of $Jpd(i)$, $Jqd(i)$ equal to 2. For example in the sample system of Figure 3, if the measurement at leaf load node 3 is not available, but the branch (2) available, set the load measurement equal to the flow measurement. If load measurement at node 2 is not available, but the line flow measurements in branches (1), (2), and (3) available, set the load measurement as algebraic sum of there branch measurements. For $i = 2 \dots n$, due to the radial nature of the distribution system in some cases it may be possible to generate required number of data using the available measurements

E: Check for any indices $Jpd(i)$, $Jqd(i)$, $i = 2 \dots n$ of real power demand, reactive power demand are still equal to zero. If none is zero, then the state estimation is performed. If any of the value is zero step 6 is performed.

F: Scan from source node [5] to leaf node in the forward direction and calculate the possible

missing real power flow and reactive power flow. For example, in the sample system, the flow measurements in branches (1), (2), and load measurements at node 2 are available, the flow measurements in branch (3) can be computed. Set the index of $Jpf(l)$ and $Jqf(l)$ of the corresponding branches equal to 2.

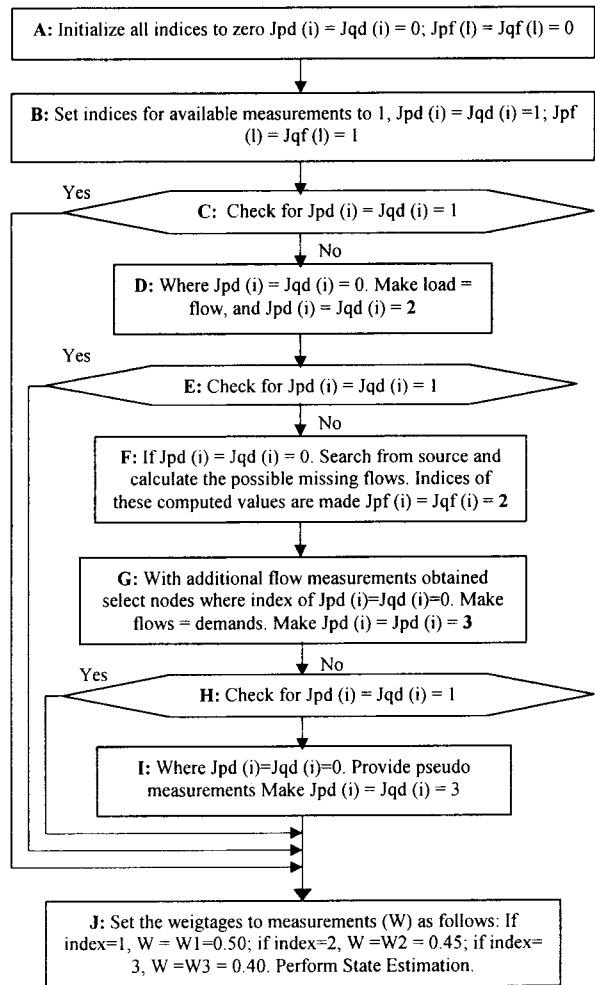


Figure 2: Basic steps in the proposed observability analysis

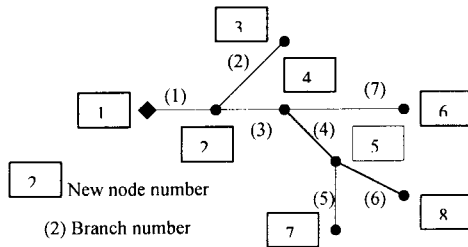


Figure 3: Sample feeder with new node numbers

Parent node	1	2	2	4	4	5	5
Child node	2	3	4	5	6	7	8

Table 1: Parent child relationship

G: With additional flow measurements obtained, select the nodes where the index of real power demand, reactive power demand, is zero, search for the branch flow information suitable to be used for load calculation. If such branch flow measurement is available then, make the load measurement equal to the flow measurement and make the corresponding index $J_{pd}(i)$, $J_{qd}(i)$ equal to 3. For example, in the sample system in Figure 3 if the measurement at leaf load node 3 not available, but the branch (2) available, we set the load measurement equal to the flow measurement. If load measurement at node 2 is not available, but the line flow measurements in branches (1), (2), and (3) available, we set the load measurement as algebraic sum of these branch measurements. Set the index $J_{pd}(i)$, $J_{qd}(i)$ of corresponding nodes equal to 3.

H: When there is no sufficient number of measurements for the state estimator, the system may not be observable. Pseudo measurements have to be provided to the estimator from the past data. The indices to these corresponding data are made 3.

I: Check for any indices $J_{pd}(i)$, $J_{qd}(i)$, $i = 2 \dots n$ are still equal to zero. If any of them are zero, then, the corresponding measurements can be replaced by pseudo-measurements and index is made 3.

J: Equal weights are provided to both measured and calculated if both are accurate. Higher weightages are provided to more accurate measurements. If W is the weight, when the index is 1, then the weight $W = W_1 = 0.50$, equal

weightage is given to both measured and calculated values. If the index is 2, the weight $W = W_2 = 0.45$, more weightage is given to calculated values than measured value. When the index is 3, it is pseudo measurement, less accurate. Hence the weight $W = W_3 = 0.40$, still more weightage is given to calculated values than the pseudo measurement value.

4. Bad Data Processing

Bad data processor detects the presence of bad data (gross error) in the measurement set. After bad data is detected, it identifies which measurements are bad. These are eliminated from the set of measurements to be utilized for the SE and suitable values are replaced. In our approach the difference between the measured and calculated values are calculated during each state estimator iterative process as explained in section 6. These difference values are monitored and get reduced as the iterations are advanced. Due to presence of bad measurements some of these values may persist to be significant. After a reasonable number of iterations (say 4 or 5), if these values corresponding to some measurements exceed a pre-specified threshold value (say 0.1), these are suspected to be bad measurements. These measurements are replaced by the calculated values or pseudo measurements and the state estimation iterative process is continued.

5. State Estimation – Iterative Scheme

Initially the node voltage magnitudes are set to the measured voltages if they are available. Otherwise voltage magnitudes are set to 1.0 per unit and voltage angles are set to 0.0, -120, 120 degrees in phase A, phase B, phase C, respectively and also all the branch currents, powers (complex) are set to (0.0,0.0) pu. $V(1)$ is the source node and its values are assumed to be known. Also its angle $\delta=0$ (taken as reference).

5.1. Backward propagation

The purpose of the backward propagation is to calculate branch currents and then the branch flows in each section. Once a branch flow is calculated this flow is weighted averaged with corresponding measured branch flow. That is, during backward propagation, voltage values are held constant and information about branch currents and averaged flows are transmitted backward along the feeder using backward walk.

In the example network of Figure 3, the backward propagation starts from branch 8-5 and proceeds along the path 7-5, 6-4, 5-4, 4-2, 3-2 and 2-1. During this propagation the load current is calculated assuming the load as the demand measurement in each node, using the equation (1) or (2) or (3), depending on the load type. For constant power loads the load current at i^{th} child node is given by equation (1). For constant current load, the load current is the same as the given load as given in equation (2) and for constant impedance load, the load current at i^{th} node is given by (3)

$$\begin{bmatrix} I_{La}(i) \\ I_{Lb}(i) \\ I_{Lc}(i) \end{bmatrix} = \begin{bmatrix} S_{La}(i) / V_a(i) \\ S_{Lb}(i) / V_b(i) \\ S_{Lc}(i) / V_c(i) \end{bmatrix}^* \quad (1)$$

$$\begin{bmatrix} I_{La}(i) \\ I_{Lb}(i) \\ I_{Lc}(i) \end{bmatrix} = \text{Constant} \quad (\text{Given}) \quad (2)$$

$$\begin{bmatrix} I_{La}(i) \\ I_{Lb}(i) \\ I_{Lc}(i) \end{bmatrix} = \begin{bmatrix} V_a(i) / Z_{La}(i) \\ V_b(i) / Z_{Lb}(i) \\ V_c(i) / Z_{Lc}(i) \end{bmatrix} \quad (3)$$

Where $I_{La}(i)$, $I_{Lb}(i)$, $I_{Lc}(i)$ are the load current at i^{th} node for constant power loads; $S_{La}(i)$, $S_{Lb}(i)$, $S_{Lc}(i)$ are the complex power of constant power load at i^{th} node; Z_{La} , Z_{Lb} , Z_{Lc} are the impedance of constant impedance load at i^{th} node

The current in branch m having parent node j and child node as i, is given by equation (4). Once the child node current is calculated, corresponding branch flow is calculated using the equation (5) and (6) and it is weighted averaged with the measured branch flow.

$$\begin{bmatrix} I_a(m) \\ I_b(m) \\ I_c(m) \end{bmatrix} = \sum_{p \in m} \begin{bmatrix} i_a(p) \\ i_b(p) \\ i_c(p) \end{bmatrix} + \begin{bmatrix} I_{La}(i) \\ I_{Lb}(i) \\ I_{Lc}(i) \end{bmatrix} \quad (4)$$

Where, branch m is connected to branch p and branch p parent node is i; $I_a(m)$, $I_b(m)$ and $I_c(m)$ = current in m^{th} branch having child node as i; $i_a(p)$, $i_b(p)$ and $i_c(p)$ = branch current in p^{th} branch.

Power flow in branch m is calculated as,

$$\begin{bmatrix} PIJC_a(m) \\ PIJC_b(m) \\ PIJC_c(m) \end{bmatrix} = \text{Real} \begin{bmatrix} V_a(j) \cdot i_a(m)^* \\ V_b(j) \cdot i_b(m)^* \\ V_c(j) \cdot i_c(m)^* \end{bmatrix} \quad (5)$$

$$\begin{bmatrix} QIJC_a(m) \\ QIJC_b(m) \\ QIJC_c(m) \end{bmatrix} = \text{Imag} \begin{bmatrix} V_a(j) \cdot i_a(m)^* \\ V_b(j) \cdot i_b(m)^* \\ V_c(j) \cdot i_c(m)^* \end{bmatrix} \quad (6)$$

If the flow measurements are not available then the values are calculated prior to the iteration. Then the difference between the measured and calculated values is obtained. If this error exceeds the normal threshold value then it is suspected as bad data. Thus bad data is detected and replaced by calculated values. The error between measured and calculated flows and loads are given in Equations (7), (8), (9) and (10). These are monitored during each iteration. After 4 or 5 iterations when the error is more than the threshold value the corresponding measurements are eliminated.

$$\begin{bmatrix} PIJE_a \\ PIJE_b \\ PIJE_c \end{bmatrix} = \begin{bmatrix} PIJC_a - PIJM_a \\ PIJC_b - PIJM_b \\ PIJC_c - PIJM_c \end{bmatrix} \quad (7)$$

$$\begin{bmatrix} QIJE_a \\ QIJE_b \\ QIJE_c \end{bmatrix} = \begin{bmatrix} QIJC_a - QIJM_a \\ QIJC_b - QIJM_b \\ QIJC_c - QIJM_c \end{bmatrix} \quad (8)$$

$$\begin{bmatrix} PIE_a \\ PIE_b \\ PIE_c \end{bmatrix} = \begin{bmatrix} PIC_a - PIM_a \\ PIC_b - PIM_b \\ PIC_c - PIM_c \end{bmatrix} \quad (9)$$

$$\begin{bmatrix} QIE_a \\ QIE_b \\ QIE_c \end{bmatrix} = \begin{bmatrix} QIC_a - QIM_a \\ QIC_b - QIM_b \\ QIC_c - QIM_c \end{bmatrix} \quad (10)$$

Where $PIJE_a$ is the error between measured and calculated real power flows, $QIJE_a$ is the error

between measured and calculated reactive power flows, PIEa is the error between measured and calculated real power loads, and QIEa is the error between measured and calculated reactive power loads

The weightages (W) to measurements are set as follows: If index =1, W = W1= 0.50; if index = 2, W = W2 = 0.45; if index = 3, W = W3 = 0.40. Then the weighted average flows are calculated using the equations (11) and (12),

$$\begin{bmatrix} PIJW_a(m) \\ PIJW_b(m) \\ PIJW_c(m) \end{bmatrix} = \begin{bmatrix} \{(1-W)PIJC_a+(W)PIJM_a\} \\ \{(1-W)PIJC_b+(W)PIJM_b\} \\ \{(1-W)PIJC_c+(W)PIJM_c\} \end{bmatrix} \quad (11)$$

$$\begin{bmatrix} QIJW_a(m) \\ QIJW_b(m) \\ QIJW_c(m) \end{bmatrix} = \begin{bmatrix} \{(1-W)QIJC_a+(W)QIJM_a\} \\ \{(1-W)QIJC_b+(W)QIJM_b\} \\ \{(1-W)QIJC_c+(W)QIJM_c\} \end{bmatrix} \quad (12)$$

Where PIJW(m), QIJW(m) are the weighted average real and reactive power flows in branch m; PIJC, QIJC are the calculated value of real and reactive power flow in branch m; PIJM, QIJM are the measured value of real and reactive power flow in branch m

5.2. Forward propagation.

The purpose of forward propagation is to calculate the voltage and load at each node starting from the source node of the feeder. The feeder substation source voltage is set to its measured value. During forward propagation, the branch currents calculated based on the averaged flows, are used to calculate the nodal voltages and hence the loads at each node. The node voltages are calculated using the equation (13), starting from feeder source and proceeding along the forward path.

$$\begin{bmatrix} V_a(j) \\ V_b(j) \\ V_c(j) \end{bmatrix} = \begin{bmatrix} V_a(i) \\ V_b(i) \\ V_c(i) \end{bmatrix} - \begin{bmatrix} Z_{aam} & Z_{abm} & Z_{acm} \\ Z_{ab,m} & Z_{bbm} & Z_{bcm} \\ Z_{ac,m} & Z_{bcm} & Z_{ccm} \end{bmatrix} \begin{bmatrix} I_a(m) \\ I_b(m) \\ I_c(m) \end{bmatrix} \quad (13)$$

Where $I_a(m)$, $I_b(m)$ and $I_c(m)$ are up-dated current values based on weighted average power

flows of branch m, connected between i^{th} and j^{th} nodes.

5.3. Convergence criteria

The steps outlined in backward and forward propagation are followed during each iteration of voltage computations. The convergence criterion is that, voltage magnitudes of real and imaginary parts of complex voltage at each node are compared with its previous iteration values. Therefore the voltage mismatch for j^{th} node during k^{th} iteration is given by equation (14).

$$\Delta V^k(j) = V^k(j) - V^{k-1}(j) \text{ for a, b, and c phases} \quad (14)$$

$$\text{Real } |(\Delta V(j))| < \epsilon, j \text{ all the nodes} \quad (15)$$

$$\text{Imag } |(\Delta V(j))| < \epsilon, j \text{ all the nodes} \quad (16)$$

$$\text{Real power flow } |(PIC-PIM)| < \epsilon \quad (17)$$

$$\text{Reactive power flow } |(QIC-QIM)| < \epsilon \quad (18)$$

$$\text{Real power injection } |(PIJC-PIJM)| < \epsilon \quad (19)$$

$$\text{Reactive power flow } |(QIJC-QIJM)| < \epsilon \quad (20)$$

If both the equation (15) and (16) are satisfied the iterative process is stopped. Once the voltages are estimated all the branch currents and the real and reactive power flows, losses, effect of unbalanced can be calculated. In addition to voltage, the absolute errors of measured and calculated values of real and reactive power flows as in equation (17) and (18), and real and reactive power injection in branch as shown in Equation (19) and (20) are checked.

6. System Studies and Results

Based on the proposed algorithm a computer program in C language was developed for unbalanced three-phase radial distribution systems. The algorithm was tested on distribution systems with a large number of nodes and multiple feeders. Results of the studies on a sample 7-node system and a practical 19-node distribution feeder are presented.

6.1. Unbalanced Three Phase 7-Node System

Based on the proposed algorithm the computer program developed was tested on an unbalanced three-phase 7-node sample system shown in Figure 4.

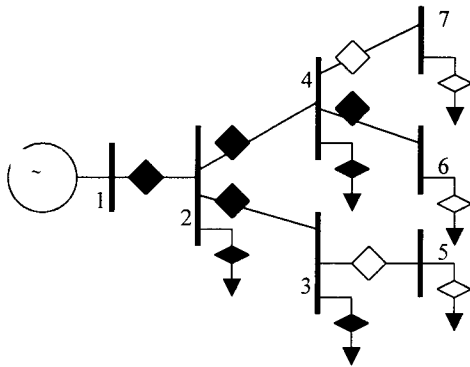


Figure 4: Load measurements at nodes 5, 6, 7 and the flows in branches (3-5) and (4-7) are not available

Table 2a: Input load measurements at nodes

Node No	Input data – demand measurements			
	Phase A	Phase B	Phase C	Index
2	0.1441	0.1141	0.1742	1
3	0.2162	0.1862	0.2462	1
4	0.1622	0.1321	0.1922	1
5	0.0000	0.0000	0.0000	0
6	0.0000	0.0000	0.0000	0
7	0.0000	0.0000	0.0000	0

Table 2b : Output load measurements at nodes

Node No	Observability analysis algorithm output			
	Phase A	Phase B	Phase C	Index
2	0.1441	0.1141	0.1742	1
3	0.2162	0.1862	0.2462	1
4	0.1622	0.1321	0.1922	1
5	0.0972	0.0631	0.1339	3
6	0.0724	0.0420	0.1033	2
7	0.1157	0.0802	0.1562	3

The total real and reactive power generations are 829.461kW and 399.593 kVAR respectively. The total real and reactive power loads are 792.300 kW and 383.600 kVAR respectively. Assume that the load measurements at nodes 5, 6, 7 and the flows in branches (3-5) and (4-7) are not available. The load power measurements at node 5, 6 and 7 are unknown and are given index 0 as shown in table 2a. The observability routine detects the index 0 and computes the values. The load at node 6 is replaced by the flow measurement in branch (4-6) and its index is made 2. The flows in branches (4-7) and (3-5) are computed from the available measurements at node 4 and 3. The loads at nodes 5 and 7 are

replaced by these computed flows as shown in table 2b. The index is made 3. The flow measurements in branches (3-5) and (4-7) are not available as shown in table 3a. These are computed from other available values at node 3 and node 4. The computed values are replaced and index is made 2 as shown in table 3b. Bad data was created in the real power flow measurements in branches (3-5) and (4-7). The presence of bad data was detected at iteration 2. By the end of the state estimation iteration process the bad data is eliminated as shown in Figure 5 at the end of iteration 4. These are detected, removed and replaced by the corresponding pseudo values.

Table 3a: Input flow measurements in the branches

From Node	To Node	Input data – Flow measurements in pu.			
		Phase A	Phase B	Phase C	Index
1	2	0.8292	0.6252	1.0513	1
2	3	0.3134	0.2493	0.3802	1
2	4	0.3503	0.2543	0.4517	1
3	5	0.0000	0.0000	0.0000	0
4	6	0.0724	0.0420	0.1033	1
4	7	0.0000	0.0000	0.0000	0

Table 3b: Output flow measurements in the branches

From Node	To Node	Observability analysis algorithm output			
		Phase A	Phase B	Phase C	Index
1	2	0.8292	0.6252	1.0513	1
2	3	0.3134	0.2493	0.3802	1
2	4	0.3503	0.2543	0.4517	1
3	5	0.0972	0.0631	0.1339	2
4	6	0.0724	0.0420	0.1033	1
4	7	0.1157	0.0802	0.1562	2

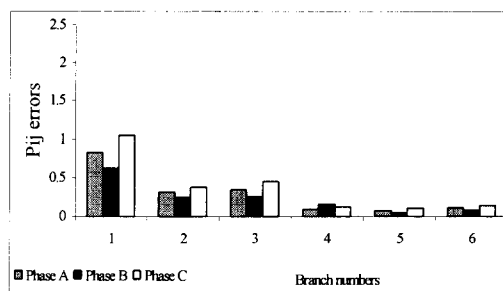


Figure 5: Bad data removed in branch (3-5) and (4-7) after iteration 4

6.2 Practical Unbalanced Three Phase 19 Node Distribution System Results

This practical distribution network consists of 19 nodes and 18 branches as shown in figure 6. The r/x ratio of the network is about 3.52. The input data consists of three-phase 18 number of MW and MVAR load demand measurements, 18 number network flow measurements. Bad data was created in the real power flow measurements in branches (4-6), (14-18), and (10-12) or branch numbers (5), (14) and (15). The presence of bad data is shown in Figure 7 at iteration 2. By the end of the state estimation iteration process the bad data is eliminated as shown in Figure 8 at the end of iteration 6. These are detected, removed and replaced by the corresponding pseudo values. The estimated voltage profile in the three phases of the state estimator with observability analysis and bad data processing is shown in Figure 9.

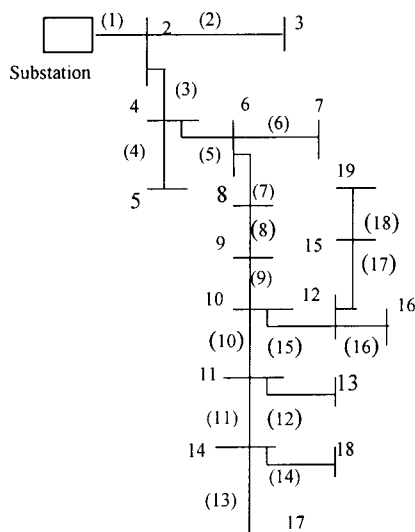


Figure 6: A practical distribution feeder

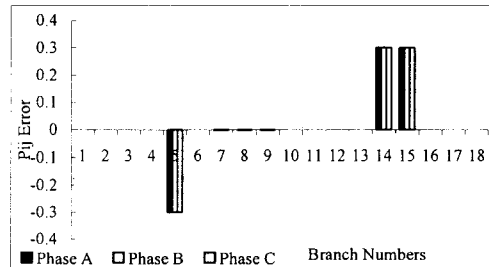


Figure 7: 19-node system with positive and negative errors in branches 5,14 and 15 at iteration 2.

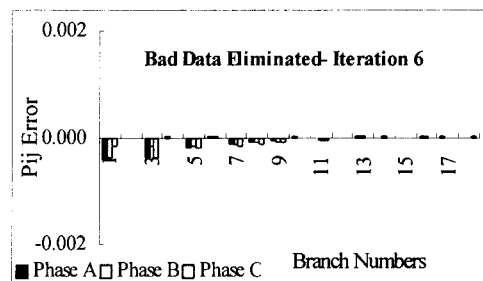


Figure 8: 19-node system errors eliminated in branches 5,14 and 15 at the end of iteration 6.

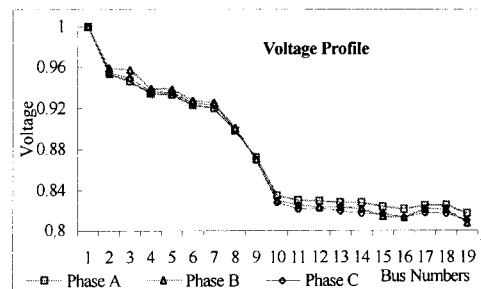


Figure 9: Estimated voltage profile in phases A, B, C for the 19-node system.

7. Conclusions

An efficient and robust three-phase state estimation solution algorithm incorporating network observability analysis and bad data processing has been presented. The algorithm

based on forward and backward propagation is suitable for unbalanced three-phase radial distribution networks. The new methodology has been tested to analyze practical distribution networks having a higher r/x ratio of lines. The proposed method has worked well regardless of the feeder r/x ratio. Results on sample and practical system studies have been presented.

8. References

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