Integration of Loss Function in Two-Dimensional Deterministic Tolerance Synthesis

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Abstract

The problems of deterministic tolerance synthesis are generally solved using a combinatorial optimization problem, for instance, an integer programming approach, a design of experiments, and the Taguchi method. Researchers generally consider one-dimensional deterministic tolerance synthesis; however, the problems of two or three-dimensional deterministic tolerance synthesis sometimes happen. This paper deals with two-dimensional deterministic tolerance synthesis. Since the objectives of designing the tolerances are minimizing the direct manufacturing cost and minimizing the sensitivity of tolerances to variations in manufacturing processes and the service environment, the optimization models provided in this paper include the objectives. It is assumed when a product performance deviates from the customer identified target value, loss is assumed to incur quadratically. In short, the concept of Taguchi's loss function is applied two-dimensional deterministic tolerance synthesis. Three general approaches, an integer programming approach, a design of experiments, and the Taguchi method, are utilized and illustrated by six numerical examples.

Keywords: quality engineering, tolerance synthesis, design of experiment, Taguchi loss function

1. Introduction

Fierce competition in the international marketplace is driving companies to seek ever-increasing product quality as well as reducing costs. The selection of tolerance has a profound impact on the manufacturing processes, product costs, and functional quality. Hence, manufacturers have treated tolerance as a very important topic and realized that a proper selection of design tolerance is a key element in their effort to increase productivity, control product quality, and yield significant cost savings. Tolerance is defined as the range between a specification limit and the nominal dimension. Traditionally, to assign tolerances to components and assemblies, designers relied on experience, handbooks, and standard information. These assignment decisions, usually made at the design stage, are often based on insufficient data or incomplete models. However, the decisions must be taken with full consideration of their significant influence on manufacturing methods, production costs, and product quality.

A survey of the literature indicates that two basic models in tolerance design have been considered. The first one is tolerance analysis -- the component tolerances are specified, and the resulting assembly variation and yield are calculated. The latter case called tolerance synthesis, which we consider through this paper, involves the allocation of the specified assembly tolerances among the component dimensions of an assembly to ensure a specified yield. Tolerance synthesis is formulated as an optimization problem by treating cost minimization as the objective function and stackup conditions as the constraints. The literature on tolerance synthesis has been reviewed by Voelcker [1], Juster [2], and Chase and Parkinson[3].

Two types of objectives have generally been used in the design of tolerance: (1) minimization of the direct manufacturing cost, and (2) minimization of the sensitivity of tolerances to variations in manufacturing processes and the service environment. The first objective was considered by Kusiak and Feng [4] in the case of one dimensional deterministic tolerance synthesis. This paper extends the idea of Kusiak and Feng [4] by considering two-dimension deterministic tolerance synthesis and introducing the concept of loss functions to the models. The objective of introducing the concept of loss functions is to satisfy the objectives: minimization of the direct manufacturing cost, and minimization of the sensitivity of tolerances to variations in the manufacturing process. This paper gives a comparative study between applying the concept of loss function and not applying the concept among three methods, Integer Programming (IP), Design of Experiments (DOE), and Taguchi Method(TM).

2. Two Dimensional Deterministic Tolerance Synthesis

From a general viewpoint of geometry, three dimensions, such as length, width and height are considered when configuring a physical object. Based on the study of literature in the area of tolerance synthesis, researchers generally consider one dimension (or one side). However, in typical manufacturing processes, two or more dimensions may be considered. This paper considers two-dimensional deterministic tolerance synthesis. The example of two-dimensional deterministic tolerance synthesis is the process of molding. The general form of two-dimensional deterministic tolerances is shown in Figure 1, where x_{ij} represents the dimension element of row i and column j ($i = 1 \ldots m$ and $j = 1 \ldots n$).

3. Bivariate Quadratic Loss Function

Traditionally, the quality evaluation system classified a product as nonconforming if the quality characteristic of the product fails to meet the predetermined specification limits and then a certain amount of economic loss is incurred; otherwise, it is classified as conforming and no loss is incurred. In other words, products are evaluated as good or bad on a go/no-go basis. Contrarily, this system does not adequately reflect customers' perception of quality. Typically, the exact form of quality loss function to evaluate the quality of a product does not exist. Various loss functions [5-9] have been discussed in the literature of statistical decision theory. However, a simple quadratic loss function is treated as a good evaluation in many situations [10-13]. The simple quadratic loss function (nominal the best) is

$$L(x) = \beta (x - \tau)^2, \qquad (1)$$

where β is a loss coefficient and τ is a target value for quality characteristic x. Kapur and Cho [14] proposed the model for the bivariate **loss** function as follows:

$$L(x_1, x_2) = \beta_{11} (x_1 - \tau_1)^2 + \beta_{12} (x_1 - \tau_1)(x_2 - \tau_2) + \beta_{22} (x_2 - \tau_2)^2, \qquad (2)$$

where β_{ij} is a loss coefficient associated between quality characteristics i and j. The loss coefficient β_{ij} can be determined by using a regression method [6]. The expected value of $L(x_1, x_2)$, E[L] is then given by

$$E[L] = \beta_{11} [(\mu_1 - \tau_1)^2 + \sigma_1^2] + \beta_{12} [(\mu_1 - \tau_1)(\mu_2 - \tau_2) + \sigma_{12}] + \beta_{22} [(\mu_2 - \tau_2)^2 + \sigma_2^2]. \qquad (3)$$

Assuming that $\mu_1 = \tau_1$ and $\mu_2 = \tau_2$, Equation [3] becomes

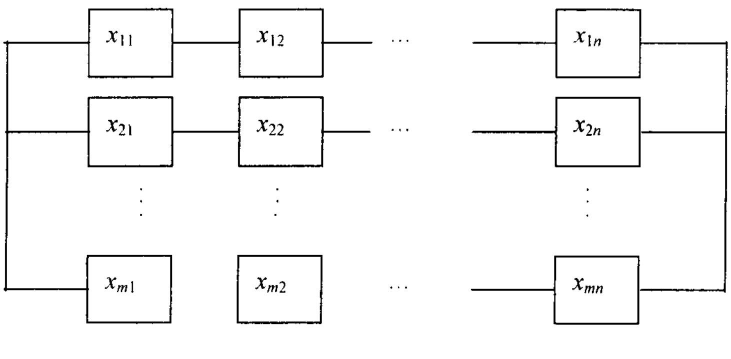

Figure 1 General form of two-dimensional deterministic tolerances

$$E[L] = \beta_{11}[\sigma_1]^2 + \beta_{12}[\sigma_{12}]^2 + \beta_{22}[\sigma_2]^2. \quad (4)$$

Generally, manufacturers accept tolerance at three sigma limits ($t = \pm 3\sigma$) and based on the statistical method, σ_{ij} is calculated by

$$\sigma_{ij} = \rho_{ij} * \sigma_1 \bullet \sigma_2 .$$

where ρ_{ij} is the correlation coefficient between the quality characteristic i and j. Hence, the expected loss in terms of tolerance is given by

$$E[L] = \beta_{11}[t_1/3]^2 + \beta_{12}\rho_{12}[t_1/3][t_2/3] + \beta_{22}[t_2/3]^2, \quad (5)$$

where t_1 and t_2 are the tolerance of dimension 1 and 2; ρ_{12} is the correlation coefficient between dimension 1 and 2.

Assuming that two dimensions are independent, ρ_{12} is zero. Hence, the middle term of Equation [5] is zero and the expected loss becomes

$$E[L] = \beta_{11}[t_1/3]^2 + \beta_{22}[t_2/3]^2 . \quad (6)$$

4. Integer Programming Approach

Ostwald and Huang [15] first introduced the integer programming (IP) approach to discrete tolerance synthesis. To solve a large-scale deterministic tolerancing problem using linear programming, Lee and Woo [16] proposed a branch and bound algorithm. Kim Knott [17] proposed the pseudo-Boolean approach to determine the least cost tolerances. Kusiak and Feng [4] applied the IP approach to deterministic tolerance synthesis problems. Assuming that all components are independent and applying a tolerance-cost analysis, the two-dimensional deterministic tolerance synthesis problems can be formulated as the following 0-1 IP model.

Minimize $$TC = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o} C_{ijk}\, x_{ijk} \quad (7)$$

Subject to

$$\sum_{i=1}^{m}\sum_{k=1}^{o} t_{ijk}\, x_{ijk} \le T_j \qquad \forall j$$

$$\sum_{k=1}^{o} x_{ijk} = 1 \qquad \forall i,j$$

$$x_{ijk} = 0, 1 \qquad \forall i,j,k$$

where TC = total tolerance cost,

C_{ijk} = manufacturing cost of process k used to produce dimension (i,j),

t_{ijk} = three sigma normal variation of process k used o produce dimension (i,j),

T_i = single side tolerance stackup limit for dimension i,

T_j = single side tolerance stackup limit for dimension j,

x_{ijk} = 1 if process k is selected for dimension (i,j) and 0 otherwise,

m = number of dimension i,

n = number of dimension j, and

o = number of process k.

The objective is to minimize the total tolerance cost. The first two constraints are to ensure that the total tolerances in dimension i and j do not exceed the tolerance stackup limits of its dimension. The next constraint is to ensure that exactly one process is selected in its dimension. The last constraint ensures the integrality of x_{ijk}.

Incorporating the quadratic loss concept, the model transforms to

Minimize

$$TC = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{o} C_{ijk}\, x_{ijk} + \sum_{i=1}^{m}\beta_i a_i^2 + \sum_{j=1}^{n}\beta_j b_j^2$$

(8)

Subject to

$$\sum_{j=1}^{n}\sum_{k=1}^{o} t_{ijk} X_{ijk} \le T_i \qquad \forall i$$

$$\sum_{i=1}^{m}\sum_{k=1}^{o} t_{ijk} \quad x_{ijk} \le T_j \qquad \forall j$$

$$\sum_{k=1}^{o} X_{ijk} = 1 \qquad \forall i,j$$

$$a_i = \sum_{j=1}^{n}\sum_{k=1}^{o} t_{ijk}\, x_{ijk}/3 \qquad \forall i$$

$$b_j = \sum_{i=1}^{m}\sum_{k=1}^{o} t_{ijk}\, x_{ijk}/3 \qquad \forall j$$

$$x_{ijk} = 0, 1 \qquad \forall i,j,k$$

where β_i = the loss coefficient for the dimension i, β_j = the loss coefficient for the dimension j, a_i = the standard deviation of dimension i, and b_j = the standard deviation of dimension j.

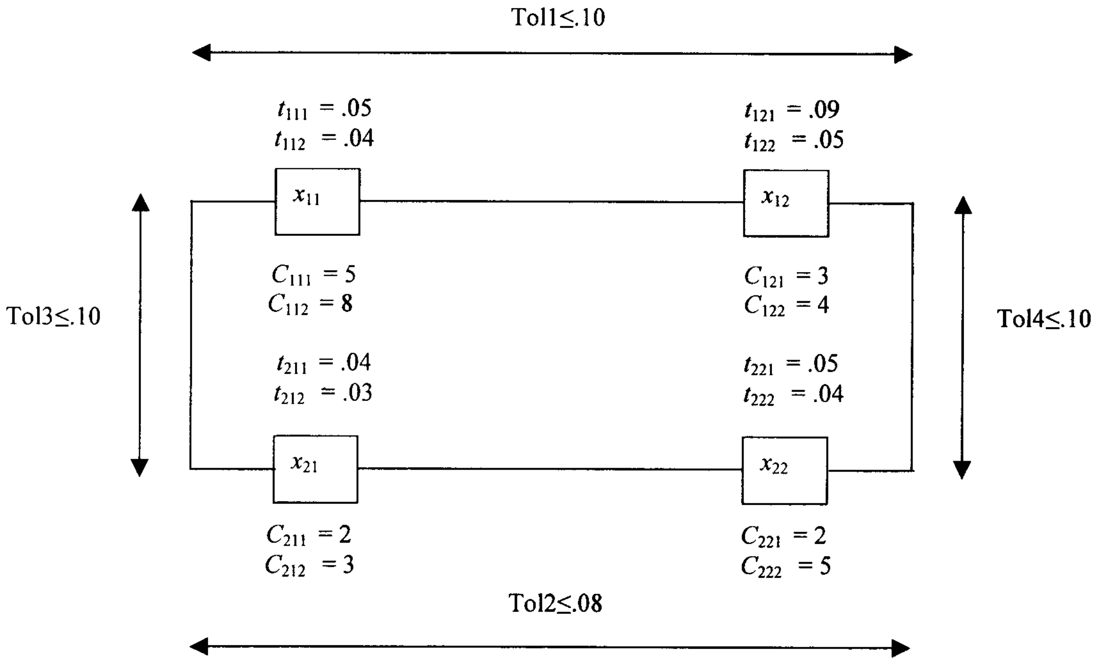

Figure 2 Representation of example 1

In Equation [8], the additional constraints are to transfer tolerances into its standard deviation, assuming that tolerance equals to three standard deviations. Then the additional constraints are utilized in the objective function. In this model, the quadratic loss function is applied in order to evaluate loss into the model in the case that the performance falls within the tolerance limits. Generally, the models can be utilized directly, however in some situations, the models need to be properly applied.

Example 1: Select tolerance for two dimensional-chain with two process alternatives each shown in Figure 2. x_{ij} where i = 1, 2 and j = 1, 2 are considered as factors. Suppose that each dimensional tolerance can be obtained independently using two process alternatives, where t_{ijk} denotes the three standard deviation of process k used to generate dimension (i, j), C_{ijk} denotes the corresponding cost. Assume that all loss coefficients are equal to 1.

Applying the first and second optimization model (Equations [7-8]), the following optimization model are generated in models as shown in Equations [9-10], respectively. Note that for calculation purpose, the first four constraints are multiplied by 100.

Minimize

$$TC = 5x_{111}+8x_{112}+3x_{121}+4x_{122}+2x_{211}+3x_{212}+2x_{221}+5x_{222} \qquad (9)$$

Subject to

$$\begin{aligned}
&5x_{111}+4x_{112}+9x_{121}+5x_{211} && \leq 10\\
&4x_{211}+3x_{212}+5x_{221}+4x_{222} && \leq 8\\
&5x_{111}+4x_{112}+4x_{211}+3x_{212} && \leq 10\\
&9x_{121}+5x_{122}+5x_{221}+4x_{222} && \leq 10\\
&x_{111}+x_{112} && = 1\\
&x_{211}+x_{212} && = 1\\
&x_{121}+x_{122} && = 1\\
&x_{221}+x_{222} && = 1\\
&x_{ijk} = 0, 1 && \forall\, i, j, k
\end{aligned}$$

Solving the optimization model in Equation [9], using LINDO software the solution is $x_{111} = x_{122} = x_{212} = x_{222} = 1$ and other x's equal zero, which corresponds to the following tolerances: t_{11} * = 5, t_{12} * = 5, t_{21} * = 4, and t_{22} * = 3, where t_{ij} * denotes the optimal tolerance for row i and column j. Integrating the quadratic loss concept in the previous model, the model becomes:

Minimize

$$TC = 5x_{111}+8x_{112}+3x_{121}+4x_{122}+2x_{211}+3x_{212} +2x_{221}+5x_{222}+a_1^2+a_2^2+a_3^2+a_4^2 \quad (10)$$

Subject to

$$5x_{111} + 4x_{112} + 9x_{121} + 5x_{211} \le 10$$
$$4x_{211} + 3x_{212} + 5x_{221} + 4x_{222} \le 8$$
$$5x_{111} + 4x_{112} + 4x_{211} + 3x_{212} \le 10$$
$$9x_{121} + 5x_{122} + 5x_{221} + 4x_{222} \le 10$$
$$x_{111} + x_{112} = 1$$
$$x_{211} + x_{212} = 1$$
$$x_{121} + x_{122} = 1$$
$$x_{221} + x_{222} = 1$$

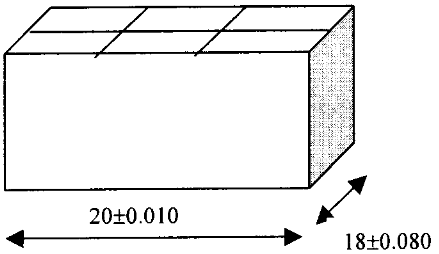

Figure 3 Example

$$a_1 = (5x_{111} + 4x_{112} + 9x_{121} + 5x_{211}) / 3$$
$$a_2 = (4x_{211} + 3x_{212} + 5x_{221} + 4x_{222}) / 3$$
$$a_3 = (5x_{111} + 4x_{112} + 4x_{211} + 3x_{212}) / 3$$
$$a_4 = (9x_{121} + 5x_{122} + 5x_{221} + 4x_{222}) / 3$$
$$a_i \ge 0 \qquad i = 1, 2, 3, 4$$
$$x_{ijk} = 0, 1 \qquad \forall\, i, j, k$$

Solving the model in Equation [10] using LINGO software, the solution is $x_{111} = x_{122} = x_{212} = x_{222} = 1$ and other x's equal zero, which corresponds to the following tolerances: $t_{11}* = 4$, $t_{12}* = 5$, $t_{21}* = 3$, and $t_{22}* = 4$.

Example 2: Select tolerance for two dimension-chain with three process alternatives each shown in Figure 3. Suppose that each dimensional tolerance can be obtained independently using two process alternatives shown in Figure 4, where t_{ijk} denotes the three sigma-standard deviation of process k used to generate dimension (i, j), C_{ijk} denotes the corresponding cost. Assuming that the loss coefficients for row tolerance are 2 and the coefficients for column tolerance are 1.

Applying optimization model Equations [7-8], the following optimization models are generated as shown in Equations [11-12], respectively. Note that for calculation purposes, the first five constraints are multiplied by 1000.

Minimize

$$TC = 5x_{111}+8x_{112}+3x_{121}+5x_{122}+2x_{131}+5x_{132} +2x_{211}+3x_{212}+3x_{221}+5x_{222}+x_{231}+3x_{232} \quad (11)$$

Subject to

$$5x_{111}+4x_{112}+3x_{121}+2x_{122}+6x_{131}+3x_{132} \le 10$$
$$4x_{211}+3x_{212}+5x_{221}+4x_{222}+4x_{231}+3x_{232} \le 10$$
$$5x_{111}+4x_{112}+4x_{211} + 3x_{212} \le 8$$
$$3x_{121} + 2x_{122} + 5x_{221} + 4x_{222} \le 8$$
$$6x_{131} + 3x_{132} + 4x_{231} + 3x_{232} \le 8$$
$$x_{111} + x_{112} = 1$$
$$x_{121} + x_{122} = 1$$
$$x_{131} + x_{132} = 1$$
$$x_{211} + x_{212} = 1$$
$$x_{221} + x_{222} = 1$$
$$x_{231} + x_{232} = 1$$
$$x_{ijk} = 0, 1 \qquad \forall\, i, j, k$$

Solving the Equation [11] model, using LINDO software, the solution is $x_{111} = x_{122} = x_{132} = x_{212} = x_{222} = x_{232} = 1$ and other x's equal zero, which corresponds to the following tolerances and processes: $t_{11}* = 5$, $t_{12}* = 2$, $t_{13}* = 3$, $t_{21}* = 3$, $t_{22}* = 4$, and $t_{23}* = 3$. Integrating the quadratic loss concept to Equation [11], the model becomes:

Minimize

$$TC=5x_{111}+8x_{112}+3x_{121}+5x_{122}+2x_{131}+5x_{132} +2x_{211}+3x_{212}+3x_{221}+5x_{222}+x_{231}+3x_{232} +2a_1^2+2a_2^2+b_1^2+b_2^2+b_3^2 \quad (12)$$

Subject to

$$5x_{111}+4x_{112}+3x_{121}+2x_{122}+6x_{131}+3x_{132} \le 10$$
$$4x_{211}+3x_{212}+5x_{221}+4x_{222}+4x_{231}+3x_{232} \le 10$$
$$5x_{111}+4x_{112}+4x_{211} + 3x_{212} \le 8$$
$$3x_{121} + 2x_{122} + 5x_{221} + 4x_{222} \le 8$$
$$6x_{131} + 3x_{132} + 4x_{231} + 3x_{232} \le 8$$
$$x_{111} + x_{112} = 1$$
$$x_{121} + x_{122} = 1$$
$$x_{131} + x_{132} = 1$$
$$x_{211} + x_{212} = 1$$
$$x_{221} + x_{222} = 1$$
$$x_{231} + x_{232} = 1$$
$$a_1 = (5x_{111}+4x_{112}+3x_{121}+2x_{112}+6x_{131}+3x_{132})/3$$
$$a_2 = (4x_{211}+3x_{212}+5x_{221}+4x_{222}+4x_{231}+3x_{232})/3$$
$$b_1 = (5x_{111}+4x_{112}+4x_{211}+3x_{212})/3$$
$$b_2 = (3x_{121}+2x_{122}+5x_{221}+4x_{222})/3$$
$$b_3 = (6x_{131}+3x_{132}+4x_{231}+3x_{232})/3$$
$$a_i, b_j \ge 0 \qquad i = 1, 2, \quad j = 1, 2, 3$$
$$x_{ijk} = 0, 1 \qquad \forall\, i, j, k$$

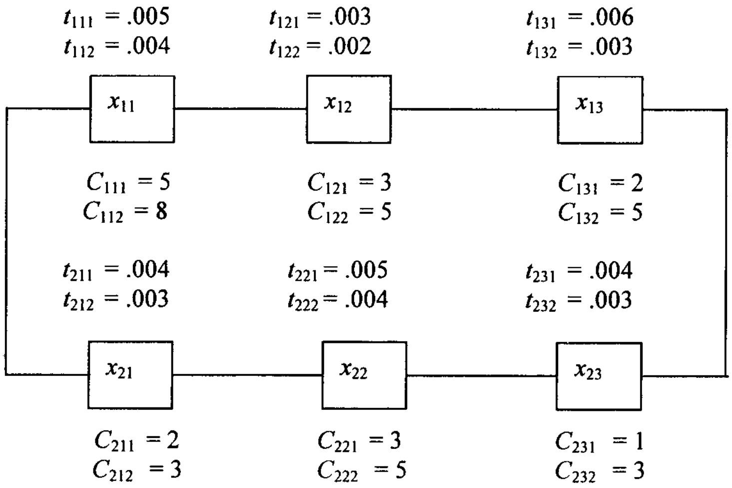

Figure 4 Representation of tolerances for example 2

Solving the model in Equation [12] using LINGO software, the solution is: $x_{112} = x_{122} = x_{132} = x_{212} = x_{222} = x_{232} = 1$ and other x's equal zero, which corresponds to the following tolerances and processes: $t_{11}* = 4$, $t_{12}* = 2$, $t_{13}* = 3$, $t_{21}* = 3$, $t_{22}* = 4$, and $t_{23}* = 3$. If the tolerance allowances are loosened by two each, the model becomes:

Minimize

$$TC = 5x_{111}+8x_{112}+3x_{121}+5x_{122}+2x_{131}+5x_{132} +2x_{211}+3x_{212}+3x_{221}+5x_{222}+x_{231}+3x_{232} \quad (13)$$

Subject to

$$\begin{aligned}
&5x_{111}+4x_{112}+3x_{121}+2x_{122}+6x_{131}+3x_{132} &&\le 12\\
&4x_{211}+3x_{212}+5x_{221}+4x_{222}+4x_{231}+3x_{232} &&\le 12\\
&5x_{111}+4x_{112}+4x_{211}+3x_{212} &&\le 10\\
&3x_{121}+2x_{122}+5x_{221}+4x_{222} &&\le 10\\
&6x_{131}+3x_{132}+4x_{231}+3x_{232} &&\le 10\\
&x_{111}+x_{112} &&= 1\\
&x_{121}+x_{122} &&= 1\\
&x_{131}+x_{132} &&= 1\\
&x_{211}+x_{212} &&= 1\\
&x_{221}+x_{222} &&= 1\\
&x_{231}+x_{232} &&= 1\\
&x_{ijk} = 0, 1 &&\forall\, i, j, k
\end{aligned}$$

Solving the model in Equation [13], using LINDO software, the solution is $x_{111} = x_{121} = x_{132} = x_{212} = x_{221} = x_{231} = 1$ and other x's equal zero, which corresponds to the following tolerances and processes: $t_{11}* = 5$, $t_{12}* = 3$, $t_{13}* = 3$, $t_{21}* = 3$, $t_{22}* = 5$, and $t_{23}* = 4$. Integrating the quadratic loss concept in Equation [13], the model becomes:

Minimize

$$TC = 5x_{111}+8x_{112}+3x_{121}+5x_{122}+2x_{131}+5x_{132} +2x_{211}+3x_{212}+3x_{221}+5x_{222}+x_{231}+3x_{232} +2a_1^2+2a_2^2+b_1^2+b_2^2+b_3^2 \quad (14)$$

Subject to

$$\begin{aligned}
&5x_{111}+4x_{112}+3x_{121}+2x_{122}+6x_{131}+3x_{132} &&\le 12\\
&4x_{211}+3x_{212}+5x_{221}+4x_{222}+4x_{231}+3x_{232} &&\le 12\\
&5x_{111}+4x_{112}+4x_{211}+3x_{212} &&\le 10\\
&3x_{121}+2x_{122}+5x_{221}+4x_{222} &&\le 10\\
&6x_{131}+3x_{132}+4x_{231}+3x_{232} &&\le 10\\
&x_{111}+x_{112} &&= 1\\
&x_{121}+x_{122} &&= 1\\
&x_{131}+x_{132} &&= 1\\
&x_{211}+x_{212} &&= 1\\
&x_{221}+x_{222} &&= 1\\
&x_{231}+x_{232} &&= 1\\
&a_1 = (5x_{111}+4x_{112}+3x_{121}+2x_{112}+6x_{131}+3x_{132})/3\\
&a_2 = (4x_{211}+3x_{212}+5x_{221}+4x_{222}+4x_{231}+3x_{232})/3\\
&b_1 = (5x_{111}+4x_{112}+4x_{211}+3x_{212})/3\\
&b_2 = (3x_{121}+2x_{122}+5x_{221}+4x_{222})/3\\
&b_3 = (6x_{131}+3x_{132}+4x_{231}+3x_{232})/3\\
&a_i, b_j \ge 0 \qquad i = 1, 2, \quad j = 1, 2, 3\\
&x_{ijk} = 0, 1 \qquad \forall\, i, j, k
\end{aligned}$$

Solving the model in Equation [14], using LINGO software, the solution is $x_{112} = x_{122} = x_{132} = x_{212} = x_{222} = x_{232} = 1$ and other x's equal zero, which corresponds to the following tolerances and processes: $t_{11}* = 4$, $t_{12}* = 2$, $t_{13}* = 3$, $t_{21}* = 3$, $t_{22}* = 4$, and $t_{23}* = 3$.

5. Design of Experiments Approach

Kusiak and Feng [4] applied the design of experiments (DOE) approach, which is a statistical tool, for the deterministic tolerance synthesis. In their research, both full and fractional factorial design are applied. The detailed discussion of the DOE approach can be found in Montgomery [18]. Applying the DOE approach to the tolerance synthesis problem, a component dimension with tolerances can be considered as a factor, e.g. x_{ij}, which means the component dimension of row i and column j. Since the tolerance synthesis problem requires the selection of a set of processes, the number of process alternatives can be considered as a level of each factor. In case of not considering the loss concept, the objective of the deterministic tolerance synthesis problem is to minimize the manufacturing cost. The direct manufacturing cost is considered as a response. Hence, the response function corresponds to the objective function in IP approach. In case of considering the loss concept, the objective is to minimize the manufacturing cost as well as loss. The summation of both is considered as a response. Since the DOE approach cannot cope with the tolerance stackup constraint, additional elements are corporated into the design in order to deal with the constrains. A set of process alternatives is selected in order to minimize the response while meeting the constraints of the component tolerances and the tolerance stackup.

This paper then proposes a step procedure to apply DOE approach to two dimensional determines tolerance synthesis as follows: (1) consider the dimensions with tolerances as factors, (2) consider the total manufacturing cost as a response when not considering loss concept or consider the summation of manufacturing cost and loss as a response when considering loss concept, (3) select a proper design for the experiment (4) incorporate the 'constraint' columns to represent the tolerance stackup constraints (5) calculate the tolerance stackup and response for each combination (row) of factors and (6) select the row with the smallest response value that satisfies the tolerance stackup constraints as the best set of processes.

Example 3: Applying DOE approach to Example 1. The number of levels is 2 as two alternatives for each factor. The numbers –1 and 1 denote the low and high levels of tolerance, respectively. A full factorial design is shown in Table 1.

Similar to Example 1, the summations of tolerances are multiplied by 100 for calculation purpose. Using the four constraints (Tol1≤10,Tol2≤8,Tol3≤10, and Tol4≤10), the minimum cost is 14 in case of not considering loss concept. The associated cost (=14) is calculated by the summation of four cost components, which are 5 for x_{11}, 4 for x_{12}, 3 for x_{21}, and 2 for x_{22}. This solution corresponds to the processes (see Table 1): Factor x_{11} and x_{22} are at the high level ($t_{11} = 5$ and $t_{22} = 5$), factor x_{12} and x_{21} are at the low level ($t_{12} = 5$ and $t_{21} = 3$). After integrating the quadratic loss concept in the design, the solution is changed and the minimum total cost (summation of cost and loss) is 48.89. The cost is calculated by the summation of four cost components, which are 8 for x_{11}, 4 for x_{12}, 3 for x_{21}, and 5 for x_{22}. The associated loss is calculated by the summation of the squared one-third tolerances. In this example, four types of tolerances are considered. They are Tol 1, Tol 2, Tol 3, and Tol 4. The associated loss is 28.89, which is $(9/3)^2+(7/3)^2+(7/3)^2+(9/3)^2$. The corresponding process is that all processes are at the low level of tolerance ($t_{11} = 4$, $t_{12} = 5$, $t_{21} = 3$, and $t_{22} = 4$). Both solutions are identical with the solutions in example 1.

Example 4: Applying DOE in example 2 The number of levels is 2 as two alternatives for each factor. The numbers –1 and 1 denote the low and high levels of tolerance for each factor, respectively. There are 6 factors in this case (x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}). To reduce the number of experiments, a fractional factorial design (2^{6-3}) is planned as shown in Table 2. The number of experiments in this case is 8; however, there is no feasible result under the five constraints (Tol 1≤ 10, Tol 2 ≤ 10, Tol 3 ≤ 8, Tol 4 ≤ 8, and Tol5 ≤ 8). Then we extend the number of experiments to 16(2^{6-2}). The result in this experiment does not meet all five constraints(Table 3). Then, we decide to extend to the full factorial design and find that the solutions between the IP and design of experiment approaches are identical.

Table 1 Factorial experiment for example 3

X_{11}	X_{12}	X_{21}	X_{22}	Tol 1	Tol 2	Tol 3	Tol 4	No Loss	Loss	
								Cost	Loss	TC
-1	-1	-1	-1	9	7	7	9	20	28.89	48.89*
1	-1	-1	-1	10	7	8	9	17	32.67	49.67
-1	1	-1	-1	13	7	7	13	19	48.44	67.44
1	1	-1	-1	14	7	8	13	16	53.11	69.11
-1	-1	1	-1	9	8	8	9	19	32.22	51.22
1	-1	1	-1	1	8	9	9	16	36.22	52.22
-1	1	1	-1	13	8	8	13	18	51.78	69.78
1	1	1	-1	14	8	9	13	15	56.67	71.67
-1	-1	-1	1	9	8	7	10	17	32.67	49.67
1	-1	-1	1	10	8	8	10	14*	36.44	50.44
-1	1	-1	1	13	8	7	14	16	53.11	69.11
1	1	-1	1	14	8	8	14	13	57.78	70.78
-1	-1	1	1	9	9	8	10	16	36.22	52.22
1	-1	1	1	10	9	9	10	13	40.22	53.22
-1	1	1	1	13	9	8	14	15	56.67	71.67
1	1	1	1	14	9	9	14	12	61.56	73.56

Table 2 2^{6-3} Factorial design for example 4

X_{11}	X_{12}	X_{13}	X_{21}	X_{22}	X_{23}	Tol 1	Tol 2	Tol 3	Tol 4	Tol 5	Cost
-1	-1	-1	1	1	1	9	13	8	7	7	24
1	-1	-1	-1	-1	1	10	11	8	6	7	24
-1	1	-1	-1	1	-1	10	11	7	8	6	25
1	1	-1	1	-1	-1	11	11	9	7	6	23
-1	-1	1	1	-1	-1	12	11	8	6	9	25
1	-1	1	-1	1	-1	13	11	8	7	9	21
-1	1	1	-1	-1	1	13	11	7	7	10	22
1	1	1	1	1	1	14	13	9	8	10	16

Table 3 2^{6-2} Factorial design for example 4

X_{11}	X_{12}	X_{13}	X_{21}	X_{22}	X_{23}	Tol 1	Tol 2	Tol 3	Tol 4	Tol 5	Cost
-1	-1	-1	-1	-1	-1	9	10	7	6	6	29
1	-1	-1	-1	1	-1	10	11	8	7	6	24
-1	1	-1	-1	1	1	10	12	7	8	7	23
1	1	-1	-1	-1	1	11	11	8	7	7	22
-1	-1	1	-1	1	1	12	12	7	7	10	22
1	-1	1	-1	-1	1	13	11	8	6	10	21
-1	1	1	-1	-1	-1	13	10	7	7	9	24
1	1	1	-1	1	-1	14	11	8	8	9	19
-1	-1	-1	1	-1	1	9	12	8	6	7	26
1	-1	-1	1	1	1	10	13	9	7	7	21
-1	1	-1	1	1	-1	10	12	8	8	6	24
1	1	-1	1	-1	-1	11	11	9	7	6	23
-1	-1	1	1	1	-1	12	12	8	7	9	23
1	-1	1	1	-1	-1	11	11	9	6	9	22
-1	1	1	1	-1	1	12	12	8	7	10	21
1	1	1	1	1	1	13	13	9	8	10	16

6. Taguchi Method

The Taguchi Method (TM) has been a useful method in the statistical design and analysis of experiments over the years. Kusiak and Feng [4] applied the TM approach to the deterministic tolerance synthesis. The methodology of TM approach is shown as follows: (1) Identify a response, and classify the product (or process) parameters as (a) control (or design) parameters, or (b) noise (or uncertain) parameters. Control (design) parameters are the product (process) characteristics whose nominal settings can be specified by the product (process) designer. Noise parameters are the variables that cause performance variation. (2) Arrange these control and noise parameters in the orthogonal array developed by Taguchi [12] and Taguchi et.al. [13]. The selection of orthogonal array depends on (a) the number of factors and their interactions of interest, (b) the number of levels for the factors of interest and (c) the desired experimental resolution or cost limitations. We usually design by using the first two items which determine the smallest orthogonal array that it is possible to use, but this will automatically be the lowest-resolution, lowest cost experiment. We may choose to run a larger experiment, which makes a higher resolution but is more expensive. Finally, we conduct experiment and collect data. (3) Incorporate the 'constraint' columns in the orthogonal array to represent the constraints and calculate the tolerance stackup. (4) Calculate the responses for each combination (row). The responses both cost and loss can be obtained by using the similar procedure as the DOE approach. (5) Select the smallest response value that satisfies the constraints.

Example 5: Applying the TM to example 1 Sixteen experiments are planned in example 3 using full factorial design. We apply the TM approach to make the experiment smaller. Four factors are defined as the control factors and the two tolerances are defined as the noise factors. Then applying $L_8(2^7)$ orthogonal array to this problem, the result is shown in Table 4. The result is that the optimal is not met. In this problem, the optimal solution may be met if the column in orthogonal array is changed. However, the results of this method and DOE approach when considering loss concepts are the same.

Example 6: Applying the TM to example 2. In this problem, we define six factors as control factors and two tolerances as noise factors. Then applying $L_8(2^7)$ orthogonal array to this problem, the result is shown in Table 5. The number of experiments in Taguchi method is 8 compared with 2^6(64) experiments in the full factorial design. Table 5 shows that a feasible solution, which is the low level of tolerance in all factors and costs 29. Compared with the result of example 2, the optimal solution gives the cost of 26. Hence, in this particular problem, TM method can not give the optimal solution. However, when integrating the quadratic loss concept, the solution is optimal (TC=62.56).

7. Conclusion and Discussion

This paper presents two-dimensional deterministic tolerance synthesis by applying three approaches, which are IP, DOE, and TM in two scenarios: with and without the loss concept. The loss concept is used to satisfy the objective of minimizing the sensitivity of tolerance to variation.

Comparing IP, DOE, and TM approaches, it can be concluded that IP approach is the most effective method because it always gives the optimal solution. However, IP approach cannot be applied to a probabilistic case, while DOE and TM approach can easily be applied. Using the full factorial design of DOE approach will gain the optimal solution as well. While the

Table 4 Taguchi method for example 5

No.	Noise factors				Constraints				Cost	Loss	TC
	X_{11}	X_{12}	X_{21}	X_{22}	Tol 1	Tol 2	Tol 3	Tol 4			
1	1	1	1	1	9	7	7	9	20	28.89	48.89
2	1	1	1	2	9	8	7	10	17	32.67	49.67
3	1	2	2	1	13	8	8	13	18	51.78	69.78
4	1	2	2	2	13	9	8	14	15	56.67	71.67
5	2	1	2	1	10	8	9	9	16	36.22	52.22
6	2	1	2	2	10	9	9	10	13	40.22	53.22
7	2	2	1	1	14	7	8	13	16	53.11	69.11
8	2	2	1	2	14	8	8	14	13	57.78	70.78

Table 5 Taguchi mehod for example 6

Noise factors						Constraints					Cost	Loss	TC
X_{11}	X_{12}	X_{13}	X_{21}	X_{22}	X_{23}	Tol 1	Tol 2	Tol 3	Tol 4	Tol 5			
1	1	1	1	1	1	9	10	7	6	6	29	33.56	62.56
1	1	2	2	2	2	12	13	8	7	10	21	58.44	79.44
2	2	1	1	2	2	11	12	8	8	7	20	49.11	69.11
2	2	2	2	1	1	14	11	9	7	9	20	58.67	78.67
1	2	1	2	1	2	10	12	8	7	7	24	45.11	69.11
1	2	2	1	2	1	13	11	7	8	9	22	53.78	75.78
2	1	1	2	2	1	10	12	9	7	6	23	45.56	68.56
2	1	2	1	1	2	13	11	8	6	10	21	54.44	75.44

fractional factorial design and TM approach may not provide the optimal solution. Therefore, company between DOE and TM approaches, DOE approach is superior to the TM approach but the DOE approach is more costly than the TM approach.

8. References

[1] Voelcker, H., A Current Perspective on Tolerancing and Metrology, 1993 International Forum Dimensional Tolerancing and Metrology, ASME, USA, pp. 49-60, 1993.

[2] Juster, N., Modeling and Representation of Dimensions and Tolerances, A Survey: Computer-Sided Design, Vol. 24, No. 1, pp. 3-17, 1992.

[3] Chase, K., and Parkinson, A., A Survey of Research in the Application of Tolerance Analysis to the Design of Mechanical Assemblies, Research Engineering Design, Vol. 1, No. 3, pp. 23-37, 1991.

[4] Kusiak, A., and Feng, C.-X., Deterministic Tolerance Synthesis: A Comparative Study, Computer-Aided Design, Vol. 27, No. 10, pp. 759-768, 1995.

[5] Berger, J. O., Statistical Decision Theory and Bayesian Analysis, New York: Springer-Verlag, 1985.

[6] Chen, G., and Kapur, K.C., Quality Evaluation System Using Loss Function, International Industrial Engineering Conference & Societies' Manufacturing and Productivity Symposium Proceedings, pp. 363-368, 1989.

[7] DeGroot, M. H., Optimal Statistical Decisions, New York, McGraw-Hill, 1970.

[8] Spring, F. An Alternative to Taguchi's Loss Function, ASQC Congress Transactions, Vol. 38, pp. 36-39, 1991.

[9] Spiring, F. A., and Yeung, A.S., A General Class of Loss Functions with Industrial Applications, Journal of Quality Technology, Vol. 30, No. 2, pp, 152-162, 1998.

[10] Phadke, M. S., Quality Engineering Using Robust Design, New York, Pretice-Hall Englewood Cliffs, 1989.

[11] Taguchi, G., Introduction to Quality Engineering, Michigan, American Supplier Institute, 1986.

[12] Taguchi, G., System of Experimental Design, New York: UNIPUB/Kraus International Publications, 1987,

[13] Taguchi, G., Elsayed, E.A., and Hsiang, T.C., Quality Engineering in Production Systems, New York, McGraw-Hill, 1989.

[14] Kapur, K.C., and Cho, B.R., Economic Design of the Specification Region for Multiple Quality Characteristics, IIE Transactions, Vol. 28, pp. 237-248, 1996.

[15] Ostwald, P. F., and Huang, J., A Method for Optimal Tolerance Selection, Journal of Engineering for Industry, pp. 558-565, 1977.

[16] Lee, W.-J, and Woo, T. C., Tolerances: Their Analysis and Synthesis, Journal of Engineering for Industry, Vol. 112, pp. 113-121, 1990.

[17] Kim, S. H., and Knott, K., A Pseudo-Boolean Approach to Determining Least Cost Tolerances, International Journal of Production Research, Vol. 26, No. 1, pp. 157-167, 1988.

[18] Montgomery, D., Design and Analysis of Experiments, USA:, John Wiley, 1991.