Deterministic Tolerance Synthesis with a Consideration of Nominal Values

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Abstract

This paper focuses on tolerance synthesis, which involves the allocation of the specified assembly tolerances among the component dimensions of an assembly to ensure a specified yield. Even though the issue of tolerance synthesis has been discussed widely, most research often assumes that component alternatives have equal nominal values. Therefore, the nominal values are negligible. However, there may be situations where the nominal values are different. In such cases, the nominal values should be considered. This paper attempts to include stackup and component nominal values to the deterministic tolerance synthesis. The objective of this paper is to integrate the nominal values of component and assemblies within the framework of tolerance synthesis. A numerical example is given to illustrate the model.

Keywords: quality engineering, tolerance synthesis, nominal values, Taguchi's loss function

1. Introduction

Tolerances are defined as the range between a specification limit and the nominal dimension. Traditionally, to assign tolerances to components and assemblies, designers have relied on experience, handbooks, and standard information. These assignment decisions, usually made at the design stage, and were often based on insufficient data or incomplete models. Due to fierce competition in marketplaces, companies seek ever-increasing product quality as well as reducing costs. A careful analysis and assigning of tolerances can significantly reduce manufacturing costs.

There are two basic processes in tolerance design: analysis and synthesis. In tolerance analysis, the component tolerances are specified and the resulting assembly variation and yield are calculated. Tolerance synthesis involves the allocation of the specified assembly tolerances among the component dimensions of an assembly to ensure a specified yield. The literature on tolerance synthesis, which is emphasized in this paper, has been reviewed by Voelcker [1], Juster [2], Chase and Parkinson [3].

Even though the issue of tolerance synthesis has been discussed widely, most research often assumes that nominal values of component alternatives are equal. Therefore, the nominal values are negligible. However, there may be situations where the nominal values are different. In such cases, nominal values of both component alternatives and assemblies should be considered. This paper attempts to include the nominal values of both types in the deterministic tolerance synthesis. Furthermore, a well-known concept of Taguchi's loss function is integrated in the model. Taguchi [4] believes that a shipped product generates loss to the customer and the loss depends on the deviation of the performance and the preferable value. This concept then applies to the model of deterministic tolerance synthesis in this paper and then a numerical example is given.

2. Model formulation for tolerance synthesis

2.1 Notation

The notation for this paper is listed as follows:

y	quality performance
k	positive loss coefficient
i	component i
j	alternative j
n	number of components

m number of alternatives
τ preferable value of the assembly
μ assembly mean
x_{ij} nominal value of component i with alternative j
t_{ij} tolerances of component i with alternative j
σ standard deviation of assembly
T allowable tolerance of assembly
TC total cost
A_C value of loss at either specification limit
C_{ij} cost of component i with alternative j
Z_{ij} 0-1 Integer

2.2 The problem

Based on the study of literature in the area of tolerance synthesis, researchers generally assume that nominal values of component alternatives are equal and concentrate in allocating the component tolerances to ensure the specified yield. Ostwald and Huang [5] are believed to be the first users of the integer programming (IP) approach to discrete tolerance synthesis. Using the IP approach, the deterministic tolerance synthesis problem can be formulated as the following 0-1 integer programming model.

$$\text{Minimize } TC = \sum_{i=1}^{n}\sum_{j=1}^{m} C_{ij}Z_{ij} \tag{1}$$

Subject to

$$\sum_{i=1}^{n}\sum_{j=1}^{m} t_{ij}Z_{ij} \le T \tag{2}$$

$$\sum_{j=1}^{m} Z_{ij} = 1 \qquad \forall i \tag{3}$$

$$Z_{ij} = 0, 1 \quad \forall i, j \tag{4}$$

The objective of the model (Eq.1) is to minimize the total component costs. The first and second constraints as shown in Eqs. (2-3) are to ensure that the total tolerance does not exceed the tolerance assembly limit, and exactly one alternative is selected to generate each tolerance. The last constraint (Eq. 4) ensures the integrality of Z_{ij}. Even though the optimization model shown in Eqs. (1-4) is well known, it is not practical in the case that nominal values of component alternatives are different. In that case, the nominal value should be a consideration to select components of the assembly. The objective of this paper is then to provide an optimization model to solve deterministic tolerance synthesis in the case of different nominal values of component alternatives. Moreover, the concept of Taguchi's loss function is integrated to model.

2.3 Taguchi's Loss Function

Taguchi [4] defines quality (using Gauss's quadratic function) as the loss a product imparts to society from the time the product is shipped. This paper integrates Taguchi's loss function to the tolerance assembly in order to satisfy the objective of minimizing the total cost and sensitivity of tolerances to variations. Three types which are Smaller The Better (STB), Larger The Better (LTB), and Nomina l The Best (NTB) are defined as follows:

$$L(y) = \begin{cases} ky^2 & for \quad STB \\ k/y^2 & for \quad LTB, \\ k(y-\tau)^2 & for \quad NTB \end{cases} \tag{5}$$

where k can be calculated by

$$k = A_C / T^2 \tag{6}$$

To illustrate the loss function, Figure 1 shows the loss to the customer in the case of NTB case, which is the focus of this paper. The expected Taguchi's loss is then given by

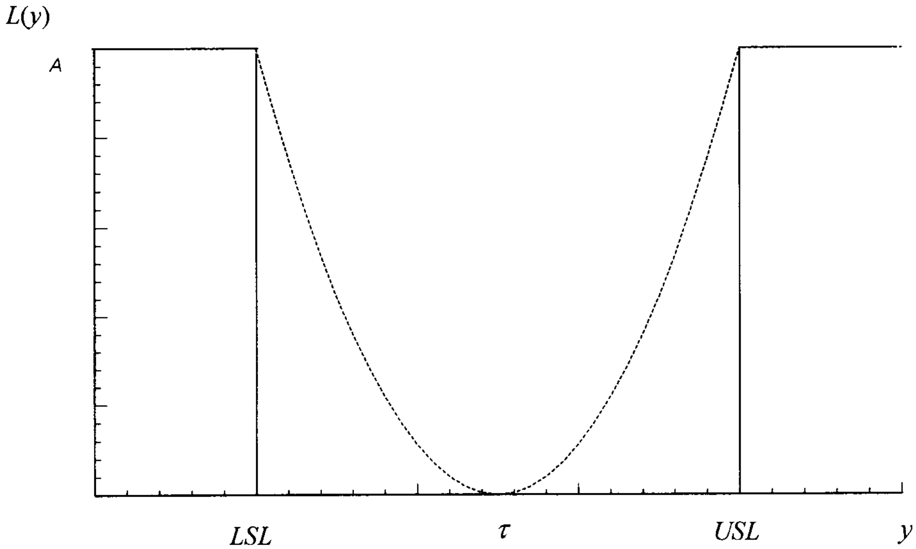

Figure 1 Taguchi's loss function

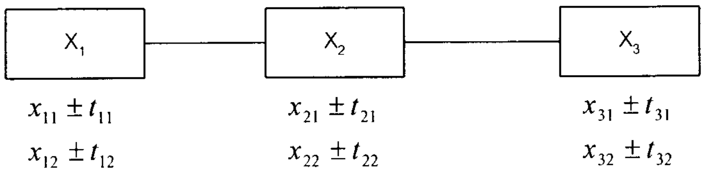

Figure 2 A general form of deterministic tolerance

$$E[L(y)] = k(\mu - \tau)^2 + k\sigma^2. \qquad (7)$$

2.4 Model Development

The general form of deterministic tolerance with the consideration of the nominal values can be generated in Figure 2, which includes nominal value, and tolerance of each alternative. Furthermore, each alternative provides its cost (C_{ij}) This paper aims to examine which alternative should be selected for each component with the consideration of the nominal values by considering the concept of Taguchi's loss function. The reason for integrating the loss function is to allow a deviation of the assembly mean and preferable value. Moreover, the concept of the loss function leads to obtain the low variation and tolerance.

Assuming that the nominal value is the mean of each alternative and considering the first part of the expected loss in Eq.(7), the bias $(\mu - \tau)$ of the assembly can be determined by $\sum_{i=1}^{n}\sum_{j=1}^{m} x_{ij}Z_{ij} - \tau$, where Z_{ij} is 1 when alternative j of component i is selected and 0 when otherwise. Consider the second part of the Eq. (7), which is the variation of assembly, and assume that manufacturers accept the three-sigma limits ($\pm 3\sigma$). Therefore, the standard deviation of each component alternative is $t_{ij}Z_{ij} / 3$. The variation of the assembly is then $\sum_{i=1}^{n}\sum_{j=1}^{m} (t_{ij}Z_{ij} / 3)^2$. The expected loss is then turned to be

$$E[L(y)] = k\left(\sum_{i=1}^{n}\sum_{j=1}^{m} x_{ij}Z_{ij} - \tau\right)^2 + k\sum_{i=1}^{n}\sum_{j=1}^{m} (t_{ij}Z_{ij} / 3)^2 \qquad (8)$$

Ostwald and Huang [5] first introduced an integer programming (IP) approach to discrete tolerance synthesis. Monte and Datseris [6] and Kusiak and Feng [7] discussed an extension of the IP approach to solve a large-scale deterministic tolerancing problem using linear programming. Lee and Woo [8] proposed a branch and bound algorithm, and Kim and Knott [9] proposed a pseudo-Boolean approach to determine least cost tolerances. In this paper, the IP approach is applied to the problem discussed in the previous section and shown in Figure 3. The objective of the model is to minimize both total costs and sensitivity of tolerances to variations. The constraints are similar to the those of the traditional model (Eqs.(2-4)).

3. An example

Consider a chain of three components with three alternatives each, as shown in Figure 4. Select the best alternative for each component that gives the assembly nominal value closing 100 and the tolerance does not exceed 18.

Applying the optimization model shown in Figure 2, the model is shown in Figure 5. Solving the model in Figure 5, using the LINGO software, the solutions become $Z_{13} = Z_{22} = Z_{32} = 1$, while other Z's are zero. That means the third, second, and second alternatives (40±7, 25±3, 37±2) are selected for component 1, 2, and 3, respectively. The solution gives the cost of 275.89, assembly nominal value of 102, and tolerance of 12. The solution (=275.89) involves the component costs (=265) and the quality loss to the customer (=10.89). In the case of the traditional condition, the nominal value of the assembly is fixed and the sum of the nominal values of the component is equal to the nominal value of the assembly, it results infeasible. That means there is no chance to select component alternatives to obtain exactly the assembly nominal value while getting the specified tolerance. If the manufacturer concentrates in the quality of the assembly, the objective function then turns to be a minimization of the quality loss as shown in Figure 6. Its result is $Z_{12} = Z_{23} = Z_{32} = 1$, while other Z's are zero.

That means the second, third, and second alternatives (32±2, 30±5, 37±2) are selected for components 1, 2, and 3, respectively. Then the assembly mean comes up 99 while tolerance is 9 and incurs the component cost of 320 and quality loss of 4.67. Table 1 shows the summary results of the two cases discussed above. It can be seen that lower component cost generally gives higher quality loss. Therefore, a trade-off between those costs should be carefully considered. The process of deciding the component alternative would depend on the

Minimize
$$TC = \sum_{i=1}^{n}\sum_{j=1}^{m} C_{ij}Z_{ij} + k\left[\left(\sum_{i=1}^{n}\sum_{j=1}^{m} x_{ij}Z_{ij} - \tau\right)^2 + \sum_{i=1}^{n}\sum_{j=1}^{m}\left(t_{ij}Z_{ij}/3\right)^2\right]$$

Subject to
$$\sum_{i=1}^{n}\sum_{j=1}^{m} t_{ij}Z_{ij} \le T$$
$$\sum_{j=1}^{m} Z_{ij} = 1 \quad \forall i$$
$$Z_{ij} = 0,1 \quad \forall i,j$$

Figure 3 The model

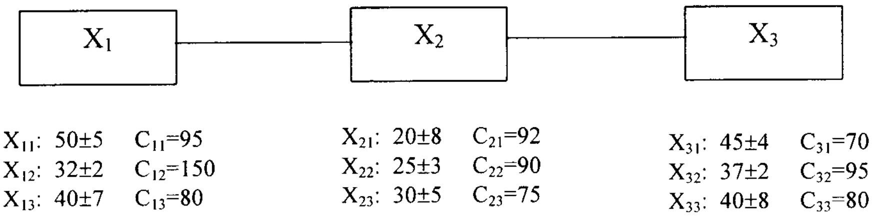

Figure 4 A case study

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MODEL:
! Select one of processes for each component that meets
 the tolerance requirement and provides low Taguchi's loss as well as costs;
 [COST] min = 95*Z11+150*Z12+80*Z13+92*Z21+90*Z22+75*Z23+70*Z31+95*Z32
              +80*Z33+(50*Z11+32*Z12+40*Z13+20*Z21+25*Z22+30*Z23+45*Z31
              +37*Z32+40*Z33-100)^2+(5*Z11+2*Z12+7*Z13)^2/9
              +(8*Z21+3*Z22+5*Z23)^2/9+(4*Z31+2*Z32+8*Z33)^2/9;
 [Tol]     5*Z11+2*Z12+7*Z13+8*Z21+3*Z22+5*Z23+4*Z31+2*Z32+8*Z33 <= 18;
 [One]     Z11+Z12+Z13 = 1;
 [Two]     Z21+Z22+Z23 = 1;
 [Three]   Z31+Z32+Z33 = 1;
 @BIN(Z11); @BIN(Z12); @BIN(Z13); @BIN(Z21); @BIN(Z22); @BIN(Z23);
 @BIN(Z31); @BIN(Z32); @BIN(Z33);
 END
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Figure 5 Optimization model

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MODEL:
! Select one of processes for each component that meets
 the tolerance requirement and provides low Taguchi's loss as well as costs;
 [COST] min = (50*Z11+32*Z12+40*Z13+20*Z21+25*Z22+30*Z23+45*Z31
              +37*Z32+40*Z33-100)^2+(5*Z11+2*Z12+7*Z13)^2/9
              +(8*Z21+3*Z22+5*Z23)^2/9+(4*Z31+2*Z32+8*Z33)^2/9;
 [Tol]     5*Z11+2*Z12+7*Z13+8*Z21+3*Z22+5*Z23+4*Z31+2*Z32+8*Z33 <= 18;
 [One]     Z11+Z12+Z13 = 1;
 [Two]     Z21+Z22+Z23 = 1;
 [Three]   Z31+Z32+Z33 = 1;
 @BIN(Z11); @BIN(Z12); @BIN(Z13); @BIN(Z21); @BIN(Z22); @BIN(Z23);
 @BIN(Z31); @BIN(Z32); @BIN(Z33);
 END
```

Figure 6 Optimization model when considering the quality loss

Table 1 Summary of the example

Objective	Component Cost	Quality Loss	Total Costs
Both component costs and quality loss	265	10.89	275.89
Quality loss	320	4.67	324.67

manufacturer. If the manufacturer focuses on both cost and quality, the model in Figure 4 should be used but if the manufacturer concentrates on the quality of a product more than the manufacturing cost, the quality may be given intensive attention and only the quality loss should be considered in the objective function.

4. Conclusion

Typical research in this area assume that alternatives of a component provide equal nominal values. However, this may not always be the case. This paper then gives an intensive extension of deterministic tolerance in the case that component alternatives have different nominal values. To select the processes for each dimension, Taguchi's loss function is integrated. A zero-one programming model is provided and a numerical example is given to illustrate the model. The example shows that the assembly mean may not equal the preferable mean, however the loss function is used to narrow the bias and variation of the assembly.

5. Reference

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