Noninteracting Control Design in Plasticating Extrusion Processes

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Abstract

This paper concentrates on the control design of a noninteracting multi-input multi-output system in a plasticating extrusion process. With the input-output information, the system identification provides the dynamics of the extrusion process under operating conditions with no in-depth understanding of what takes place inside an extruder. The model obtained has a strong interaction between process variables. The noninteracting system design is applied such that each input affects one and only one output. The PI controllers are then designed separately in order to yield the desired specifications in each loop. The simulation results show the effectiveness of the proposed methodology.

1. Introduction

The most common plasticating extruders are single-screw extruders as shown in Fig 1.

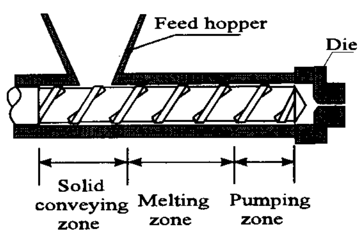

Fig. 1 A schematic diagram of a plasticating extruder. Three zones are illustrated: solid conveying, melting, pumping[1,2].

Plastic pellets are fed into the extruder through a feed hopper. From the solid conveying zone, they are compressed in the channel of the screw and dragged forward to the melting zone. Generated by friction between the pellets and the barrel surface, heat causes the pellets to melt. The melt is pressurized by means of a drag mechanism along the pumping zone and forced into a forming die. Finally, it is cooled to hold the shape. To form the precise dimensions of the product, the process variables, such as the flow rate and temperature of the extrudate, need to be controlled.

Controlling process variables in the extrusion processes is regarded as the challenging multivariable control problem. This is because unique features of the processes, such as nonlinearities and heavy interaction of variables, are presented in the processes. Consequently, it is found that the conventional PID controllers, which are widely used, are difficult to tune and they may cause instabilities.

To enhance the capability of the conventional PID controllers, the proposed methodology is applied to such a control problem as follows. Modelling of physical phenomena around the operating conditions can first be handled by the system identification[3]. After obtaining the dynamic model, the noninteracting control design in the next section will be implemented such that each input influences one and only one output. Now, the conventional PID controllers can be implemented in order to obtain the desired specifications.

In this paper, the modelling part is not omitted and more details can be seen in [3,4]. The design of a noninteracting system is presented and applied to the plasticating extrusion process in section 3. Finally the conclusion is stated.

2. Noninteracting Control Design

In this section, the linear state feedback control design[5] is implemented in a multi-input multi-output system in such a way that each input affects one and only one output.

Consider the time-invariant linear system with an equal number m of inputs and outputs:

$$\dot{x} = Ax + Bu \quad (1)$$
$$y = Cx \quad (2)$$

where x is the $n \times 1$ state vector, u is the $m \times 1$ input vector, y is the $m \times 1$ output vector, A, B and C are the $n \times n$, $n \times m$ and $m \times n$ system matrices respectively.

To develop noninteracting system, let's propose a state feedback control law:

$$u = -K_d x + F_d v \quad (3)$$

where v is the new $m \times 1$ input vector.
Therefore, the $m \times m$ square transfer matrix between y and v for the state feedback of Eq. (1), (2) and (3) is

$$\frac{y(s)}{v(s)} = G(s) = C[sI - A + BK_d]^{-1} BF_d \quad (4)$$

Referring to Eq.(4), the noninteracting problem is that of determining matrices K_d and F_d so that G is a diagonal and nonsingular matrix.
Thus, let the i^{th} row of C be c_i and a set of m integers be given by

$$d_i = \min_j \left\{ j \middle| c_i A^j B \neq 0, j = 0,1,\ldots,n-1 \right\}$$
or
$$d_i = n - 1 \text{ if } c_i A^j B = 0 \text{ for all } j \quad (5)$$

One set of noninteracting matrices is

$$F_d = P^{-1} \quad (6)$$
and
$$K_d = P^{-1} \begin{bmatrix} c_1 A^{d_1+1} \\ \vdots \\ c_m A^{d_{m+1}} \end{bmatrix} \quad (7)$$

where P is defined by

$$P = \begin{bmatrix} c_1 A^{d_1} B \\ c_2 A^{d_2} B \\ \vdots \\ c_m A^{d_m} B \end{bmatrix} \quad (8)$$

To avoid unacceptable performance of the noninteracting system, one may design a state feedback matrix K_p which is used to place the eigenvalues of the noninteracting system to desired locations. This can be done by treating the noninteracting system as the open-loop system in Eq. (1) and (2) where $\tilde{A} = A - BK_d$, $\tilde{B} = BF_d$ and $\tilde{C} = C$. The pole placement method[6] is then applied in order to yield specified closed-loop poles. A block diagram of a closed-loop system is shown in Fig 2.

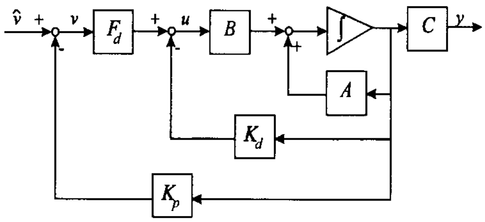

Fig. 2 The closed-loop system uses K_d and F_d to attain noninteraction and K_p to achieve the pole placement.

Thus, the closed-loop transfer matrices of the noninteracting system between y and $\tilde{v}$ can be written as

$$\frac{\hat{y}(s)}{\hat{v}(s)} = \hat{G}(s) = \hat{C}[sI - \hat{A} + \hat{B}K_p]^{-1}\hat{B} \tag{9}$$

In the next section, the noninteracting system design will be applied to the plasticating extrusion process developed by [7].

3. Example

This example demonstrates the regulating of the flow rate of the coolant in the cooling jacket u_1 and the speed of the driving motor u_2 to obtain the desired temperature of the extrudate y_1 and the flow rate of extrudate y_2 whereas the temperature of coolant is considered the disturbance of the system d. This interacting process is described by the block diagram in Fig. 3 and the values of parameters are listed in the Appendix.

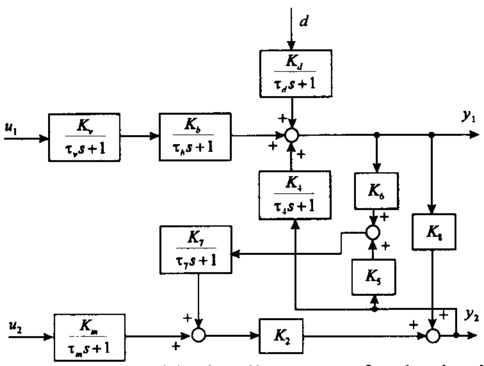

Fig. 3 The block diagram of plasticating extrusion process.

According to the gain and the transfer function blocks in Fig. 3, the state representation of the process without dynamic disturbance can be expressed by Eq. 1 and 2 where the system matrices are

$$A = \begin{bmatrix} -0.5 & 0 & 0 & 0 & 0 \\ 2.5696 & -0.1266 & 0 & 0 & 0 \\ 0 & 0.0028 & -0.1238 & 0.0046 & 0.0046 \\ 0 & 0 & 0 & -0.5 & 0 \\ 0 & 1.4114 & 1.4114 & 0.0950 & -0.5717 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.5 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0.5 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0.0867 & 0.0867 & 0.1414 & 0.1414 \end{bmatrix}$$

To illustrate the interactions of the variables, the unit step signals of u_1 and u_2 are inputted to the process and the responses of y_1 and y_2 are shown in Fig. 4 (a) and 4(b) respectively. It is noted that both inputs u_1 and u_2 can affect not only y_1 but also y_2.

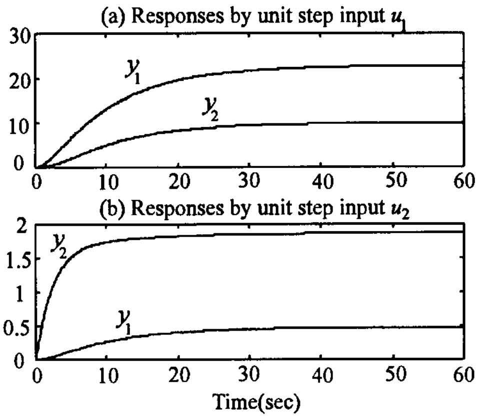

Fig. 4 The responses y_1 and y_2 of the process in two cases: (a) unit step input u_1 and (b) unit step input u_2

To design a noninteracting system, Let all the states be measurable. Referring to Eq. (5)-(8), noninteracting matrices F_d and K_d are determined as:

$$K_d = \begin{bmatrix} -1.2532 & 0.0122 & 0.0122 & -0.0005 & -0.0005 \\ 0.3151 & 0.2671 & 0.2671 & -0.0804 & -0.1138 \end{bmatrix}$$

$$F_d = \begin{bmatrix} 0.7783 & -0.0253 \\ 0 & 1.4144 \end{bmatrix}$$

To yield acceptable dynamics of a noninteracting system, let's propose the state feedback K_p is specified such that the closed loop poles of the system are located at $s = -0.6, -0.7, -0.8, -0.9, -2.1$. Hence, the gain matrix K_p can be obtained as

$$K_p = \begin{bmatrix} 7.8410 & 1.6482 & 1.3870 & 0.0320 & 0.0424 \\ 0.0978 & 0.1271 & 9.6269 & 0.1777 & 0.1777 \end{bmatrix}.$$

With these matrices obtained, the performance of noninteracting system design are shown in Fig 5. It is clearly seen that input u_1 affects only output y_1 in Fig 5(a) while input u_2 affects only output y_2 in Fig 5(b)

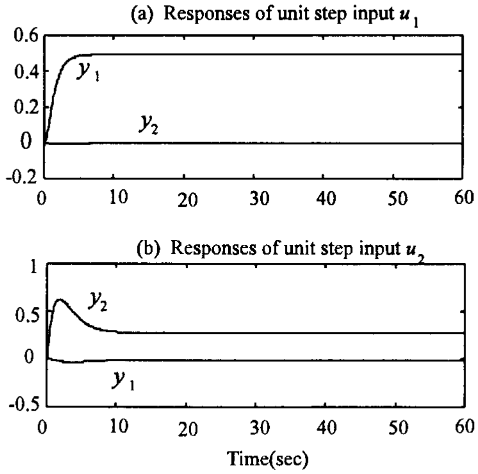

Fig. 5 The perfomance of noninteracting system design for unit-step inputs: (a) unit step input u_1 and (b) unit step input u_2

Fig. 6 shows the undesired effect of unit-step disturbance d which acts on the noninteracting system. It is seen that the outputs y_1 and y_2 deviate from the desired outputs (the zero level).

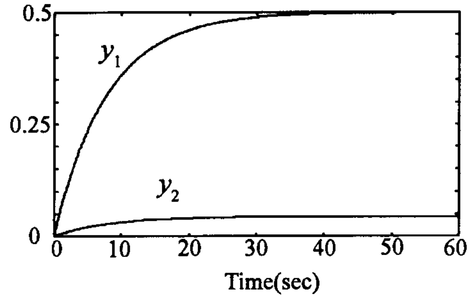

Fig . 6 The response y_1 and y_2 due to unit-step disturbance d

Now, the PI control law is used to minimize the difference between the actual outputs (y_1, y_2) and the reference inputs (y_{r1}, y_{r2}) when disturbance d acts on the system or the new reference inputs may be required. The diagram of PI controllers is illustrated in Fig. 7.

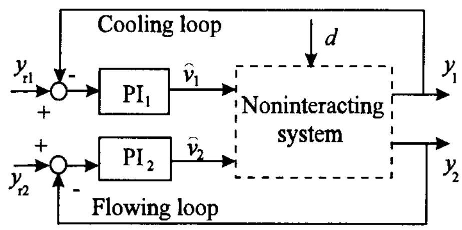

Fig. 7 The incorporation of PI controllers to noninteracting system

From Fig. 7, the relation of the extrudate temperature and the cooling flow rate is included in a "cooling loop". Meanwhile, the relation of the extrudate flow rate and the motor speed is included in a "flowing loop".

The proportional and integral gains of two PI controllers for each loop are used as 0.75 and 0.5 respectively. The performances of the PI controllers when the unit-step disturbance acts on the noninteracting system are illustrated in Fig. 8 and when the unit-step responses are required in Fig. 9.

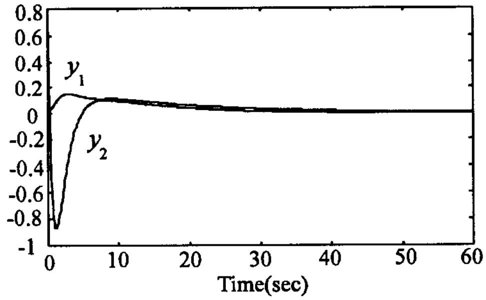

Fig. 8 The performance of PI controllers are demonstrated when the unit-step disturbance d acts on the system.

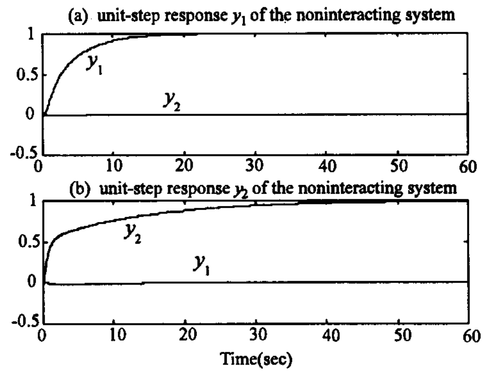

Fig. 9 The performance of PI controllers are demonstrated when the reference inputs are required: (a) unit-step y_{r1} and (b) unit-step y_{r2}.

4. Conclusion

With system identification techniques, one can obtain suitable linear model structures, which describe the nonlinearity of interacting extrusion processes under operating conditions. The noninteractingt control design presented here is applied in order that uncoupling can be achieved. A change in one reference input affects only one output. Consequently, typical PID controllers can be designed easily due to an absence of interaction.

From the simulation results in the section 3, the noninteracting system, which is designed, performs well. Obviously, it can be seen from Fig. 4 that both outputs y_1 and y_2 of the extrusion process are affected by the inputs u_1 and u_2 without the decoupling. On the other hand, when the process is decoupled, only one input can influence only one output. This is shown in Fig. 5. The PI Controllers are separately designed in each loop. In Fig. 8, the controllers can maintain the actual outputs at the reference points when the disturbance signal is injected to the system. Additionally, they can drive two outputs to reach the new reference points at the steady state in Fig 9.

5. Appendix

The parameters of plasticating extrusion process in Fig. 3 are listed as follows:

$$K_v = 1 \quad \frac{\mathrm{lb_m}}{\mathrm{in\text{-}hr}}$$

$$K_b = 20.3 \quad \frac{\mathrm{hr\text{-}F}}{\mathrm{lb_m}}$$

$$K_d = 0.5$$

$$K_m = 10 \quad \mathrm{ft\text{-}lb_f}$$

$$K_2 = 0.1414$$

$$K_4 = 0.2551 \quad \frac{\mathrm{hr\text{-}F}}{\mathrm{lb_m}}$$

$$K_5 = 57.57 \quad \frac{\mathrm{in\text{-}lb_f - hr}}{\mathrm{lb_m}}$$

$$K_6 = 115.99 \quad \frac{\mathrm{in\text{-}lb_f}}{\mathrm{F}}$$

$$K_7 = 0.0175 \quad \frac{\mathrm{lb_m}}{\mathrm{in\text{-}lb_f - hr}}$$

$$K_8 = 0.0867 \quad \frac{\mathrm{lb_m}}{\mathrm{hr\text{-}F}}$$

$$\tau_v = 2 \quad \mathrm{sec}$$

$$\tau_b = 7.9 \quad \mathrm{sec}$$

$$\tau_d = 7.9 \quad \mathrm{sec}$$

6. References

[1] Michaeli, W. (1995), Plastic Processing: an Introduction Hanser Garder Publications.

[2] Baird, D. G. and Collias, D. I. (1995), Polymer Processing: Principles and Design, Butterworth-Heinemann Publisher.

[3] Leephakpreeda, T., and Batur, C. (1997), Modelling of Local Crystallinity in Polymer Extrusion Process, Research and Development Journal of The Engineering Institute of Thailand, Vol. 7, No 2, pp. 76-81.

[4] Defaye, G. and Caralp, L. (1991), Simultaneous Diameter and Thickness Control in Tube Extrusion, International Polymer Processing VI, pp. 188-194.

[5] Falb, P. L. and Woloich, W. A. (1967), Decoupling in the Design and Synthesis of Multivariable Control Systems, Vol. AC-12, No. 6, pp.651-659.

[6] Brogan, W. L. (1991), Modern Control Theory, 3rd Ed., Prentice-Hall, Inc.

[7] Kaya, A. and Rice, L. S. (1985), Measurement and Control Problem For Extrusion Process, Proceeding of American Control Conference, TP-85-9.