Characterizations of the Array Antenna Pattern Synthesis Performing the Tapered Minor Lobes for Radar and Low Noise Applications

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Abstract

This paper presents the characterizations of the discrete array antenna pattern synthesis that performs the tapered minor lobes for the applications of radar and low noise systems. Some kinds of orthogonal polynomials i.e., Legendre, Hermite and the second kind Tschebyscheff polynomials are used to synthesize the array antenna pattern. The results are compared with the conventional discretized Taylor one parameter and Taylor $\bar{n}$ methods. Additionally, these tapered minor lobe array patterns are also comparatively demonstrated together with the array antenna pattern yielding the uniform minor lobe distributions viz., the first kind Tschebyscheff array. The array designs using some orthogonal polynomials are rigorously described. The array characteristics such as radiation pattern, the maximum to minimum current ratio, beamwidth; both half power beamwidth and first null beamwidth, directivity, the nearest to the furthest minor lobe ratio, and beam efficiency are illustrated. The advantages and disadvantages of each method are discussed.

1. Introduction

Following the dramatic growth of the wireless applications such as radar and communication with low noise becomes drastically, the antenna plays an important role as the key device in transmitting and receiving the signal. Generally, it is desirable for the antenna to achieve the maximum directivity, narrow beamwidth and low side lobe ratio [1]. The array antenna is one of the most suitable candidates that can fulfill these requirements. The array antenna pattern synthesis has been extensively investigated to realize the current distributions weighted to the array elements from which the radiation characteristics are specified. Furthermore, to apply the array antenna in radar and low noise systems the tapered minor lobe properties are necessary. The nearest to the furthest minor lobe ratio and the beam efficiency must be sufficiently high because interfering or spurious signals will be decreased further when they enter through those tapered minor lobes. Therefore, the significant contributions from interfering signal will be through the pattern in the vicinity of the main lobe. Moreover, in low noise systems, the tapered minor lobe pattern plays a vital role in order to diminish the radiation accepted through them from the relatively hot ground [1]. Historically, the uniform array is the simplest way to determine the current distribution because each element is excited identically. The directivity is maximum at the expense of very high side lobe ratio. Binomial array [2], in which the currents are determined from the coefficients of Binomial expansion or Pascal's triangle, is the candidate to solve the drawback of the uniform array. It is apparent that the side lobe ratio of Binomial array is extremely low while the degradation of the directivity occurred [3]. Dolph [4] proposed using the first kind Tschebyscheff polynomial to synthesize the array antenna pattern to compromise between the uniform and Binomial arrays. The side lobe ratio of the first kind Tschebyscheff array is lower than the uniform array and the directivity is higher than the Binomial one. However, it was found that the Dolph-Tschebyscheff array has the uniform minor lobe distribution which leads to the loss of the beam efficiency and the zero in dB of the nearest to the furthest minor lobe ratio. Subsequently, Rashid [5] presented the possibilities of mathematical features to synthesize the array antenna pattern using Legendre polynomial. In addition, it was pointed out later [6] that Legendre array provides the tapered minor lobe which is suitable for the application of radar and low noise systems. The investigations of the discrete array antenna pattern synthesis accomplishing the taper minor lobe especially the far-out minor lobe are consequently conducted [7]. The authors describe the feasibility to employ some orthogonal polynomials to synthesize the array antenna pattern [8]. As an alternative choice to achieve the array antenna with the tapered minor lobe at which the far-out minor lobe decreases more rapidly, the authors propose the second kind Tschebyscheff array [9]. The comparative study of these aforementioned synthesis is reported [10]. From the mathematical viewpoint, it is apparent that Hermite polynomial which is a kind of orthogonal polynomial is able to synthesize the array antenna pattern [11]. It will be revealed in this paper that the nearest to the furthest minor lobe ratio is extremely high and the beamwidth is relatively wide comparing with some other orthogonal polynomial arrays. This paper presents the characterizations of the array antenna pattern synthesis providing the tapered minor lobes for utilizing in radar and low noise systems. The array antenna pattern synthesis using some orthogonal polynomials is substantially summarized. The array characteristics such as radiation pattern, the maximum to minimum current ratio, half power beamwidth, first null beamwidth, directivity, the nearest to the furthest minor lobe ratio and beam efficiency are carried out. The comparisons among the array antenna pattern synthesis possessing the tapered minor lobe using some orthogonal polynomials and the conventional discretized Taylor one parameter and Taylor $\bar{n}$ methods are illustrated. The merits and demerits of each method are discussed.

2. Array Factor

To synthesize the array antenna pattern with the non-uniform amplitude excitations, the array factor should be first considered. Let us assume that there is a linear array of isotropic elements. These elements are aligned along z axis and symmetry with the center of the array and have equi-distance. When the number of the elements is even, an array factor (AF) can be written as [1]

$$AF_{2N}(\theta) = \sum_{n=1}^{N} I_n \cos[(2n-1)\frac{\pi d}{\lambda}\cos\theta]. \qquad (1)$$

An array factor for the odd number of the elements can be expressed as

$$AF_{2N+1}(\theta) = \sum_{n=1}^{N+1} I_n \cos[2(n-1)\frac{\pi d}{\lambda}\cos\theta], \qquad (2)$$

where I_n is the amplitude current excitation coefficient, $2N$ and $2N+1$ are the number of even and odd elements, respectively, d is the spacing between each element, λ is the wavelength at the operating frequency and θ is the angle between the field direction and the z axis.

3. Array Antenna Pattern Synthesis Using Some Orthogonal Polynomials

The mathematical properties of some orthogonal polynomials which are necessary to treat the array antenna pattern synthesis are reported. These properties are very useful in the computer programming to characterize the array antenna pattern. The array design procedure of the pattern synthesis is described, subsequently.

3.1 Mathematical Properties of Some Orthogonal Polynomials

Some orthogonal polynomials i.e., Legendre, Hermite, the first and the second kinds Tschebyscheff polynomials will be utilized to synthesize the array antenna pattern in this paper. Since these four polynomials are types of orthogonal polynomials, they can be defined on the interval $a \le x \le b$ with respect to the weight function, $w(x)$, as [12]

$$\int_a^b w(x) f_n(x) f_m(x) dx = 0, \qquad (3)$$

where $f_n(x)$ and $f_m(x)$ are systems of polynomials of degrees n and m, respectively. The weight function is a real and non-negative value which is the constant factor in each polynomial. Alternatively, another form of the orthogonal polynomial, referred to as standardization, can be written as

$$\int_a^b w(x) f_n^2(x) dx = h_n(x), \qquad (4)$$

where $h_n(x)$ is the function of x and it can be represented in each polynomial type. The functions $w(x)$, $h_n(x)$, a and b of Legendre, Hermite, the first and second kinds Tschebyscheff polynomials are tabulated in Table 1.

Table 1 Weight functions and limit of integration

$f_n(x)$	$w(x)$	$h_n(x)$	a	b
$P_n(x)$	1	$\frac{2}{2n+1}$	-1	1
$H_n(x)$	e^{-x^2}	$\sqrt{\pi}2^n n!$	$-\infty$	∞
$T_n(x)$	$(1-x^2)^{-\frac{1}{2}}$	$\begin{cases}\frac{\pi}{2}, & n \ne 0 \\ \pi, & n = 0\end{cases}$	-1	1
$U_n(x)$	$(1-x^2)^{\frac{1}{2}}$	$\frac{\pi}{2}$	-1	1

where $f_n(x)$, $P_n(x)$, $H_n(x)$, $T_n(x)$ and $U_n(x)$ denote the general expression of the orthogonal polynomial, Legendre function, Hermite function, the first and second kinds Tschebyscheff functions, respectively. These four kinds of functions also satisfied the orthogonal's differential equation as

$$c_2(x)\frac{d^2y}{dx^2} + c_1(x)\frac{dy}{dx} + c_0(n)y = 0, \qquad (5)$$

where $c_2(x)$, $c_1(x)$, $c_0(n)$ are the coefficient functions. $c_2(x)$ and $c_1(x)$ are independent of n, only function of x but $c_0(n)$ depends only on n. These coefficient functions for four kinds of orthogonal polynomials are shown in Table 2.

Table 2 Coefficient functions for four kinds of orthogonal polynomials

$f_n(x)$	$c_2(x)$	$c_1(x)$	$c_0(n)$
$P_n(x)$	$1-x^2$	$-2x$	$n(n+1)$
$H_n(x)$	1	$-2x$	$2n$
$T_n(x)$	$1-x^2$	$-x$	n^2
$U_n(x)$	$1-x^2$	$-3x$	$n(n+2)$

Next, in order to find the polynomials of any orders from the polynomials in which orders are given, the recurrence relation is used. Normally, the polynomials of higher orders would be determined from the polynomials of lower orders. The general form of recurrence relation is

$$a_1(n) f_{n+1}(x) - a_2(n) x f_n(x) + a_3(n) f_{n-1}(x) = 0, \qquad (6)$$

where $a_1(n)$, $a_2(n)$ and $a_3(n)$ can be defined as in Table 3.

Table 3 Coefficients of recurrence relation

$f_n(x)$	$a_1(n)$	$a_2(n)$	$a_3(n)$
$P_n(x)$	$n+1$	$2n+1$	n
$H_n(x)$	1	2	$2n$
$T_n(x)$	1	2	1
$U_n(x)$	1	2	1

In addition, Rodrigue's formula is the alternative form of the orthogonal polynomials in which the polynomials of order n are in the form of derivative of the weight and the coefficient functions as

$$f_n(x) = \frac{1}{a_n p(x)} \frac{d^n}{dx^n}\left[p(x)\{g(x)\}^n\right], \qquad (7)$$

where $p(x)$ and $g(x)$ are functions of x, independent of n and a_n is the function of n. These coefficients are illustrated as in Table 4.

Table 4 a_n, $p(x)$ and $g(x)$

$f_n(x)$	a_n	$p(x)$	$g(x)$
$P_n(x)$	$(-1)^n 2^n n!$	1	$1-x^2$
$H_n(x)$	$(-1)^n$	e^{-x^2}	1
$T_n(x)$	$(-1)^n 2^n \frac{\Gamma(n+\frac{1}{2})}{\sqrt{\pi}}$	$(1-x)^{-\frac{1}{2}}$	$1-x^2$
$U_n(x)$	$(-1)^n 2^{n+1} \frac{\Gamma(n+\frac{3}{2})}{(n+1)\sqrt{\pi}}$	$(1-x)^{\frac{1}{2}}$	$1-x^2$

3.2 Array Design Using Some Orthogonal Polynomials

After Legendre, Hermite, the first and the second type Tschebyscheff polynomials are well-studied, the next step is to apply these polynomials to synthesize the array antenna pattern. The summation of the cosine term of (1) and (2) for the case of even and odd elements will be expanded. The order of harmonic cosine term is equal to the total number of the elements minus one and the argument of the cosine term is the positive integer times of the fundamental frequency that can be written in the form

$$\cos(ku) = \cos^k(u) - \binom{k}{2}\cos^{k-2}(u)\sin^2(u) + \binom{k}{4}\cos^{k-4}(u)\sin^4(u) - \dots - \binom{k}{k-2}\cos^2(u)\sin^{k-2}(u) + \sin^k(u), \quad (8)$$

where $\binom{k}{n} = \frac{k!}{n!(k-n)!}$ and $\sin^2(u) = 1-\cos^2(u)$.

Consequently, the design procedure of orthogonal polynomial array will be summarized. Assume that the number of elements, the spacing between the elements in terms of wavelength and the ratio of the major to the first minor lobe intensity ratio are known. To obtain the array factor the following step can be applied. From the known number of elements, we can select the array factor from (1) or (2) which corresponds to the even or odd number of elements.

1. Select the appropriate cosine term function from (8) and substitute in the expanded array factor.

2. Find the order of the orthogonal polynomial by subtracting the total number of elements by one.

3. Solve the root for the derivative of the orthogonal polynomial to determine the level of the first maximum ripple (y_n) to form the point of the peak of the main beam and the first side lobe.

4. Equate the orthogonal polynomial with the major to the first minor lobe intensity ratio (R_n). The side lobe of the array pattern can be established from 0 dB to the first null point (measured from the main beam) and the main beam is formed from the first null point to x_m region as depicted in Fig.1.

5. Normalize x, to ensure that the magnitude of cosine term is not more than unity, by dividing by x_m.

6. Equate the expanded array factor to the orthogonal polynomial, the amplitude current excitation coefficient I_n will be obtained.

After the current excitation coefficients are known, by substituting into (1) or (2) corresponding to the even or odd number of elements, the complete expression of the array factor can be realized.

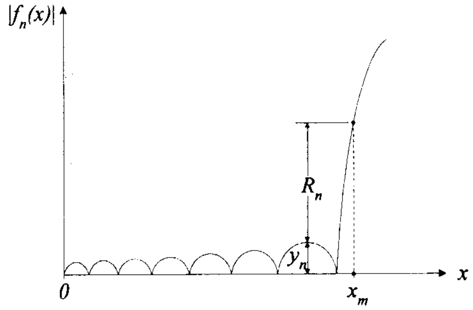


Fig. 1 How to form the main beam and the minor lobes

4. Array Pattern Synthesis using Conventional Discretized Taylor One Parameter and Taylor $\bar{n}$ Methods

Some other array antenna pattern syntheses that perform the tapered minor lobe were proposed by Taylor. They are Taylor one parameter and Taylor $\bar{n}$ methods. Taylor one parameter method was first introduced in Taylor's unpublished classic memorandum and the details are widely described by many authors [13]. In practice, Taylor one parameter method is more applicable to the line source distribution. To apply this method to the discrete array antenna pattern synthesis, the aperture distribution must be sampling (discretized). The amplitude current excitation coefficient can be calculated from the source distribution, which is given as

$$I_n = I_0\left(\pi B\sqrt{1-\xi^2}\right), \tag{9}$$

where $I_0(x)$ is the modified Bessel function of the first kind of order zero, related to the ordinary Bessel function of the first kind of order zero $(J_0(x))$ as

$$I_0(x) = J_0(jx) \tag{10}$$

and ξ is the normalized distance along the overall length of the array, which is defined as

$$\xi = \frac{2z_n}{(M-1)d}, \tag{11}$$

where z_n is the dimension along the array with the origin at the center of array and M is the total number of elements. The constrained value of ξ is between -1 and 1. B is the constant, which can be determined from the side lobe ratio. The B parameter is also called weighting parameter or one parameter. For the specified side lobe ratio, the formulation for the relation between the side lobe ratio R_n(dB) and the weighting parameter is given as [13]

$$R_n(\text{dB}) = 13.26 + 20\log\left(\frac{\sinh \pi B}{\pi B}\right), \tag{12}$$

where the cardinal number 13.26 means the side lobe ratio of the uniform distribution which occurs when the value of the weighting parameter vanishes. In the design of the radiation pattern, we would generally start with the required side lobe ratio and subsequently determine the weighting parameter. For this reason, (12) it is not appropriate for computing the weighting parameter because it is an inverse problem. Blanton [14] presented the alternative expression for solving the weighting parameter as the straightforward problem from the hyperbola equation as

$$B = b\sqrt{\frac{(R_n(dB)-c)^2}{a^2} - 1}, \tag{13}$$

where a and b represent the hyperbola's semi-transverse and semi-conjugate axes, respectively. c is the displacement (in dB) of the center of the hyperbola measured from the origin. In the case of the line source distribution the values of a, b and c are, respectively, 22.96, 0.9067 and -9.7. The sampling point of the aperture distribution are set to be

$$z_n = -\left(\frac{M+1}{2}\right)d + nd, n = 1,2,3,\ldots,P, \tag{14}$$

where

$$P = \begin{cases} \frac{M}{2}, & \text{M = even} \\ \frac{M+1}{2}, & \text{M = odd} \end{cases} \tag{15}$$

Additionally, Taylor also proposed the line source distribution with the tapered minor lobe referred to as Taylor $\bar{n}$ method [15]. To apply this method to the discrete array antenna pattern synthesis, various techniques were employed. They are discretization of the source expression (aperture sampling) [16], applying the null matching [17] and using Villeneuve method [18]. These three methods yield similar results as reported in reference [16]. Aperture sampling and null matching are easier than Villeneuve method but the accuracy is slightly lower than Villeneuve one. When compared between aperture sampling and null matching, it is evident that null matching gives better results for small arrays and aperture sampling gives slightly more accurate results for large arrays. However, as the simplest way to determine the current distribution the aperture sampling will be used in this paper. The current distribution of Taylor $\bar{n}$ can be expressed as

$$I_n = \frac{1}{(M-1)d}\left[1 + 2\sum_{p=1}^{\bar{n}-1} F(p,A,\bar{n})\cos\left(\frac{2\pi p z_n}{(M-1)d}\right)\right], \tag{16}$$

where the space factor of the Taylor $\bar{n}$ distribution can be written as

$$F(p,A,\bar{n})=\begin{cases}\dfrac{[(\bar{n}-1)!]^2}{(\bar{n}-1+p)!(\bar{n}-1-p\pi)!} \\ \times\prod_{m=1}^{\bar{n}-1}[1-(\frac{p}{u_m})^2] & |p|<\bar{n} \\ 0 & |p|\geq\bar{n}.\end{cases} \tag{17}$$

The location of the null can be obtained by using

$$u_m=\begin{cases}\pm\pi\sigma\sqrt{A^2+(m-\frac{1}{2})^2} & 1\leq m<\bar{n} \\ \pm m\pi & \bar{n}\leq m<\infty.\end{cases} \tag{18}$$

The scaling factor is

$$\sigma=\frac{\bar{n}}{\sqrt{A^2+(m-\frac{1}{2})^2}}, \tag{19}$$

where the constant A can be found from

$$A=\frac{1}{\pi}\cosh^{-1}R_n. \tag{20}$$

5. Numerical Example of the Array Antenna Pattern Synthesis

5.1 Using Some Orthogonal Polynomials

To demonstrate the principle, the linear discrete broadside array of 10 elements with the major to first minor lobe intensity ratio of 20.00 dB (which equals 10.00 in dimensionless) with half wavelength of antenna spacing is illustrated. To form the maximum value of the main lobe, the maximum value of the first ripple (y_n) and the major to the first minor lobe intensity ratio (R_n) are multiplied. The value of x_m can be determined by solving the root of the following characteristic equation

$$k_9x^9+k_7x^7+k_5x^5+k_3x^3+k_1x+k_0=0, \tag{21}$$

where y_n, R_n, k_9, k_7, k_5, k_3, k_1 and k_0 in case of this demonstration are shown as in Table 5. The value of x_m is substituted to normalize in the cosine term so that

Table 5 Coefficients of characteristic equation (21)

$f_n(x)$	$P_n(x)$	$H_n(x)$	$T_n(x)$	$U_n(x)$
y_n	0.41	428152.00	1.00	2.25
R_n	10.00	10.00	10.00	10.00
k_9	94.96	512.00	256.00	512.00
k_7	-201.09	-9216.00	-576.00	-1024.00
k_5	140.77	48384.00	432.00	672.00
k_3	-36.09	-80640.00	-120.00	-160.00
k_1	2.46	30240.00	9.00	10.00
k_0	-4.10	-4281520.00	-10.00	-22.58
x_m	1.04	3.56	1.06	1.03

$$\cos(\frac{\pi d}{\lambda}\cos\theta)=\frac{x}{x_m}. \tag{22}$$

By following step 6 in the design procedure, the amplitude current excitation coefficients I_n can be obtained as tabulated in Table 6. After the amplitude current excitation coefficients are revealed, by substituting that current into (1), the complete expression of the array factor can be realized.

Table 6 Amplitude current excitation coefficients of six arrays

$f_n(x)$	I_1	I_2	I_3	I_4	I_5
$P_n(x)$	1.95	1.81	1.54	1.22	1.00
$H_n(x)$	6.91	6.72	5.60	3.31	1.00
$T_n(x)$	1.56	1.44	1.21	0.93	1.00
$U_n(x)$	2.29	2.13	1.84	1.44	1.00
$O_n(x)$	2.88	2.64	2.19	1.62	1.00
$N_n(x)$	1.47	1.34	1.06	0.89	1.00

By plotting this array factor as a function of the angle, it can be shown in Fig.2.

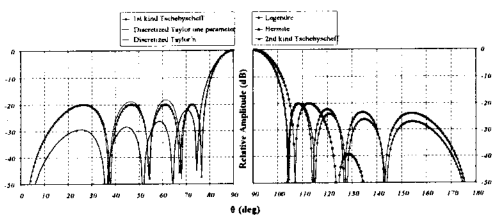


Fig. 2 Radiation pattern of six arrays

5.2 Using Discretized Taylor One Parameter and Taylor $\bar{n}$ Method

The linear array of 10 elements of the major to the first minor lobe intensity ratio of 20.00 dB with half wavelength spacing of the conventional discretized Taylor one parameter and Taylor $\bar{n}$ methods is also carried out to compare with those using some orthogonal polynomials. By using the expression of (9) for Taylor one parameter and (16) for Taylor $\bar{n}$ method, the current distribution of each element can be determined straightforwardly. The results of the current distributions of these conventional methods are also shown in Table 6, where $O_n(x)$ represents Taylor one parameter method and $N_n(x)$ is Taylor $\bar{n}$ method. The array factor corresponding to these currents are plotted together with the synthesis by using some orthogonal polynomial methods as illustrated in Fig.2. The discussions for the results of the radiation patterns will be mentioned in the subsequent section.

5.3 Comparison of Radiation Characteristics

To evaluate the merit of the antenna for applying to the radar and low noise system, the beam efficiency is a very significant parameter. It is defined as the ratio of the power transmitted (received) within the main beam to the power transmitted (received) by the antenna. For the discrete linear broadside array, beam efficiency can be formulated as [1]

$$\text{Beam Efficiency}(BE) = \frac{\int_{\theta_1}^{\pi/2} |AF(\theta)|^2 \sin\theta d\theta}{\int_0^{\pi/2} |AF(\theta)|^2 \sin\theta d\theta}, \tag{23}$$

where θ_1 is the half angle of the cone where the first null occurs. Directivity is another important parameter of the antenna which must be considered. It is defined as the ratio of the maximum intensity of the antenna to the radiation intensity of the isotropic source. Determination of directivity is done by using [1]

$$\text{Directivity}(D_0) = \frac{2|AF(\pi/2)|^2}{\int_0^{2\pi} |AF(\theta)|^2 \sin\theta d\theta}. \tag{24}$$

In order to compare the tapered minor lobe distribution characteristics, the nearest to the furthest minor lobe ratio is defined as the ratio of the level of the nearest minor lobe to the furthest minor lobe when the nearest and the furthest minor lobes are referred to with respect to the main lobe position.

In case of 10 elements discrete broadside linear array of $\lambda/2$ spacing with 20.00 dB side lobe ratio, the first null angle (θ_1), beam efficiency (BE), half power beamwidth $(HPBW)$, first null beamwidth $(FNBW)$ and directivity (D_0) are determined and the results are shown in Table 7. It is evident that these array types yield different characteristics , for example the discretized Taylor one parameter provides the highest beam efficiency whereas the discretized Taylor $\bar{n}$ performs the narrowest beamwidth and maximum directivity. The details of the analysis of the array characteristics will be summarized in the next section.

Table 7 Radiation characteristics of six arrays

$f_n(x)$	θ_1 (deg)	BE (%)	$HPBW$ (deg)	$FNBW$ (deg)	D_0 (dBi)
$P_n(x)$	75.79	97.86	11.02	28.42	9.76
$H_n(x)$	73.00	99.01	13.05	34.00	9.10
$T_n(x)$	76.39	96.30	11.17	27.22	9.84
$U_n(x)$	75.54	88.43	11.75	28.93	9.70
$O_n(x)$	74.75	99.12	12.25	30.50	9.55
$N_n(x)$	76.50	95.45	11.00	27.00	9.85

6. Array Characteristics

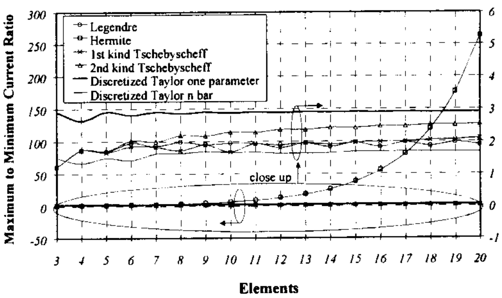
In this section, the array characteristics will be illustrated. The radiation patterns of six arrays are first revealed for the case of 10 elements of the 20.00 dB side lobe ratio. After that the other array characteristics such as the maximum to minimum current ratio, the beamwidth, the nearest to the furthest minor lobe ratio, the directivity and the beam efficiency are illustrated. There are two cases to demonstrate; Fig.3(a) through Fig.8(a) demonstrates when the side lobe ratio is fixed at 20.00 dB, but the number of elements is varied, and Fig.3(a) through Fig.8(a) demonstrates when the number of elements is fixed at 10, but the side lobe ratio is varied.

6.1 Radiation Pattern

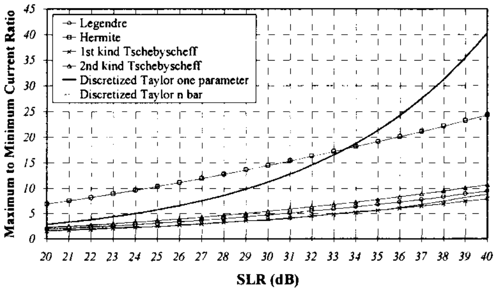
To gain more insight into the synthesis of the discrete array yielding the tapered minor lobes, the radiation patterns of Legendre, Hermite and the second kind Tschebyscheff array are illustrated and compared with the conventional tapered minor lobe array pattern i.e., discretized Taylor one parameter and discretized Taylor $\bar{n}$ methods as depicted in Fig.2. The results of these array antenna patterns possessing the tapered minor lobe are also compared with the array pattern performing the uniform minor lobe viz., the first kind Tschebyscheff array. It can be seen that for specified 20.00 dB side lobe ratio, Taylor one parameter belongs to the first side lobe around 2 dB lower than 20.00 dB because this method is originally applicable to continuous line source distribution. Therefore, the decrease of side lobe ratio occurs when this method is applied to discrete array. In the similar fashion, although the first side lobe of Taylor $\bar{n}$ can be controlled to be 20.00 dB as desired, the second and the third side lobes exhibit slightly higher than the first one. This circumstance is caused from the error of the discretization and the results will be better when the number of the elements is sufficiently large. The levels of all minor lobes of the first kind Tschebyscheff array are identical as known. The minor lobes of the discretized Taylor $\bar{n}$ are almost identical because this method is originally derived from Dolph-Tschebyscheff one. For Taylor one parameter, it is evident that the minor lobe decreases drastically. When compared with three kinds of orthogonal polynomial arrays, it is clear that Hermite array has the lowest level of the far-out minor lobe. The second kind Tschebyscheff array possesses slightly more tapered minor lobe than Legendre array. However, it is noted that Hermite and discretized Taylor one parameter arrays have extremely low furthest minor lobe at the expense of the wide beamwidth. Although Legendre and the second kind Tschebyscheff arrays yield the less tapered minor lobe than Hermite and discretized Taylor one parameter arrays, their beamwidths are close to the first kind Tschebyscheff one which is near optimum.

6.2 Maximum to Minimum Current Ratio

The maximum to minimum current ratio is another parameter to characterize the array pattern. Fig.3(a) illustrates the maximum to minimum current ratio as a function of the number of elements, it is explicit that Hermite array has very high maximum to minimum current ratio. The nominal value of the ratio more than 270 is observed when the number of the elements exceeds 20. The ratio higher than this value is too high for fabrication. For other array types, it is obvious that discretized Taylor one parameter array gives the highest current ratio followed by the second kind Tschebyscheff array. Legendre and the first kind Tschebyscheff arrays have very similar results whereas the discretized Taylor $\bar{n}$ provides the lowest current ratio. Fig.3(b) shows the maximum to minimum current ratio for various side lobe ratios. When the side lobe ratio is less than 33 dB, Hermite array yields the highest current ratio. However, the current ratio of the discretized Taylor one parameter becomes the highest for the side lobe ratio lower than 33 dB. For the other arrays, the second kind Tschebyscheff array possesses the highest ratio and follows, in order, by Legendre, discretized Taylor $\bar{n}$ and the first kind Tschebyscheff arrays. From the view point of the current ratio, the feeder structure for the case of the discretized Taylor one parameter and Hermite arrays is difficult to construct when the extremely low side lobe ratio is desired.



(a)

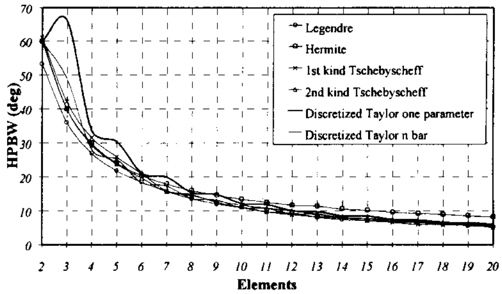


(b)

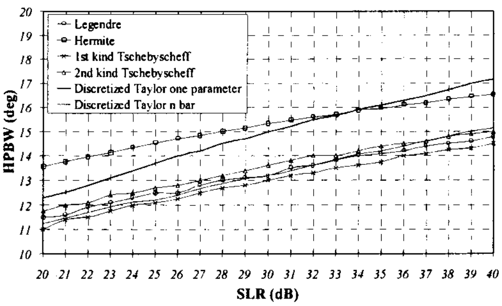
Fig.3 (a) Maximum to minimum current ratio for various number of elements (b) Maximum to minimum current ratio for various side lobe ratios

6.3 Beamwidth

The beamwidth; both half power beamwidth and first null beamwidth of six arrays are illustrated in Fig.4 and Fig.5, respectively. Figs.4(a) and 5(a) illustrate the half power beamwidth and the first null beamwidth as a function of the number of elements. These beamwidths have the same trends, i.e., the discretized Taylor one parameter and discretized Taylor $\bar{n}$ arrays give the wider beamwidth than the other ones. It is noted that the beamwidth of these two arrays in the event of the odd elements exhibits wider beamwidth than the even elements because the aperture sampling techniques cause error for the small number of elements. This difference becomes decreased when the number of the elements is higher. The first kind Tschebyscheff array performs the narrowest beamwidth as expected. Legendre and the second kind Tschebyscheff arrays have very similar results of the beamwidth. However, the beamwidth of Hermite array is the highest when the number of the elements exceed 10. For various side lobe ratios, half power beamwidth and first null beamwidth are depicted as shown in Figs.4(b) and 5(b), respectively. These results are similar to each other. For small side lobe ratio, Hermite array has the widest beamwidth. In contcast the discretized Taylor one parameter array provides the widest beamwidth for the large side lobe ratio. The beamwidth are descending as follows; the second kind Tschebyscheff array, Legendre array, the discretized Taylor $\bar{n}$ array and the first kind Tschebyscheff array. The first null beamwidth of the discretized Taylor $\bar{n}$ array for the side lobe ratio exceeds 37 dB is rapidly increased because the first null becomes very shallow until its level is equal to the level of the side lobe. Hence, the first null point is shifted to be the second one. This situation always takes place for the extremel side lobe of the discretized Taylor $\bar{n}$ array.

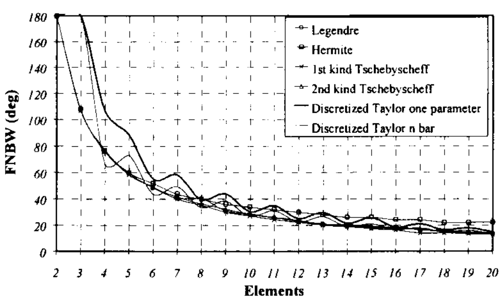


(a)

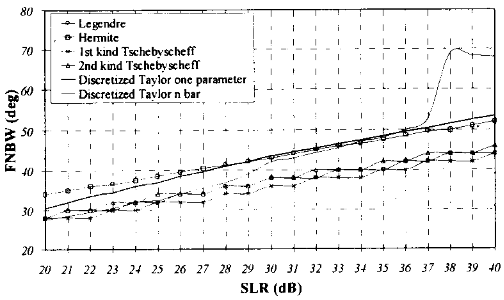


(b)

Fig.4 (a) Half power beamwidth for various number of elements (b) Half power beamwidth for various side lobe ratios



(a)

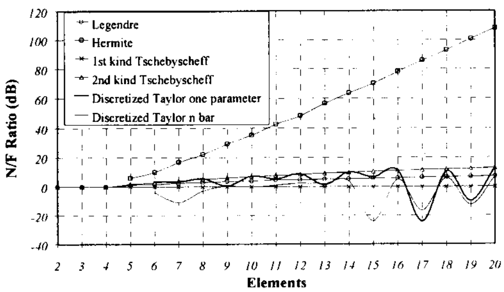


(b)

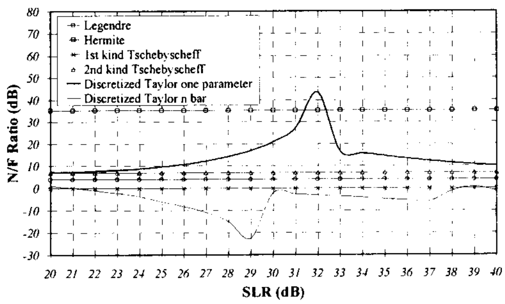
Fig.5 (a) First null beamwidth for various number of elements (b) First null beamwidth for various side lobe ratios

6.4 The Nearest to the Furthest Minor Lobe (N/F) Ratio

The nearest to the furthest (measured with respect to the main beam) minor lobes ratio is defined to compare the tapered minor lobe characteristics. N/F ratio versus the number of elements and side lobe ratios are plotted in Figs.6(a) and 6(b), respectively. As expected, Hermite array provides the highest N/F ratio and is followed, in order, by the second kind Tschebyscheff array, Legendre array and the first kind Tschebyscheff array, respectively. N/F ratio of the discretized Taylor one parameter and discretized Taylor $\bar{n}$ arrays are non-uniform because the degradation of the side lobe from the sampling occurred. It is mentioned that for the number of elements less than 6, some array antenna pattern exhibit no side lobe. Therefore, N/F ratio does not appear in Fig.6(a). From Fig.6(b), it is apparent that although the side lobe ratio is varied, Hermite, the second kind Tschebyscheff, Legendre, the first kind Tschebyscheff arrays have the constant N/F ratio of 35, 7, 5 and 0 dB, respectively. The discretized Taylor one parameter and discretized Taylor $\bar{n}$ arrays possess the non-uniform N/F ratio. The N/F ratio is changed as the side lobe ratio is varied. Their N/F ratios do not converge to any certain value according to the deterioration of the side lobe from the sampling process.



(a)

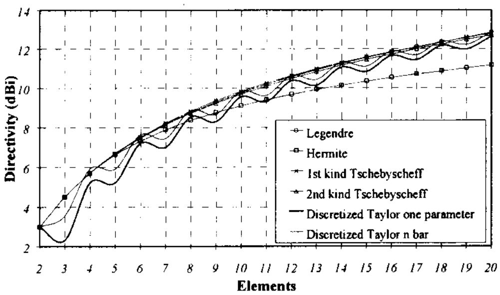


(b)

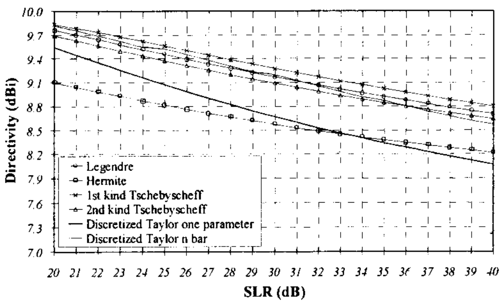
Fig.6 (a) The nearest to the furthest minor lobe ratio for various number of elements (b) The nearest to the furthest minor lobe ratio for various side lobe ratios

6.5 Directivity

Directivity of all arrays for various number of elements and side lobe ratios are plotted as shown in Figs.7(a) and 7(b), respectively. The values of the directivity correspond to the beamwidth i.e., the narrow beamwidth leads to the high directivity and vice versa. It is evident that the directivity of the second kind Tschebyscheff, Legendre and the first kind Tschebyscheff arrays are almost identical. The discretized Taylor $\bar{n}$ and discretized Taylor one parameter array possess the directivity slightly lower than those three arrays. Furthermore, the directivity of these discretized Taylor arrays of even elements is higher than the odd elements. The reason is that the current distribution at the end of the array for odd elements is not distributed in the same manner as in even elements which affects the directivity. The directivity of Hermite array is the lowest as expected due to very wide beamwidth. When the side lobe is varied at the fixed number of elements as illustrated in Fig.7(b), the directivity of all arrays are similar. They are decreased from higher to lower value as follows; the first kind Tschebyscheff, Legendre, the discretized Taylor $\bar{n}$, the second kind Tschebyscheff, the discretized Taylor one parameter and Hermite arrays, respectively.



(a)

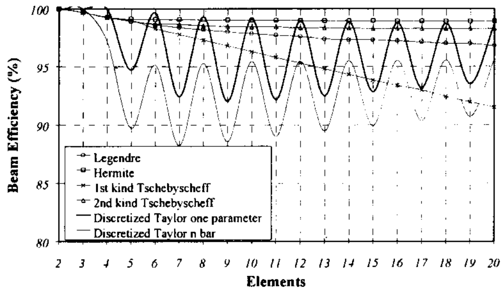


(b)

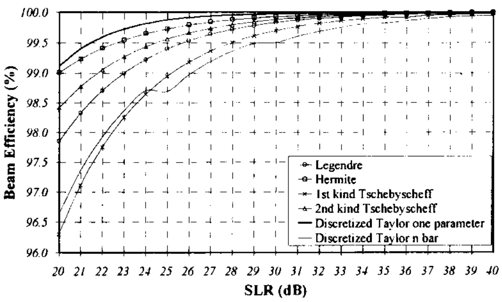
Fig.7 (a) Directivity for various number of elements (b) Directivity for various side lobe ratios

6.6 Beam Efficiency

Beam efficiency is defined as the ratio of the power, distributed to the main lobe, to the total radiated power. For some applications, it is desirable to yield the tapered minor lobe as well as the high beam efficiency. Fig.8(a) illustrates the beam efficiency for various number of elements. It is evident that beam efficiency of Hermite array is the highest and it is followed by the discretized Taylor one parameter array of the even elements, the second kind Tschebyscheff array, Legendre array, the discretized Taylor $\bar{n}$ array of the even elements, the first kind Tschebyscheff array, the discretized Taylor one parameter array of odd elements and the discretized Taylor $\bar{n}$ array of the odd elements, respectively. Alternatively, when the side lobe ratio is varied at the constant number of elements as shown in Fig.8(b), it is seen that beam efficiency of all the arrays is greater than 96%. The beam efficiency tends to 100% when the side lobe ratio approaches 40 dB. The discretized Taylor one parameter array achieves the highest beam efficiency followed, in order, by Hermite, the second kind Tschebyscheff, Legendre and the first kind Tschebyscheff or the discretized Taylor $\bar{n}$ arrays. The beam efficiency of the discretized Taylor $\bar{n}$ is not decreased in the same fashion as the other arrays due to the change of the null angle as described in the previous section.



(a)



(b)

Fig.8 (a) Beam efficiency for various number of elements (b) Beam efficiency for various side lobe ratios

7. Conclusions

Array characteristics of the Legendre, Hermite, the second kind Tschebyscheff, discretized Taylor one parameter, discretized Taylor $\bar{n}$ and the first kind Tschebyscheff arrays are comparatively studied. The radiation patterns, the maximum to minimum current ratio, beamwidth, directivity, the nearest to the furthest minor lobe ratio and beam efficiency are characterized and compared. It is obvious that the first kind Tschebyscheff array has uniform minor lobe distributions. Thus, the noise or spurious signals can enter through those tapered minor lobes. It is not suitable for low noise systems. For the arrays with the tapered minor lobe, Hermite array has the highest N/F ratio but a very high maximum to minimum current ratio, especially when the number of elements exceed 20, this drops our further interest in applications. The discretized Taylor one parameter and the discretized Taylor $\bar{n}$ arrays are alternative ways to accomplish the appropriate characteristics for applying to radar and low noise systems. However, since these two methods are originally applicable for the continuous line source, the discretization for small number of elements maybe causes some errors and leads to the non-uniform characteristics. Eventually, Legendre and the second kind Tschebyscheff arrays yield the tapered minor lobes with narrow beamwidth, low maximum to minimum current ratio, high directivity, high N/F ratio and high beam efficiency. Even though their characteristics are not dominant as in the other arrays, it is acceptable from the view point of all average properties. However, it can be expressed that it is possible to use all of the wrays presented in this paper in radar and communication with low noise systems. Furthermore, for practical applications the choice to use these array types can be made according to the individual characteristic requirements. For instance, if the number of elements is small, the side lobe is low, and a high N/F ratio as well as high beam efficiency are necessary, Hermite array is the most appropriate.

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