PAPR Reduction in OFDM Systems

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Abstract

Orthogonal Frequency Division Multiplexing or OFDM is a form of multi-carrier modulation technique. High spectral efficiency, robustness to channel fading, immunity to impulse interference, uniform average spectral density, capability of handling very strong echoes and less nonlinear distortion are among the favorite properties of OFDM. Even though there are many advantages of OFDM, it has two main drawbacks: high Peak to Average Power Ratio (PAPR) and frequency offset. In this paper, the issue of PAPR in OFDM is discussed. Two new algorithms are proposed to reduce the PAPR. The first algorithm is carried out by selecting the input sequences properly using a lookup table and the second by scaling the input envelope for subcarriers before they are transformed to the time domain by Inverse Fast Fourier Transform (IFFT). Simulation results show that the PAPR can be reduced significantly with both the schemes.

Keywords: Orthogonal Frequency Division Multiplexing (OFDM), Peak-to-Average Power Ratio (PAPR), QPSK.

Introduction

After more than thirty years of research and developments carried out in different places, orthogonal frequency division multiplexing (OFDM) has been widely implemented in high speed digital communications. Due to the recent advancements in digital signal processing (DSP) and very large scale integrated circuits (VLSI) technologies, the initial obstacles of OFDM implementations do not exist any more. Meanwhile, the use of Fast Fourier transform (FFT) algorithms has eliminated arrays of sinusoidal generators and coherent demodulation required in parallel data systems and made the implementation of the technology cost effective. In recent years OFDM has gained a lot of interest in diverse digital communication applications. This has been due to its favorable properties like high spectral efficiency and robustness to channel fading. Today, OFDM is mainly used in digital audio broadcasting system (DAB) initiated by CCETT in France [1], and digital video broadcasting system (DVB) [2] enabling an end-to-end digital transmission system, which is spectrally efficient and rugged against channel distortions. This can be used for services such as HDTV, offering increased capacity for program broadcasting. In the conventional serial data transmission system, the information symbols are transmitted sequentially where each symbol occupies the entire available spectrum bandwidth. But in an OFDM system, the information is converted to N parallel subchannels and sent at lower rates using frequency division multiplexing. The subcarrier frequency spacing is selected carefully such that each subcarrier is located on the other subcarriers' zero crossing points. This implies that there is overlapping among the subcarriers but will not interfere with each other, if they are sampled at the sub carrier frequencies. This means that all subcarriers are orthogonal.

The OFDM has many advantages such as high bandwidth efficiency, robustness to the selective fading problem, use of small guard interval, and its ability to combat the ISI problem. So, simple channel equalization is needed instead of complex adaptive channel equalization.

Apart from various advantages of OFDM, there are certain disadvantages also. The frequency offset of the subcarriers and the high PAPR are the major drawbacks of OFDM [3].

In this paper, the PAPR problem has been addressed and two new algorithms are proposed to reduce the PAPR. The rest of the paper is organized as follows. The next section provides an introduction to OFDM system, followed by a detailed analysis of PAPR, the related work on PAPR reduction, the proposed schemes and simulation results. Finally, the conclusions and scope for future work concludes the paper.

Introduction to OFDM

An OFDM symbol consists of N subcarriers by the frequency spacing of Δf. Thus, the total bandwidth B will be divided into N equally spaced subcarriers. And all the subcarriers are orthogonal to each other within a time interval of length $T = 1/\Delta f$. Each subcarrier can be modulated independently with the complex modulation symbol $X_{m,n}$, where m is a time index and n is a subcarrier index. Then within the time interval T the following signal of the m-th OFDM block period can be described by equation (1) as:

$$x_m(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_{m,n} g_n(t - mT) \quad (1)$$

where, $g_n(t)$ is defined through equation (2).

$$g_n(t) = \begin{cases} \exp(j2\pi n\Delta f t), & 0 \le t \le T \\ 0, & else \end{cases} \quad (2)$$

where, $g_n(t)$ is a rectangular pulse applied to each subcarrier [4]. The total continuous time signal $x(t)$ consisting of all the OFDM blocks is given by equation (3).

$$x(t) = \frac{1}{\sqrt{N}} \sum_{m=0}^{\infty} \sum_{n=0}^{N-1} X_{m,n} g_n(t - mT) \quad (3)$$

Now, consider a single OFDM symbol ($m = 0$) without loss of generality. This can be shown because there is no overlap between different OFDM symbols.

Since $m = 0$, $X_{m,n}$ can be replaced by X_n .Then, the OFDM signal can be described as follows:.

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi n\Delta f t} \quad (4)$$

If the bandwidth of the OFDM signal is $B = N \times \Delta f$ and the signal $x(t)$ is sampled by the sampling time of $\Delta t = \frac{1}{B} = \frac{1}{N\Delta f}$, then the OFDM signal is in discrete time form and can be written as shown in equation (5).

$$x_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi kn/N}, k = 0,1,\ldots,N-1 \quad (5)$$

where, n denotes the index in frequency domain and X_n is the complex symbol in frequency domain. Furthermore, equation (5) can be expressed using the IFFT [5].

Figure 1 shows a typical system block diagram of an OFDM system. The serial input data bit stream is converted to N parallel subchannels and mapped with a selected modulation scheme (signal mapper), resulting in N subchannels containing information in complex number form.

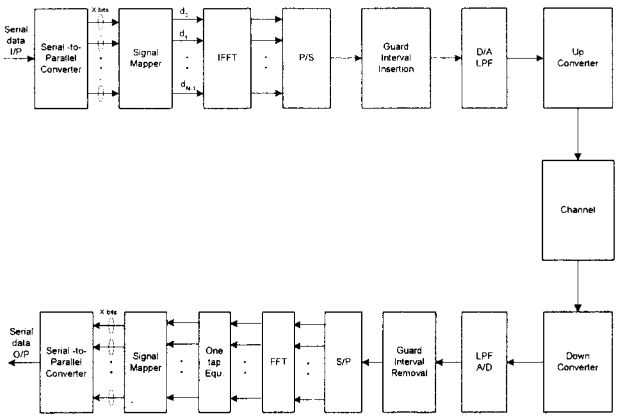

Figure 1: Basic FFT based OFDM system

These complex values are then sent to the N channel IFFT. Then, the parallel signals are converted back to a serial sequence by using a P/S device. A guard interval is inserted to reduce the effect of ISI caused by multipath propagation. Finally, the signal is converted to an analog signal and converted back up to a form suitable for transmission. At the receiver, a reverse procedure is used to demodulate the OFDM signal. In an OFDM system, only a simple equalizer, or more specifically, a single tap equalizer, is needed at the receiver to remove the ISI.

Peak-to-Average Power Ratio (PAPR)

As explained earlier, one of the major drawbacks of OFDM is the very high peak-to-average power ratio (PAPR). PAPR of OFDM increases exponentially with the number of subcarriers. If power amplifiers are not operated with large power back-offs, it is impossible to keep the out-of-band power below the specified limits. This situation leads to very inefficient amplification and expensive transmitters, so it is highly desirable to reduce the PAPR.

In this section, a detailed mathematical analysis for PAPR is presented. The Root Mean Square (RMS) magnitude of the OFDM signal is defined as the root of the time average of the envelope power ($\sqrt{\overline{P}}$), where $\overline{P}$ is defined by equation (6).

$$\overline{P} = \frac{1}{T}\int_{t=0}^{T}|x(t)|^2 dt = \frac{1}{N}\sum_{n=0}^{N-1}|X_n|^2 \quad (6)$$

where, $x(t)$ is the OFDM signal defined by equation (3). The value $\overline{P}$ in this case corresponds to a single OFDM symbol, and depends on the sequence of information carrying coefficients $\{X_n\}$. The average power of OFDM symbols can be written as $P_{av} = E\{\overline{P}\}$. Thus, the PAPR of an OFDM signal can be defined as,

$$\xi = \frac{\max\limits_{t\in[0,T)}|x(t)|^2}{P_{av}} \quad (7)$$

$$= \frac{\max\limits_{t\in[0,T)}|x(t)|^2}{E\left\{\,|x(t)|^2\right\}}$$

If the input data power is normalized, then $E\left\{\,|x(t)|^2\right\} = 1$, and we get,

$$\xi = \max_{t\in[0,T)}|x(t)|^2 \quad (8)$$

$$\xi = \max_{t\in[0,T)}\left|\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}X_n e^{j2\pi n\Delta ft}\right|^2 \quad (9)$$

$$\leq \frac{1}{N}\left|\sum_{n=0}^{N-1}X_n e^{j2\pi n\Delta ft}\right|^2 \quad (10)$$

$$\leq N \quad (11)$$

It clearly shows that the maximum PAPR is equal to the number of subcarriers N. For M-PSK modulation, there are only M^2 sequences having maximum PAPR N as described in [6]. This means, the number of sequences that gives very high PAPR is not very high. If the number of subchannels increases, the ratio of the sequence that gives very high PAPR and all distinct sequences decreases rapidly. The overall number of distinct sequences for the N subcarrier-OFDM system with M-PSK is M^N. Thus the ratio can be obtained by equation (12) and (13) as:

$$R = \frac{M^2}{M^N} \tag{12}$$

$$R = M^{2-N} \tag{13}$$

Related Works

Several algorithms [7-13] have been proposed to handle this PAPR problem. However, none of these algorithms have produced significant reduction of PAPR in OFDM systems.

Partial Transmit Sequence (PTS) was proposed in [7]. In PTS, the information bearing subcarrier block is subdivided into disjointed carrier subblocks and introduced rotation factors for each sub-block and modified the subcarrier amplitude vector. Thereby, PAPR was reduced with different rotation factors for different sub-blocks. This needs a number of iterations to find the optimum combination of factors for sub-blocks. Adaptive PTS [8] was proposed to reduce the number of iterations by setting up a desired threshold and trial for different weighing factors until the PAPR dropped below the threshold. With this approach we can reduce the number of iterations and the complexity of the system by only 0.1% loss in reduction of PAPR. In [8], 256 subcarriers are considered with QPSK. Results showed that the PAPR can be reduced by 4.1 dB and 4.0 dB without adaptive PTS and with adaptive PTS respectively. However these two approaches need to send side information to the receiver which implies a reduction in the bandwidth efficiency.

PTS with embedded side information [9] is another approach that can be combined with both conventional and adaptive PTS. This approach embeds a combined knowledge within the transmitted data, so no extra bits are sent. But these introduce word errors during detection of the sequence information.

A simple Encodable/Decodable OFDM QPSK proposed in [10] used Reed-Muller code with QPSK. This could reduce the PAPR to less than 6 dB but it could not be used with higher order signal constellations. OFDM PAPR reduction by a rotation of redundancy bit position in subblock code word scheme was proposed in [11]. In this method the redundant bit positions of subblock code words are rotated and the lowest PAPR codeword is chosen by a feed back scheme. However, the side information for bit position is required.

Oversized IFFT [12] is proposed as another scheme to solve this problem. In oversized IFFT, clipping and filtering are done by forward and inverse FFT. This can avoid out of band power but with some in band distortion, overall shrinking of constellation and with the introduction of some noise like components. Companding transform [13] compresses a large signal while enhancing a small signal that can achieve a desired PAPR but with a significant increase in the bit error rate (BER).

Proposed Methods to Reduce PAPR

I. PAPR Reduction using Lookup Tables

We have studied the effect of BPSK and QPSK modulation schemes on PAPR, for systems with different number of channels (2^k number of channels in the set), and found that QPSK is a better option as compared to BPSK. Table 1 summarizes the information about the 0 dB PAPR sequences found in the study with QPSK as the modulation format. The study is limited to a maximum of 16 channels and further work is in progress to generalize the idea and methodology for any number of channels of the form 2^k.

Using the information given in Table 1, we can find an expression for the number of sequences in each case as follows.

Let k be an integer, then

Number of channels = 2^k

Number of all possible input sequences = 4^{2^k}

Number of sequences with 0 dB PAPR = 2×4^k

Table 1: The number of QPSK input sequences that gives 0 dB PAPR

K	Number of channels	Number of all distinct input sequences	Number. Of sequences which 0 dB PAPR output
1	2	16	8
2	4	256	32
3	8	65,536	128
4	16	4,294,967,296	512

Now, consider the 8 channel system of Table 1. We can generate the 0 dB PAPR OFDM system with 8 subcarriers and information rate of 7/16, by grouping the input bit stream into a group of 6 or 8 bits. The output of the QPSK signal mapper is grouped in either 3 or 4 complex numbers (each complex number corresponds to a symbol) and each group is compared with table containing 0 dB PAPR sequences corresponding to the system. The matching sequence is sent to the 8 channel IFFT.

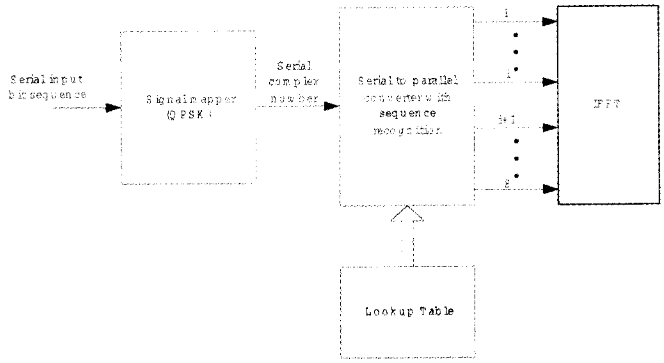

Figure 2(a): Block diagram for the input sequence selection

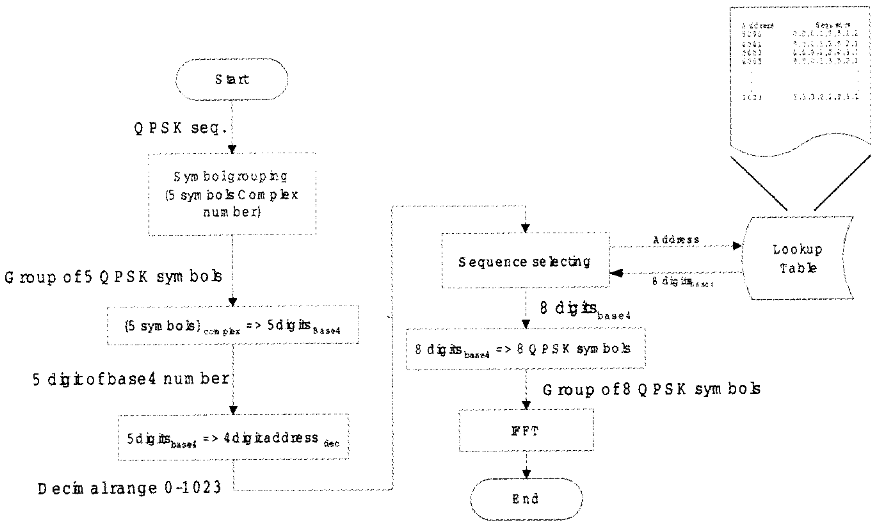

Figure 2(b): Flowchart of the lookup table approach

Figure 2 shows the block diagram of the proposed algorithm. In figure 2(a), channel 1 to i are related to the number of real information channels and the rest contain residual values to make the sequence 0 dB PAPR. As proved above, i can be either 3 or 4 depending on the serial complex number sequence. Thus, the information rate is $\log_4(128)/8$, which is equal to 7/16. Since most of the practical systems can afford to have a PAPR of more than 0 dB, we can increase the information rate until the PAPR reaches the maximum tolerable level of the system. Then the number of input sequences in the lookup table can be increased as shown in Table 2. Figure 2(b) presents the complete flowchart of the lookup table approach to reduce PAPR.

Table 2: Maximum PAPR and information rate

Maximum PAPR (dB)	Number of useful Input sequences	i (average value)	Information rate
0	128	3.5	7/16
0.97	1152	5.0	5/8
1.76	2177	5.5	11/16
2.32	2433	5.6	28/40
2.68	2945	5.7	57/80
2.92	5504	6.2	62/80
3.01	12096	6.7	67/80

An added advantage of this method is that it can detect possible errors as the system transmits only a specific set of sequences and hence any sequence out of this set can clearly be concluded as an error. An educated guess can be considered to be a coarse error correction.

Simulation Results

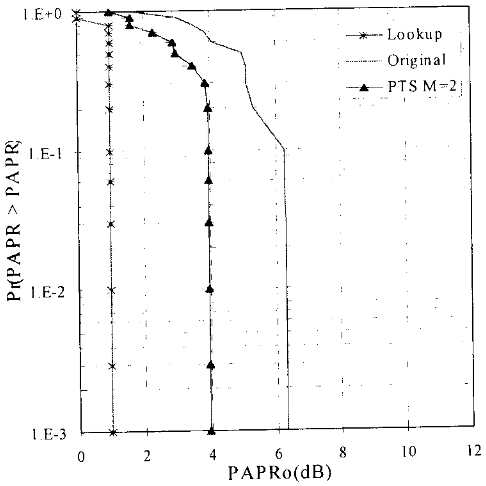

Figure 3: Comparison of CCDF of PAPR with lookup table and PTS with $M = 2$

The complementary cumulative distribution function (CCDF = Pr(PAPR > $PAPR_0$)) has been used to measure PAPR of an OFDM system. We also use CCDF to present our results. Figure 3 shows CCDF distribution for the lookup table approach. For the comparison purpose, CCDF curves of the original OFDM system and the PTS with $M = 2$ have also been included. It clearly shows that the PAPR can be reduced to 0.97 dB with the proposed scheme; whereas the PTS can reduce the PAPR only up to 4 dB. However, the information rate of the lookup table approach is lower than that of the PTS scheme.

II. PAPR Reduction using Envelope Scaling

We have studied the effect of envelope scaling by scaling factors to reduce the PAPR in the OFDM system. In our study, we use 256 subcarriers and the QPSK modulation technique so that the envelopes of all the subcarrier inputs are equal. The proposed idea is to scale the envelope of the input in some subcarriers to obtain the minimum PAPR at the output of the IFFT. The final input that gives the lowest PAPR will be sent to the system. Note that this input sequence has the same phase information as the original one, but the envelopes are different. So the receiver can demodulate the received sequence without any side information.

The detailed explanation is given as follows. In Figure 4, the input N data block is partitioned into disjoint subblocks or clusters. First define the data block as a vector as:

$X = [X_0 \quad X_1 \quad \dots \quad X_{N-1}]^T$ Then partition $\boldsymbol{X}$ into M disjoint sets, represented by the vectors $\{\boldsymbol{X}_m, m = 1,2,\dots, M\}$. All vectors $\boldsymbol{X}_m$ are then scaled with the scaling factors $s_m \in (0,1]$, $m = 1,2,...M$ and combined together before being sent to the IFFT.

$$\boldsymbol{X}' = \sum_{m=1}^{M} s_m \boldsymbol{X}_m \qquad (14)$$

$$x' = IFFT\{\boldsymbol{X}'\} \qquad (15)$$

The scaling factors are chosen to minimize the PAPR of x'. A sub-optimum iteration is introduced to reduce the complexity of the calculations.

The scaling factors are set to the one digit decimal such as 0.4, 0.5... 0.8 and for example let all the scaling factors are {0.5, 1}. After dividing the input sequence into M clusters, set all scaling factors, $s_m = 1$, calculate the PAPR of the combined signal as shown in eq.(14). Next, change the first scaling factor $(s_1 = 0.5)$ and recompute the resulting PAPR. If the new PAPR is lower than that in the previous step, retain s_1 as part of the final sequence, otherwise change it back to the one that gives the lowest PAPR (in this case it is 1). The algorithm continues in this fashion until all s_m are explored in general but in the particular case as explained above, only M iterations have been done. Note that the possible values of s_m can be more than 0.5 and 1 as explained in this example but it increases the system complexity. Figure 5 shows the flow chart of the Envelope Scaling algorithm.

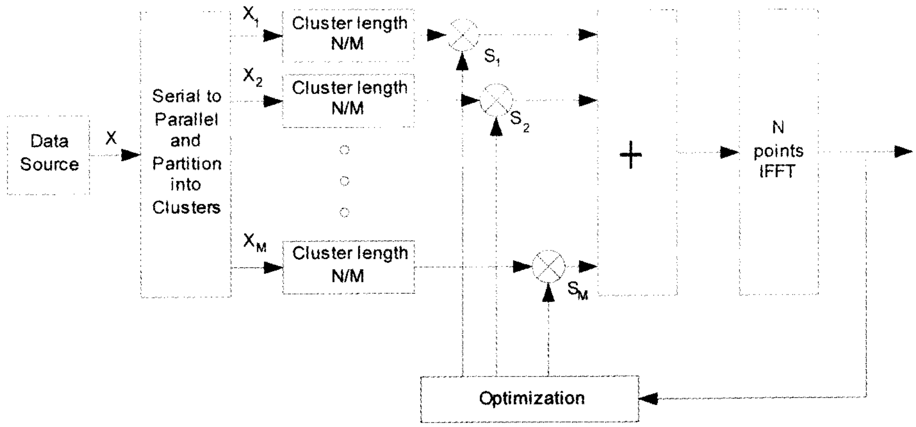

Figure 4: Block Diagram for envelope scaling

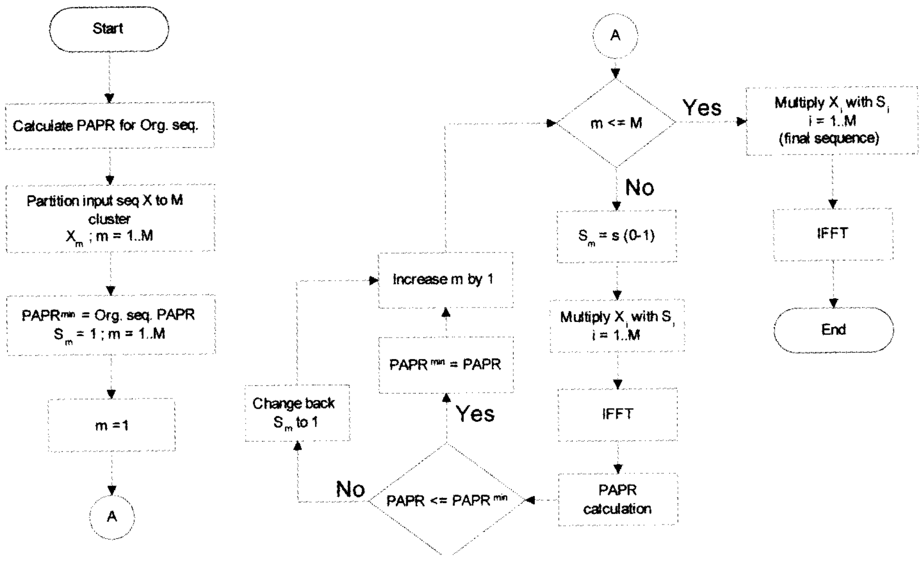

Figure 5: Flow Chart for envelope scaling

Simulation Results

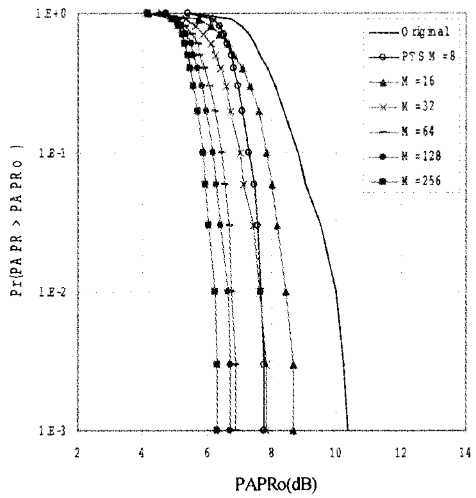

Figure 6: Comparison of CCDF of PAPR with scaling factor = 0.5 and PTS with $M = 8$

In Figure 6, the complementary cumulative distribution function (CCDF = Pr(PAPR > $PAPR_0$)) of an OFDM signal with envelope scaling approach is shown with $N = 256$ subcarriers, and QPSK as the modulation scheme for all the subcarriers. The results in Figure 6 are for varying the number of cluster M with fixed scaling factor at 0.5. It shows that by increasing the number of M, PAPR can be reduced more efficiently.

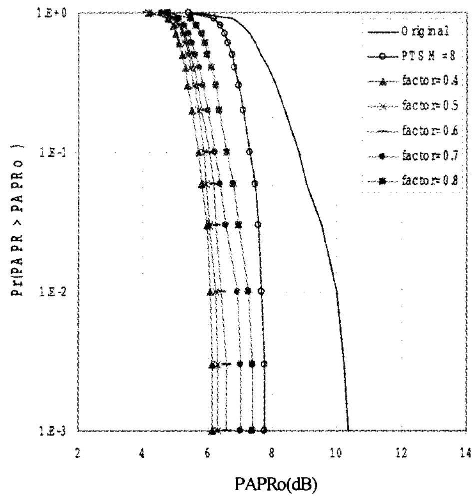

Figure 7: Comparison of the result with PTS and one scaling factor value

Figure 7 represents the CCDF distribution with different scaling factors. The value of M is fixed at 256 clusters or equal to N and varies the scaling factor. As compared to the modified PTS [14] with $M = 8$, the 1% PAPR is reduced by 1.8 dB and by 4 dB as compared to the original symbol. Note that the smaller scaling factor gives more reduction in PAPR.

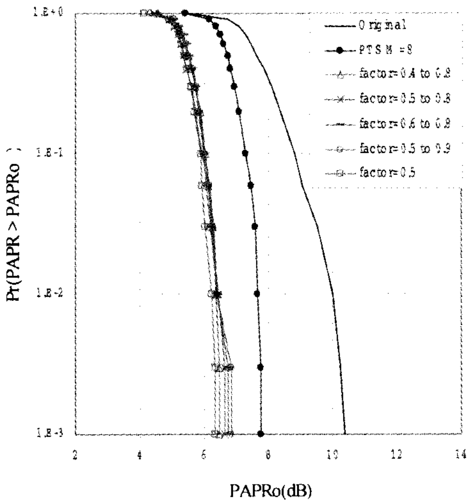

Figure 8: CCDF of PAPR with multi scaling factors

Figure 8 shows the result for the case of multiple scaling factors when M is fixed at 256 at a scaling factor of 0.5. It is clear that the performance of the multiple scaling factors is lower than that for the case of fixed scaling factor. Now the average power of the system is considered. The adaptive PTS approach is to change only the phase of the sequence, so the average power of the OFDM symbol remains the same. For envelope scaling approach, we change the envelope of the input sequence making the output of the corresponding OFDM symbol to change.

Figure 9 shows the BER performance of the OFDM system with envelope scaling and the PTS with and without Turbo Coding for a frequency selective fading channel. As we compare only the performances of the two PAPR reduction schemes, envelope scaling and PTS, the nonlinearity effect is not considered in the simulations. This clearly shows that the BER performances of both the schemes are more or less the same with channel coding (Turbo coding). With the nonlinearity effect of the high power amplifier, the BER performance of the proposed scheme must improve significantly with respect to the PTS since the PAPR performance of the proposed scheme is much better than the PTS.

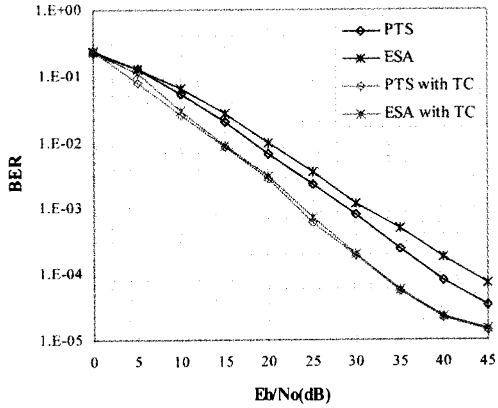

Figure 9: BER Performance of envelope scaling (N=256, scaling factor=0.5) with PTS.

Besides the significant reduction of the PAPR, the main advantage of this approach is that the receiver does not need any side information. However the complexity of the proposed scheme is higher than the PTS especially for the case of $M=N$. But a sub-optimum approach can be found if we need to reduce the complexity. The limitation of the proposed scheme is that it cannot be applied with non-constant modulation schemes.

Conclusions

In this paper, we proposed two schemes to reduce the PAPR in an OFDM system. In the first approach the input sequence is compared with the sequence that gives the lowest PAPR in the lookup table. Results show that PAPR can be reduced to 0 dB when the information rate is 7/16 and to 0.97 dB when rate is 5/8, for an OFDM system with 8 channels using QPSK modulation. In the second scheme we scale the input sequence envelope using different scaling factors before transforming the new sequence into the time domain by IFFT. Results show that the PAPR can be reduced significantly, at most 4 dB for 1% CCDF without the need of any side information for the receiver. Finally the system with single scaling factor (0.5) and the number of clusters equal to the number of subcarriers is recommended. Future work is required to extend the first algorithm to a system with large number of subchannels as well as to analyze other modulation techniques and develop an optimum algorithm for higher number of sub-channels in the second algorithm.

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