Justification of Material Selections with Fuzzy Quantifiers and OWA Operators

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Abstract

This paper presents a multi-criterion decision-making method with the fuzzy quantifier and the Ordered Weight Averaging (OWA) operator developed to support the justification of material selection in engineering design applications. All the properties of the materials available are to be defined in the fuzzy linguistic terms such as Low_Hardness, Medium_Hardness, and High_Hardness, which are given by the material experts. With these labels of the fuzzy sets to the material properties, the fuzzy quantifiers and OWA operators can be used to determine the truth value of the proposition: *Most Desired Properties are Properties of trial Material*. This truth value obtained is to be used as the most suitability index to rank all of the suitable materials for their application. The numerical examples of the material selections are used to illustrate the viability of the proposed methodology.

1. Introduction

Design engineers are usually more comfortable making imprecise verbal statements rather than quantitative estimations for their justification of multi-criterion decision making. Unintentionally, the outcomes of such decision making may be subjective or biased by their own personal feeling. The problems of material selection for multi-criterion also possess such characteristics of impreciseness. Fuzzy logic approach is much closer in spirit to such human thinking and natural language. It also provides an effective means of quantifying the inexact nature of the real world. In using the formalisms "*Q D's is M*" where Q is the fuzzy quantifier, D and M are the predicates in the fuzzy subsets, the problem of considerable interest becomes that of determining the truth of such a proposition. In the problem of the material selection, that proposition can be expressed as: "*Most Desired Properties are Properties of trial Material*". The degree of the truth of this statement can be determined by a method of Yager[1] for associating each regular fuzzy quantifier with OWA operator. The purpose in this paper is to introduce a methodology, which converts the linguistic strategy based on expert knowledge into a quantitative multi-criterion decision making strategy for the justification of the material selections with fuzzy quantifier and OWA operator.

2. Ordered Weighted Averaging (OWA) operators

In this section, the OWA operator of Yager[1] is reviewed to provide an aggregation of multi-criterion functions, which lies in between the two extremes. At one extreme is the situation in which we desire that all the criteria be satisfied. At the other extreme is the situation in which the satisfaction of at least one of the criteria is all we desire. These two cases leads to the use of "and" and "or" operators to combine the criterion function. The OWA operator can be regarded as the "orand" operator. That is, the requirement that "most" of the criteria be satisfied corresponds to one of the OWA operators.

Definition:

The OWA operator of dimension n is defined as a function $f:[0,1]^n \rightarrow [0,1]$ that has associated with a set of weights $W = \{w_1 \quad \Lambda \quad w_i \quad \Lambda \quad w_n\}$.

$$f(a_1 \quad \Lambda \quad a_i \quad \Lambda \quad a_n) = w_1 b_1 + \mathrm{K} + w_i b_i + \mathrm{K} + w_n b_n \qquad (1)$$

where b_i is the i^{th} largest of a_i's

The weights are defined on the following conditions: $w_i \in [0,1]$ and $\sum_{i-1}^{n} w_i = 1$. It should be noted that the argument of b is arranged in the descending order and $0 \leq f(b_1 \quad \Lambda \quad b_i \quad \Lambda \quad b_n) \leq 1$.

3. Fuzzy Quotient Operator

The evaluation of the truth of the following propositions[2] is presented in this section:

$$Q \ D\text{'s is } M.$$

Where Q is the regular quantifier, D and M are the fuzzy subsets of a set P.

For a discrete set of elements, the fuzzy subset is usually formed by a vector including the symbol "/" that associates the membership value d_i with its coordinate p_i such as $D = \{d_1/p_1 \quad \Lambda \quad d_i/p_i \quad \Lambda \quad d_n/p_n\}$. The quantifier Q is used to represent the amount of items for indicating a given predicate. For an example, there are two widely used quantifiers of the classical two-valued logic, that is, the universal $\forall$ and existential $\exists$. To be more natural, Zadeh[3] introduced the concept of a linguistic quantifier. It is used to quantify the additional vagueness to a given statement such as "Most" and "More or less" which can be qualitatively defined as $Q(\mu) = \mu^2$ and $Q(\mu) = \mu^{0.5}$ respectively where $\mu \in [0,1]$.

With any argument $\langle u_1 \quad \Lambda \quad u_i \quad \Lambda \quad u_n \rangle$ where $u_i \in [0,1]$, the truth value $\mathfrak{I}$ of the resulting OWA operation can be obtained as the following equation

$$\mathfrak{I} = f(u_1 \quad \Lambda \quad u_i \quad \Lambda \quad u_n) = \sum_{j=1}^{n} w_j \bar{e}_j \qquad (2)$$

where $\bar{e}_j$'s is the j^{th} largest of the u_i's.

It should be noted that the $\bar{e}_j$ are ordered such that $\bar{e}_1 \geq \bar{e}_2 \geq \mathrm{K} \geq \bar{e}_n$.

The argument u and the weight w can be determined as follows. With the definition of $d = \sum_{i=1}^{n} d_i$, the weights from Q given by a d can be determined by the following relation:

$$w_i = Q(s_i) - Q(s_{i-1}) \qquad (3)$$

where

$$s_i = \frac{1}{d} \sum_{k=1}^{i} \underline{e}_k \qquad ; s_0 = 0 \qquad (4)$$

where $\underline{e}_k$ is the k^{th} smallest of the d_i's. This yields $\underline{e}_1 \leq \underline{e}_2 \leq \mathrm{K} \leq \underline{e}_n$.

For each p_i, the argument u_i can be determined from Eq. (5)

$$u_i = D(p_i)M(p_i) + \lambda(1 - D(p_i)) \qquad (5)$$

with

$$\lambda = 1 - \frac{1}{n-1} \sum_{i=1}^{n} (n-i) w_i \qquad (6)$$

where λ is the degree of the *andness* associated with the OWA weights.

It should be noted that if Q is the *all* quantifier, $\lambda = 1$ and if Q is the *or* (*at least one*) quantifier, $\lambda = 0$.

4. Examples

A numerical example of finding the most suitable material, which possesses most of the desired properties for a design is used to illustrate how to implement the OWA operators to provide a quotient operation in fuzzy relational data bases. Suppose that the triangular fuzzy subsets as shown in Fig. (1) and Fig. (2) are used to specify the fuzziness concept of the two properties, that is the hardness and density respectively.

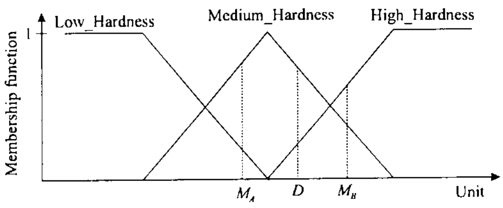


Fig. 1 Membership function of Hardness property.

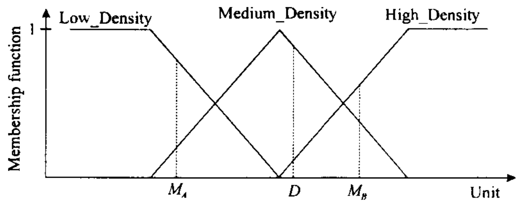


Fig. 2 Membership function of Density property.

The value of membership of the material A and B can be defined as shown in Fig. (1) and Fig. (2) and listed in Table 1.

Table 1 The membership value of materials to hardness and density properties

Fuzzy set of properties	Member -ship of Material A	Member -ship of Material B
Low_hardness p_1	0.20	0
Medium_hardness p_2	0.80	0.38
High_hardness p_3	0	0.62
Low_density p_4	0.80	0
Medium_density p_5	0.20	0.38
High_density p_6	0	0.62

According to Table 1, the fuzzy property subsets of the material A and B are respectively written as:

$$M_A = \{0.20/p_1 \quad 0.80/p_2 \quad 0/p_3 \quad 0.80/p_4 \quad 0.20/p_5 \quad 0/p_6\}$$

and

$$M_B = \{0/p_1 \quad 0.38/p_2 \quad 0.62/p_3 \quad 0/p_4 \quad 0.38/p_5 \quad 0.62/p_6\}$$

The fuzzy subset of the desired material D is defined on the property set as:

$$D = \{0/p_1 \quad 0.76/p_2 \quad 0.24/p_3 \quad 0/p_4 \quad 0.90/p_5 \quad 0.10/p_6\}$$

In this case, $Q(\mu) = \mu^2$ for "Most" quantifier. Therefore, we obtain

$\underline{e}_1 = 0$, $\underline{e}_2 = 0$, $\underline{e}_3 = 0.10$, $\underline{e}_4 = 0.24$, $\underline{e}_5 = 0.76$, $\underline{e}_6 = 0.90$

and $d = 0+0+0.10+0.24+0.76+0.9 = 2$. From Eq. (3),

$$s_1 = \frac{1}{2}[0] = 0$$

$$s_2 = \frac{1}{2}[0+0] = 0$$

$$s_3 = \frac{1}{2}[0+0+0.10] = 0.05$$

$$s_4 = \frac{1}{2}[0+0+0.10+0.24] = 0.17$$

$$s_5 = \frac{1}{2}[0+0+0.10+0.24+0.76] = 0.55$$

$$s_6 = \frac{1}{2}[0+0+0.10+0.24+0.76+0.90] = 1$$

By using Eq. (3), the weights associated with the OWA operator are:

$$w_1 = Q(s_1) - Q(s_0) = 0 - 0 = 0$$
$$w_2 = Q(s_2) - Q(s_1) = 0 - 0 = 0$$
$$w_3 = Q(s_3) - Q(s_2) = (0.05)^2 - 0 = 0.0025$$
$$w_4 = Q(s_4) - Q(s_3) = (0.17)^2 - (0.05)^2 = 0.0264$$
$$w_5 = Q(s_5) - Q(s_4) = (0.55)^2 - (0.17)^2 = 0.2736$$
$$w_6 = Q(s_6) - Q(s_5) = (1)^2 - (0.55)^2 = 0.6975$$

According to the weights obtained, the degree of andness can be determined from Eq. (6).

$$\lambda = 1 - \frac{1}{6-1}[5(0) + 4(0) + 3(0.0025) + 2(0.0264) + 1(0.2736) + 0(0.6975)] = 0.9332$$

For material A,

$$u_1 = 0(0.20) + 0.9332(1-0) = 0.9332$$
$$u_2 = 0.76(0.80) + 0.9332(1-0.76) = 0.8320$$
$$u_3 = 0.24(0) + 0.9332(1-0.24) = 0.7092$$
$$u_4 = 0(0.80) + 0.9332(1-0) = 0.9332$$
$$u_5 = 0.9(0.20) + 0.9332(1-0.90) = 0.2733$$
$$u_6 = 0.10(0) + 0.9332(1-0.10) = 0.8399$$

$\bar{e}_1 = 0.9332$, $\bar{e}_2 = 0.9332$, $\bar{e}_3 = 0.8399$, $\bar{e}_4 = 0.8320$, $\bar{e}_5 = 0.7092$, $\bar{e}_6 = 0.2733$

The truth value of material A have properties mostly similar to the desired material is

$$\Im_{M_A} = 0.9332(0) + 0.9332(0) + 0.8399(0.0025) + 0.8320(0.0264) + 0.7092(0.2736) + 0.2733(0.6975) = 0.4087$$

For material B,

$$u_1 = 0(0) + 0.9332(1-0) = 0.9332$$
$$u_2 = 0.76(0.38) + 0.9332(1-0.76) = 0.5128$$
$$u_3 = 0.24(0.62) + 0.9332(1-0.24) = 0.8584$$
$$u_4 = 0(0) + 0.9332(1-0) = 0.9332$$
$$u_5 = 0.90(0.38) + 0.9332(1-0.90) = 0.4391$$
$$u_6 = 0.10(0.62) + 0.9332(1-0.10) = 0.9019$$

$\bar{e}_1 = 0.9332$, $\bar{e}_2 = 0.9332$, $\bar{e}_3 = 0.9019$, $\bar{e}_4 = 0.8580$, $\bar{e}_5 = 0.5128$, $\bar{e}_6 = 0.4316$

The truth value of material B which has properties mostly similar to the desired material is

$$\begin{aligned}\Im_{M_B} &= 0.9332(0) + 0.9332(0) + 0.9019(0.0025) \\ &\quad + 0.8580(0.0264) + 0.5128(0.2736) \\ &\quad + 0.4316(0.6975) \\ &= 0.4662\end{aligned}$$

Therefore, the material which has most of the desired properties is the material B, since the truth value of material B is greater than that of material A.

For the further discussion, suppose that the material C in the rational database has a similar fuzzy subset of the set P in the desired material D, that is,

$M_C = \{0/p_1 \quad 0.76/p_2 \quad 0.24/p_3 \quad 0/p_4 \quad 0.90/p_5 \quad 0.10/p_6\}$

After doing the same calculation above, the truth value of the material C is 0.7789, which is the maximum truth value since the material C is identical to the desired material D.

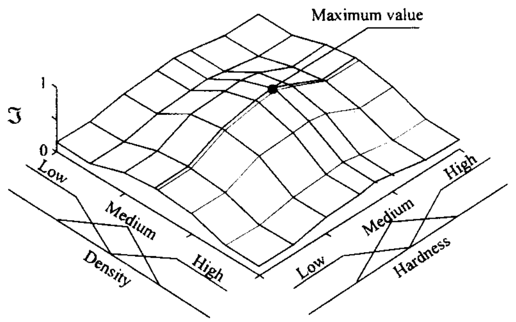


Fig. 3 Truth values against the fuzzy sets of the hardness and the density

According to Fig. 3, it can be seen that all the truth values determined are less than or equal to the maximum truth value at the coordinates of the desired properties.

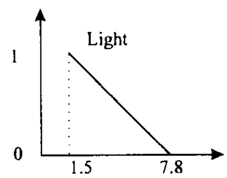
Now, the case study of material selection for a torsionally stressed cylindrical shaft in Callister[4] is considered and summarized as follows. The five candidate materials in Table 2 are presented according to their properties: density, strength and relative cost.

Table 2 Density, strength, and relative cost for five engineering materials.

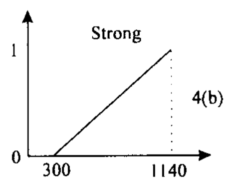
Materials	Density (Mg/m^3)	Shear strength (MPa)	Relative cost[1]
Carbon fiber-reinforced composite	1.5	1140	45
Glass fiber-reinforced composite	2.0	1060	6.5
Aluminum alloy	2.8	300	10
Titanium alloy	4.5	480	20
4340 steel	7.8	780	2

[1]The relative cost is the ratio of the prices per unit mass of the material and the low-carbon steel.

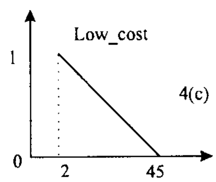
The criterion of material selection is that the most suitable material is light, strong and low cost. The definitions of membership functions of the properties, that is, light p_1, strong p_2 and low cost p_3 are given by linear relation as shown in Fig. 4(a), 4(b), and 4(c) respectively.



4(a) Membership function of "Light" fuzzy set in density



4(b) Membership function of "Strong" fuzzy set in shear strength



4(c) Membership function of "Low_cost" fuzzy set in relative cost

Fig. 4 Membership functions of the given properties. (a) "Light" fuzzy set in density (b) "Strong" fuzzy set in shear strength (c) "Low_cost" fuzzy set in relative cost.

The desired properties are defined as:

$D = \{1/p_1 \quad 1/p_2 \quad 1/p_3\}$

After applying Eqs (3), (4), (6), the degree of *Andness* associated with OWA weights is 0.7722.

Referring to Fig. 4 and using Eq. (2), the truth values of the given materials can be determined and listed in Table 4.

Table 4 Ranking the given materials according to truth value

Materials	Truth value $\Im$
Carbon fiber-reinforced composite	0.4463
Titanium alloy	0.3143
4340 steel	0.3041
Glass fiber-reinforced composite	0.1807
Aluminum alloy	0.0867

From Table 4 and using Eq. (2), the five materials are ranked according to the truth values. It can be seen that Carbon fiber-reinforced composite has the maximum truth value. This means that Carbon fiber-reinforced composite has most of the properties for the requirements especially in density and strength. However, in real-life situations, material cost may be a more important issue, which may dictate the material of choice. With this reason, the Titanium alloy, 4340 steel, and so on can be considered more suitable materials instead.

5. Conclusion

This paper has presented a quantitative approach for justification of the material selections with the fuzzy quantifiers and OWA operators. The truth value of the proposition, *Most Desired Properties are Properties of trial Material,* is determined such that it can be used to rank all candidate materials for consideration. The material corresponding to the highest truth value has the most degree of satisfaction of all the criteria. In this sense, the truth value can be used as the suitability index. The proposed methodology has been applied to the problems of the multi-criterion decision making of the material selections.

6. References

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