On Modelling and Optimizing Traffic Flow at a Busy Road Junction

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Abstract

This paper develops the model equations governing traffic queue growth and traffic waiting time for time-dependent vehicle arrival and departure rates, at a busy road junction. The equations are developed for deterministic arrival and departure rates from first principles. The paper then considers the problem of controlling the traffic flow at such a junction, through optimization of the traffic light phases. The first criterion considered for the optimization is the minimization of total vehicle waiting time. Then a new criterion for optimizing the traffic light phasing, based on attempts to equalize the queue lengths for all directions at a road junction, is considered. This new criterion is termed here the "fairness criterion". Results are obtained from simulations representing a busy junction, with time-dependent vehicle arrival rates, when the traffic lights at the junction are controlled using these criteria. The results obtained from using these two criteria are contrasted and discussed. The results obtained indicate that the fairness criterion may have some attractive features for traffic light control at busy junctions.

1. Introduction

The analysis of traffic flow, particularly for busy or over-saturated road conditions, particularly at signalized junctions, has received, and continues to receive great attention from transport researchers all over the world. A variety of modelling and analysis techniques have been used to address this problem. Recent examples are: the application of fuzzy logic (ref. [1]), the interpretation of traffic systems in terms of chaotic behaviour (ref. [2]), and the use of coordinate transforms in the development of stochastic models (ref. [3]).

The need for such analysis, and moreover, the need for the benefits which should accrue from such analysis, are especially true for cities with very serious traffic congestion problems. A notorious example of such a city is Bangkok, Thailand, where the long-suffering Bangkok commuter is subject to what appears to be, at times, arbitrary and irrational traffic control at road junctions (ref. [4]).

Practical details relating to the implementation of traffic-control devices and systems are discussed in detail in ref. [5] and in ref. [6]. A queuing model for traffic at signalized junctions is presented in ref [7]. In addition, an example of one form of optimization of traffic light phasing is presented in ref. [7], where the optimization criterion used is the minimization of total vehicle waiting time at the junction.

In this paper, a traffic queuing model for a busy junction is developed representing a generalization and extension of what is presented in ref. [7]. The optimization of traffic light phasing is then addressed, and a novel criterion for this optimization is suggested. This novel criterion is termed here the 'fairness criterion', and attempts to prioritize equalization of queue lengths at a junction rather than the minimization of the lumped total waiting time of all vehicles at a junction. This 'fairness criterion' is specifically aimed at very heavily congested junctions, and was born out of observation of the traffic conditions at a particularly congested junction in central Bangkok. At the junction in question, traffic is halted in two opposing directions for what appears to be a protracted period of time, whilst flow crosswise is permitted unhindered.

The optimization carried out in ref. [7], makes a text book assumption that the capacity of the junction exceeds arrivals for all approaches at the junction. No such assumption is made in the optimization carried out in this paper: all possible scenarios with regard to the time-dependent arrival and departure rates are incorporated into the model equations.

In the development, the principle of "Ockham's Razor" (ref. [8]) is used. In the context of mathematical modelling, one way of putting this principle is "Complex models are penalized" (Ibid.). Model equations are developed here systematically from first principles, using basic or elemental variables.

2. Model Equations

The deterministic equations for traffic queue growth at a busy road junction controlled by traffic lights will be derived from first principles using basic queuing theory. It will not be assumed that the capacity of the junction exceeds arrivals, or in other words, that the traffic queue will necessarily dissipate within one traffic light cycle. This assumption is particularly apt for modelling traffic at a busy junction in Bangkok, where traffic queues during busy periods can easily contain several hundred vehicles, in two or more approach lanes.

The queue lengths for each of the directions at a road junction will increase, decrease, or even stay the same, over a period of time, according to the stipulated arrival and departure rates, the initial queue conditions, and the control exercised over the flow by the traffic lights. The vehicle arrival rates, $\lambda_j(t)$, and departure rates, $\mu_j(t)$, will in general be functions of both time t and road direction index j. This allows the analyst to model time-dependency or periodic effects (for example, "rush-hour" scenarios) as well as directional-dependency represented by the suffix j. The value of the departure rate, $\mu_j(t)$, can incorporate effects such as the ratio of the number of vehicles having a straight departure line to the number of vehicles performing a turn at the junction, and will in general be time-dependent. It is assumed that the time spans associated with changes in $\lambda_j(t)$, and $\mu_j(t)$ are much larger than one complete traffic light cycle; in other words, if there are changes in the values of these quantities, these will only occur at the start of a traffic light cycle. For this reason, the notation indicating time-dependency no longer uses continuous time, t, but instead uses the discrete time steps, i, where $i \geq 1$, representing each of the traffic light cycles. That is, $\lambda_j(i)$, and $\mu_j(i)$ will be used for the arrival and departure rates, respectively. Ultimately the values of $\lambda_j(i)$ and $\mu_j(i)$, can be either deterministic or distributed (i.e. stochastic), depending on what is judged by the analyst to be an adequate model representation.

The model equations are first derived for one direction at a road junction, and are then expressed to cover all directions, through the use of the integer, j, as defined above. Because first of all only one road direction is considered, the suffix j will be temporarily dropped, and the notation: $\lambda(i)$ and $\mu(i)$ will be used for the sake of brevity and clarity.

For a sequence of red and green lights at one set of controlling traffic lights, the queue length at the start of the red time, can be compared with the queue length at the end of the green time. This comparison represents the change in the length of the queue through one complete cycle of the traffic lights. And based on the assumptions made above about arrival and departure rates not changing significantly within one traffic light cycle, a graph of queue length (measured in number of vehicles) versus elapsed time (measured in seconds), can be drawn.

One of the features of the model derivation presented in this paper is the use of the irregular pentagonal graphs shown in Figure 1, in order to classify, and indeed clarify, the change in queue length during one traffic light cycle. It should be emphasized that *no assumption* is being made here regarding the relevant sizes of **a**, **b** and **c.** It will *not* always be the case that $0 < \mathbf{b} \leq \mathbf{c}$: *this pentagon may degenerate*, and this is inherent in the model equations developed here. This may occur, for example, if one or more of the sides have zero length, indicating that queue dissipation has occurred within the traffic light cycle. It is possible for the pentagon to degenerate to: a rectangle if the initial queue length was non-zero and dissipation occurs; to a triangle if the initial queue length was zero and the arrival rate is non-zero, but queue dissipation occurs; and to a straight line if both the initial queue length and the arrival rate are zero. The geometrical representation presented here, is also useful in the calculation of total vehicle waiting time, because of the ease with which the areas under the pentagonal graph (including degenerate forms) can be calculated.

The length, **a**, represents the number of vehicles in the queue at the point when the traffic light has just changed to red. Then, depending on the arrival rate, $\lambda(i)$, the queue will continue to grow (except for a zero arrival rate) until the traffic light changeover to green occurs. The length, **c = a + d**, represents the queue length at the point of changeover to green. If $\lambda(i)$ and $\mu(i)$ are identical, the queue length will remain the same through the green time. If $\lambda(i)$ is greater than $\mu(i)$, the queue length will continue to increase through the green time, but now at a reduced rate. And if $\lambda(i)$ is less than $\mu(i)$, the queue length will tend to decrease for the duration of the green time, possibly dissipating altogether before the next red light occurs. The final length of the queue, for that particular traffic light cycle, is then represented by the length **b**. The value **b** then becomes the **a** value for the next traffic light cycle. The process is then repeated for the next traffic light cycle, with the construction of a new graph as shown in Figure 1. Each of the possibilities described above will be considered in the following analysis.

Consider the quantity ΔT_i defined by:

$$\Delta T_i = \frac{a_i + r_i \lambda(i)}{\mu(i) - \lambda(i)} \text{ if } \lambda(i) \neq \mu(i) \qquad (1)$$

$$\Delta T_i = \infty \text{ for } \lambda(i) = \mu(i) \qquad (2)$$

ΔT_i is positive when $\mu(i) > \lambda(i)$, and represents the current theoretical time length which the traffic light would need to remain on green, at the i^{th} traffic light cycle, for the queue to dissipate. ΔT_i is negative when $\lambda(i) > \mu(i)$, indicating that if the current arrival and departure rates were never to change, queue dispersal could not occur. Similarly, if the arrival rate and the departure rates were equal, ΔT_i as defined, will be infinite, and queue dispersal could not occur.

Next consider the length of time the traffic light remains on green for the i^{th} traffic light cycle, represented by g_i. The corresponding red time is represented by r_i, and the total traffic light cycle length by $p_i = r_i + g_i$. The red and green light times will, in general, be different from one cycle to the next. Moreover, the effective time for which a traffic light can remain on red or green is not constrained by p_i. For example, if g_i is selected to have a zero value, it then follows that the effective time for which the traffic light remains on red can, in fact, be longer then p_i. For the length of the traffic queue at the end of the i^{th} traffic, it can be shown that:

$$b_i = 0 \text{ if } g_i \geq \Delta T_i \geq 0 \qquad (3)$$

(and ΔT_i replaces g_i in the construction of the pentagon, as shown in Figure 1 for case (ii))

$$b_i = \lambda(i) p_i - \mu(i) g_i + a_i \text{ for all other } \Delta T_i \qquad (4)$$

Equation (3) represents the case where the traffic queue would be able to dissipate within the green time. Equation (4) represents the case where the queue would not be able to dissipate during the green time.

For $i \geq 2$, the next $(i+1)^{th}$ traffic light cycle begins with:

$$a_i = b_{i-1} \qquad (5)$$

This process is initiated with the initial condition, a_1, representing the traffic queue length at the start of the first traffic light cycle under consideration. The initial condition, a_1, can be any non-negative value, including, but not necessarily, zero. The length of the queue, c_i, at the start of the green time, for the i^{th} cycle, is given by:

$$C_i = a_i + r_i \lambda_i \tag{6}$$

A common index used in traffic analysis is the total vehicle waiting time (ref. [7]). The total vehicle waiting time effectively represents the integral of the difference between the arrival and departure curves. For one complete traffic light cycle, the incremental increase in total vehicle waiting time which occurs, corresponds to the area underneath the pentagonal graphs shown in Figure 1. Consider the following cases:

(i) For $0 < b_i \le C_i$, there has been no increase in the traffic queue length during the green time, but the queue has not decreased to zero length before the changeover to red occurs. The area of the pentagon, A_i, represents the increment in total traffic waiting time, and can be shown to be given by:

$$A_i = \frac{1}{2}\left[a_i(p_i + r_i) + (\lambda(i)p_i - \mu(i)g_i + a_i)g_i + \lambda(i)r_i p_i\right] \tag{7}$$

(ii) For $b_i = 0$ and $\lambda(i) \neq \mu(i)$, dissipation of the queue has occurred during the green time. In this case, the pentagon is degenerate because one of its sides has zero length. The increment in total vehicle waiting time, A_i, can be shown to be given by:

$$A_i = \frac{1}{2}\left[2a_i r_i + \frac{(a_i + r_i\lambda(i))^2}{\mu(i) - \lambda(i)} + r_i(a_i + r_i\lambda(i))\right] \tag{8}$$

And for $b_i = 0$ and $\lambda(i) = \mu(i)$, this implies that $c_i = 0$. And since $c_i = a_i + r_i\lambda_i$, then this implies that $a_i = 0$ and $r_i\lambda_i = 0$ for $a_i \ge 0$, $r_i \ge 0$ and $\lambda_i \ge 0$. If $a_i = b_i = c_i = 0$, then clearly:

$$A_i = 0 \text{ if } b_i = 0 \text{ and } \lambda(i) = \mu(i) \tag{9}$$

(iii) For $b_i > c_i$, the traffic queue length continues to increase during the green time. The increment in total vehicle waiting time, A_i, can be shown to be given by:

$$A_i = \frac{1}{2}\left[a_i(2r_i - g_i) + 3b_i g_i + \lambda(i)r_i(2r_i - p_i)\right] \tag{10}$$

For a junction comprising n approaching road directions, equations (1) to (10), can be generalized through the use of the extra suffix, j, defining the road direction index. So that, for example, the generalization of equation (4) is:

$$b_{ij} = \lambda_{ij} p_i - \mu_{ij} g_i + a_{ij}$$

$$j = 1..n \tag{11}$$

The increment in total vehicle waiting time, Δw, for the whole junction, is given by summing the values for all n road directions:

$$\Delta w = \sum_{j=1}^{n} A_{ij} \tag{12}$$

And the total vehicle waiting time for the junction at the end of the $(i+1)^{th}$ traffic light cycle T_{i+1} is given by:

$$T_{i+1} = T_i + \Delta w \tag{13}$$

3. Traffic Light Control

The traffic light phasing used at a junction, represented by the values r_i, g_i and p_i, determines the nature of queue growth at a junction. An inappropriate choice of times for these values at a busy junction can produce excessively long tailbacks in one or more of the directions at a road junction. This can have a serious knock-on effect for the road network as a whole. It seems reasonable that the phasing of the traffic lights should be subject to some form of optimization. The criterion for such an optimization might take into account the changes in traffic arrival and departure rates, the current queue lengths in all directions as well as the total number of vehicles at a junction, and maybe even other considerations such as the minimization of pollution levels. Moreover, a junction might also be considered as part of a much wider interconnected road network, and thus each individual traffic light cycle will be seen only as a component in a much larger or global optimization problem.

Figure 2 illustrates, for a particular set of time-dependent arrival and departure rates, the effect that various traffic light phases can have on total vehicle waiting time at a junction. Here, the traffic light phase lengths are fixed, and for each fixed traffic light phase, a time history curve is generated, representing the incremental increase in total vehicle waiting time incurred. Combining these time history curves for all the fixed traffic light phase lengths, produces the surface plot shown.

Although no optimization is performed during the simulations shown in Figure 2, for the traffic conditions prevailing during the simulations, one particular traffic light phasing leads *a posteriori* to the fastest queue dissipation, namely a red time of 28 seconds. The corresponding green time is 32 seconds, for a nominal traffic light cycle length of 60 s. These figures represent one pair of opposing road directions at the junction (for example, northbound and southbound, or eastbound and westbound). Now consider the general problem of optimizing and controlling the traffic light phase lengths, where the optimization can be carried out before each traffic light cycle begins.

4. Total Waiting Time and Fairness Criterion

The minimization of total vehicle waiting time, as defined by equations (7) to (13), can be used as one criterion for performing an optimization of the traffic light phases. This criterion is used in an example given in ref. [7] for fixed arrival and departure rates, with the assumption that approach capacity exceeds approach arrivals in all directions. For the results presented here, this assumption is not made, because the emphasis of this paper is on busy road junction, where at least for some period of time, the arrivals will exceed approach capacity in at least one of the road directions.

The road junction simulated here has time-dependent arrival and departure rates; these are presented in Table 1. A deterministic arrival and departure (D/D/1) queuing model (e.g. ref. [7]) is used. The length of the simulation is 120 minutes. The simulation includes "very busy", "less busy" and "quiet" periods, for all four directions at the road junction, as shown in Table 1. The initial queue lengths are here set to zero, though as noted earlier, this is not a requirement of the method, and any initial non-negative value could be used. The simulation and optimization is performed using the standard mathematical analysis software: *Mathcad* (ref. [9]). In addition, *Mathcad* is used to calculate relevant statistics (such as mean and maximum queue lengths) and to display the simulation results graphically.

Some results for using total vehicle waiting time as the basis for the optimization of the traffic light phases are shown in Figures 3(a), 4(a) and 5 (a). One motivation for seeking an alternative to total vehicle waiting time as the basis for the optimization of the traffic light phases can first be seen in Figure 3(a). The minimum waiting time algorithm holds the Eastbound/Westbound directions on red for a protracted period of time, whilst allowing the Northbound/Southbound directions to flow unceasingly during this time. The development of the queue lengths in the various directions is shown in Figure 4(a). This traffic flow pattern, whilst leading to minimal total waiting time for the junction as a whole, results in a gigantic Eastbound/Westbound tailback which persists for the majority of the simulation period.

The disparity between the queue lengths can be measured by the following criterion, which is introduced here, and termed the fairness criterion ($FAIRNESS_i$). For a junction with n road directions, it is defined by:

$$FAIRNESS_i = \sum_{\substack{p=1 \dots n \\ q=1 \dots p-1}} \left| a_{ip} - a_{iq} \right| \tag{14}$$

where a_{ij} is the queue length at the start of the i^{th} traffic light cycle for the j^{th} road direction.

For example, at a road junction with four road directions, the fairness criterion, $FJ4_i$, is given by:

$$FJ4_i = | a_{i,1} - a_{i,2} | + | a_{i,1} - a_{i,3} | + | a_{i,1} - a_{i,4} | + | a_{i,2} - a_{i,3} | + | a_{i,2} - a_{i,4} | + | a_{i,3} - a_{i,4} | \tag{15}$$

The fairness criterion will have a zero value for queues of equal lengths and large positive values when significant disparities occur. Fairness criterion values are shown as part of Figure 5(a) for traffic light control based on minimizing the total waiting time. The large disparities between the various directions, particularly for the period between about 30 to 50 minutes after the start of the simulation, are evident in the very high fairness criterion values reached during this period. Complete queue dissipation in all directions is seen to occur after about 96 minutes.

5. Fairness Criterion as basis for Optimization

Now consider the minimization of the fairness criterion as a new basis for optimizing the traffic light phasing. As this new criterion is based on the differences between the individual queue lengths, it should be more sensitive to the state of the traffic at each of the individual road directions. This criterion will seek to avoid the excessive disparity in queue lengths seen when total vehicle waiting time alone is used for the optimization, and will be generally fairer to road users in terms of the provision of green time at the junction. In addition, seeking to balance the traffic load in all directions, at each junction in a road network, intuitively feels like the most perfect solution for improving or harmonizing the overall traffic flow.

Some results for basing the optimization solely on the fairness criterion are shown in Figures 3(b), 4(b) and 5(b). It can be seen from Figure 3(b), that the fairness criterion allows traffic flow to take place from all four directions during each traffic light cycle. Unlike the total waiting time criterion, there are no directions subject to lengthy and continuous red time. The development of the queues in the various directions is shown in Figure 4(b). The fairness criterion values are shown as part of Figure 5(b). From all these graphs, it can be seen that the fairness criterion does indeed lead to lower levels of disparity between the queue lengths for the various directions, when compared to the use of the total waiting time criterion. Complete queue dissipation in all directions is seen to occur after about 110 minutes.

6. Use of both Total Waiting Time and the Fairness Criterion

One possibility could be the case where the optimization of the traffic light phasing is based on *both* total vehicle waiting time and the fairness criterion. This is not analyzed in detail in this paper, but is only very briefly considered, with a view to obtaining some indication of possible benefits for further investigation and development. For the simulation of the use of the two criteria, total vehicle waiting time is firstly used, after which a switch is made to the fairness criterion. The first 20 minutes of traffic light control were arbitrarily selected here for the use of total waiting time, after which a switch is made to the use of the fairness criterion. The graphs showing the red time, traffic queue development and fairness criterion, are shown in Figures 3(c), 4(c) and 5(c) respectively. Note, that during the initial period when total vehicle waiting time is used to control the traffic light phasing, the Eastbound/Westbound directions are held permanently on red. Following the switchover to the fairness criterion after 20 minutes, the Northbound/Southbound directions are held on red continuously for a further ten minutes, reducing the Eastbound/Westbound tailback. After this, the fairness criterion permits all directions to flow, within each traffic light cycle. Complete queue dissipation in all directions is seen to occur after about 98 minutes.

7. Comparison

All three cases considered display the most extreme conditions around the forty-five minute mark, corresponding to the busiest time at the junction. Some statistics representing the first 45 minutes are shown in Table 2.

The maximum individual queue length is about 850 vehicles when the total waiting time criterion alone is used, and this drops to about 650 vehicles when the fairness criterion alone is used. Interestingly, these figures drop even further to about 560 vehicles when both criteria are used. The mean longest queue length (meaning for all four road directions, select the longest current queue length from these directions, and then calculate the mean value over the indicated time span) for the first 45 minutes shows a similar trend. Both the fairness criterion and the mixed criterion have much lower mean values for longest queue length than when the total waiting time criterion is used.

The total number of vehicles at the junction is generally higher when the fairness criterion alone is used, whilst the total waiting time and mixed criteria have quite similar values. As would be expected, however, the use of the fairness criterion is much "fairer" than the total waiting time criterion, meaning that there is less disparity in the queue lengths for all the directions at the road junction. And moreover, as noted earlier, there are no lengthy red times when the fairness criterion alone is used.

Statistics for the first 90 minutes are shown in Table 3. The picture is more or less similar to that described above for the first 45 minutes, in that the use of the fairness criterion is shown to produce more equitable traffic conditions at the junction, for the duration of the time period considered.

8. Summary and Conclusions

Model equations for traffic queue growth and traffic light control at a busy road junction were derived for time-dependent traffic arrival and departure rates, covering all possible contingencies. These model equations were derived for deterministic arrival and departure rates from first principles. Simulation results were obtained representing a busy road junction, with the traffic light phasing optimized using total vehicle waiting time. Then a new criterion for optimizing the traffic light phasing, based on minimizing the differences between queue lengths for each of the directions at a road junction, was proposed and demonstrated. Simulation results showed that this new approach for optimizing traffic light phasing (the fairness criterion) appears to have some attractive features for reducing the length of traffic tailbacks and for reducing the disparity between queue lengths for all directions at a busy road junction. In addition, attempting to balance the traffic load for all directions, at each road junction in a road network, may be also the best solution for an overall improvement in traffic flow.

The simulations performed here, as well as the data analysis and graphing of the results were carried out using the standard mathematical analysis software package *Mathcad.*

9. References

[1] Niitymaki Jarkko., Fuzzy Logic Application to Public Transport Priorities at Signalized Intersections, Proceedings of the 3rd IMA International Conference on Mathematics in Transport Planning and Control, pp. 47-57. (Editor: Griffiths J.D), Pergamon, 1998.

[2] Low David J. and Addison Paul S., The Complex Dynamical Behaviour of Congested Road Traffic, Proceedings of the 3rd IMA International Conference on Mathematics in Transport Planning and Control, pp. 341-350. (Editor: Griffiths J.D), Pergamon, 1998.

[3] Heydecker B.G. and Verlander N.Q., Transient Delay in Oversaturated Queues Proceedings of the 3rd IMA International Conference on Mathematics in Transport Planning and Control, pp. 371-380. (Editor: Griffiths J.D), Pergamon, 1998.

[4] Exasperated from Bang Khae, Bangkok., Letters to the Editor, Nation Newspaper, Thailand, Tuesday, 26th October, Letter: ["A New Way", Memo, Police Recruitment Office: Please Raise the Minimum Requirement for a Policeman in Charge of Traffic Lights to have at Least Half a Brain.], 1999.

[5] Wright Paul H., Highway Engineering, pp.330 – 364. John Wiley and Sons, Inc, 1996.

[6] Drane Chris, Rizos Chris., Positioning Systems in Intelligent Transportation Systems, pp. 11- 13, etc. Artech House, Boston and London, 1998.

[7] Mannering Fred L. and Kilareski Walter P., Principles of Highway Engineering and Traffic Analysis, pp. 141-206, Second Edition, John Wiley and Sons, Inc, 1998.

[8] Hoffmann R., Minkin Vladmir I. and Carpenter Barry K., Ockham's Razor and Chemistry", HYLE – An International Journal for the Philosophy of Chemistry, Vol 3, pp.3-28, 1997.

[9] Mathsoft Inc., Mathcad – User's Guide Mathcad 2000, Mathsoft Inc. Cambridge, Massachusetts, 02142, USA.

Table 1: Arrival and departure rates used in simulation.

North $\lambda(t)$ vehicle/s	South $\lambda(t)$ vehicle/s	East $\lambda(t)$ vehicle/s	West $\lambda(t)$ vehicle/s	North $\mu(t)$ vehicle/s	South $\mu(t)$ vehicle/s	East $\mu(t)$ vehicle/s	West $\mu(t)$ vehicle/s	Time t (mins)
0.6	0.25	0.4	0.25	0.38	0.38	0.38	0.38	$0 \le t \le 30$
0.2	0.1	0.1	0.5	0.43	0.43	0.45	0.45	$30 < t \le 60$
0.01	0.01	0.05	0.01	0.45	0.45	0.45	0.45	$60 < t \le 120$

Table 2: Statistics for the first 45 minutes.

For Time 0 to 45 Minutes	Max. Longest Individual Queue Length	Mean Longest Individual Queue Length	Max. Number of Vehicles at Junction	Mean Number of Vehicles at Junction	Max. Fairness Criterion	Mean Fairness Criterion
(a) Total Waiting Time Criterion	852	516	1.35×10^3	837	2.85×10^3	1.71×10^3
(b) Fairness Criterion	653	400	1.56×10^3	992	2.35×10^3	1.41×10^3
Mixed Criteria: $0 \le T \le 20$:(a) $T \ge 20$:(b)	565	390	1.38×10^3	860	1.57×10^3	1.05×10^3

Table 3: Statistics for the first 90 minutes.

For Time 0 to 90 Minutes	Max. Longest Individual Queue Length	Mean Longest Individual Queue Length	Max. Number of Vehicles at Junction	Mean Number of Vehicles at Junction	Max. Fairness Criterion	Mean Fairness Criterion
(a) Total Waiting Time Criterion	852	479	1.35×10^3	737	2.85×10^3	1.52×10^3
(b) Fairness Criterion	653	347	1.56×10^3	795	2.35×10^3	1.29×10^3
Mixed Criteria: $0 \le T \le 20$: (a) $T \ge 20$:(b)	565	371	1.38×10^3	804	1.77×10^3	1.17×10^3

Figure 1: Traffic Queuing Model Variables

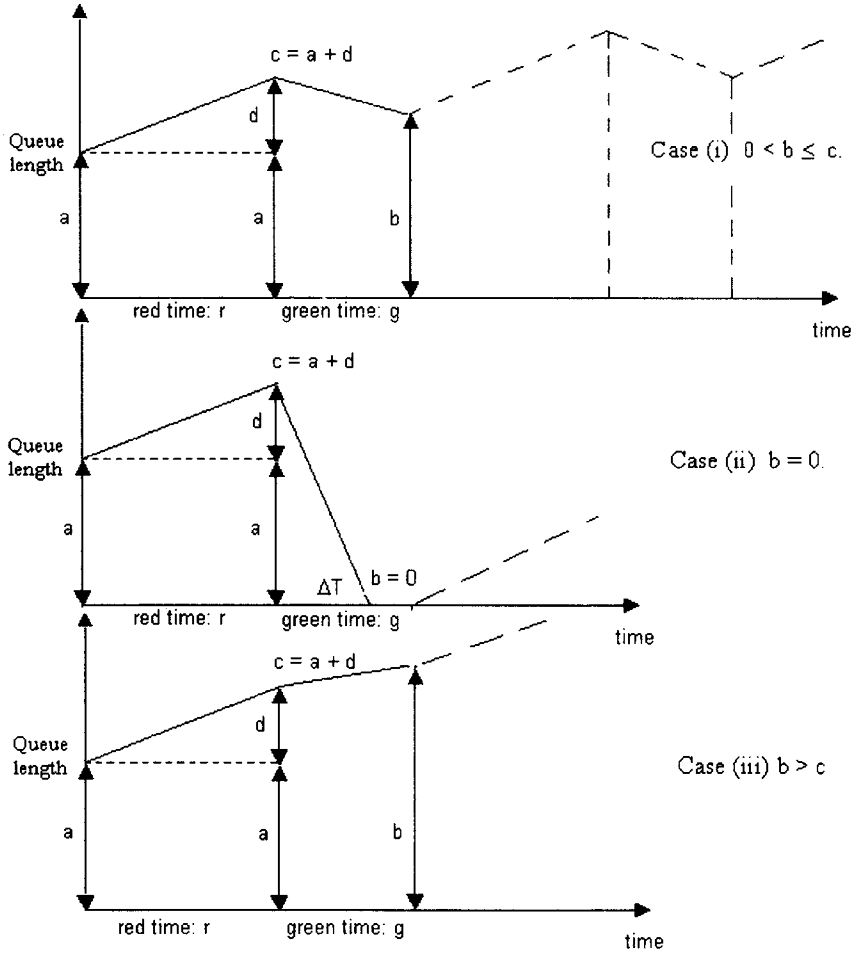

Figure 2: Effect of fixed red time on total vehicle waiting time

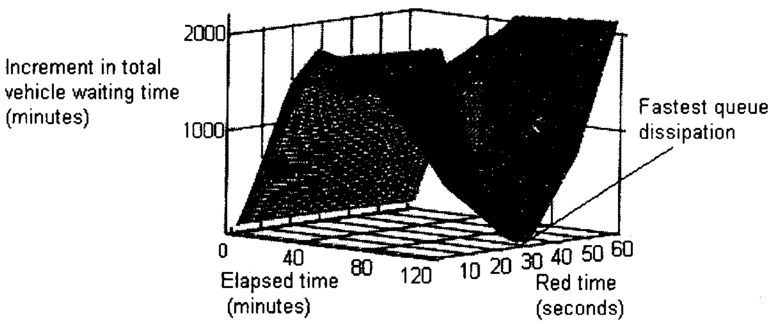

Figure 3: Red time versus elapsed time at junction:

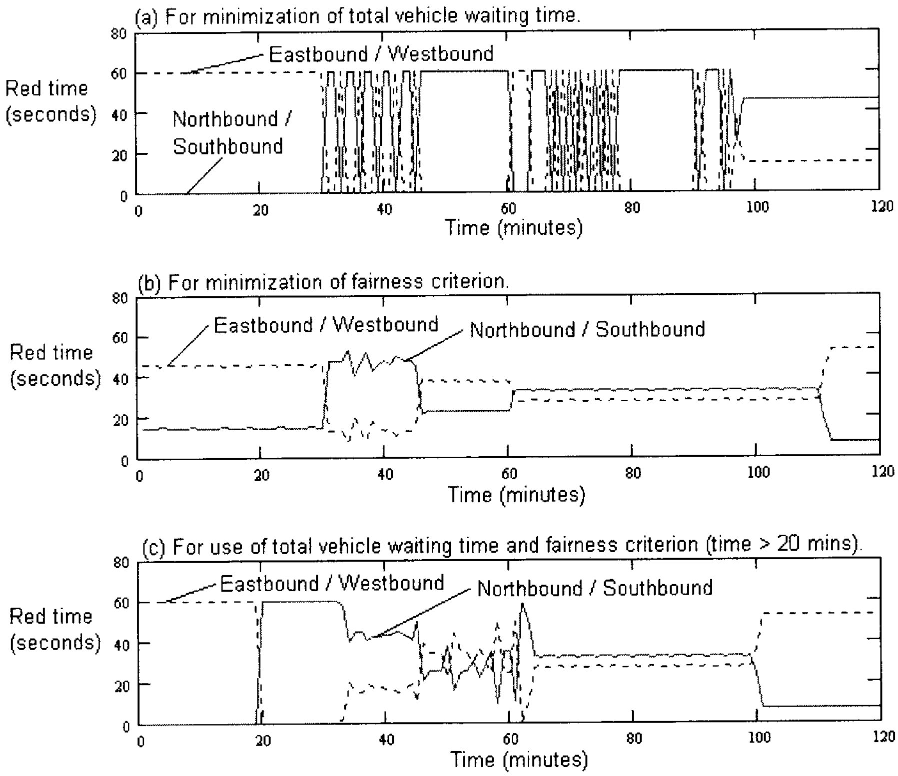

Figure 4: Queue length versus elapsed time at junction:

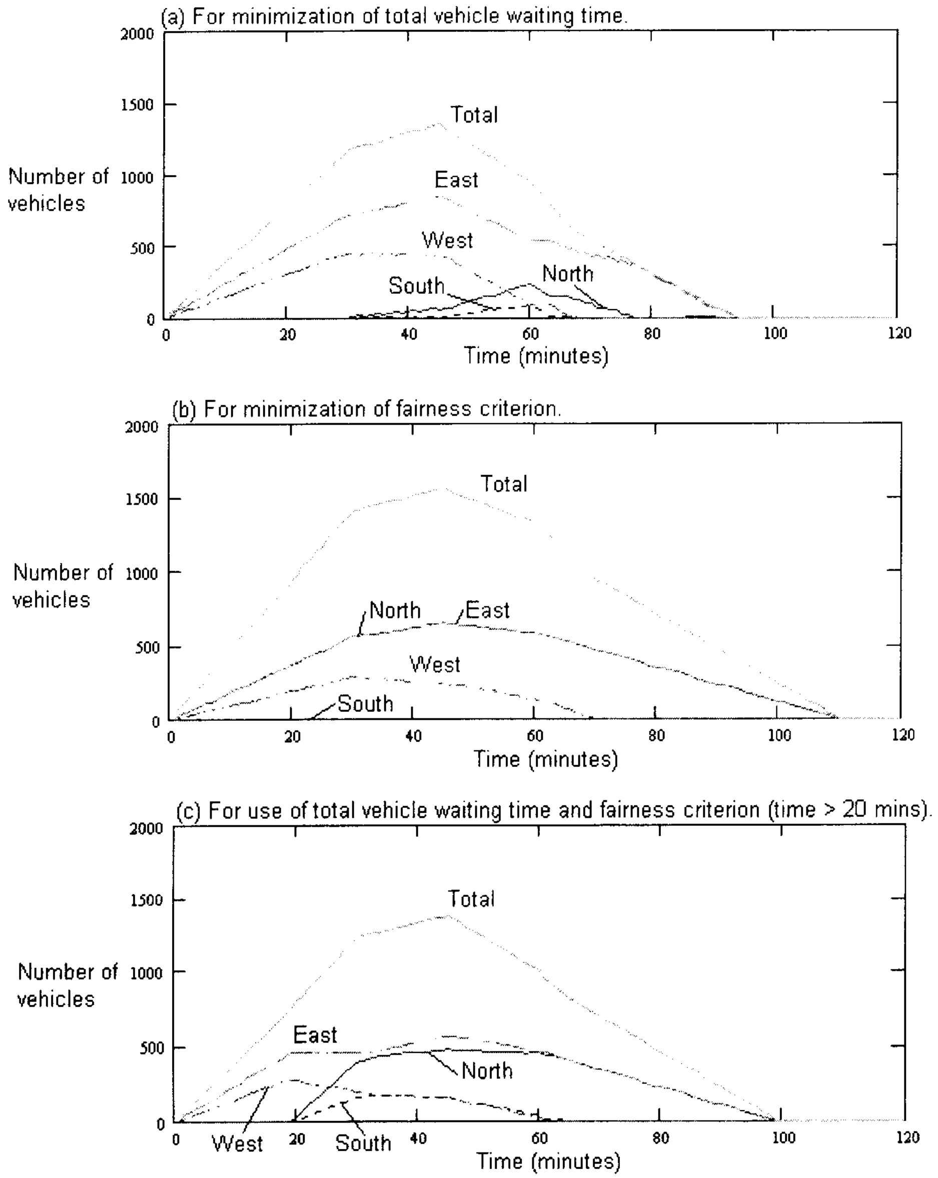

Figure 5: Fairness criterion, total queue length, and maximum queue length, versus elapsed time at junction: