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The Discrete Array Pattern Synthesis which Provides the Tapered Minor Lobes

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Abstract

The discrete array antenna pattern synthesis which provides the tapered minor lobes is presented in this paper by using Legendre and the second kind Tschebyscheff polynomials. The array characteristics are also compared with the conventional one-parameter Taylor method which is modified. A discussion of the advantages and disadvantages of each method is included.

1. Introduction

In the design of the broadside discrete array antenna, it is desirable to obtain narrow beamwidth, maximum gain and low side lobe level. To accomplish these requirements, the amplitude current excitation must be appropriately controlled. Dolph[1] proposed the method to improve the array pattern of a broadside linear array in which the elements are fed in phase but the amplitude current excitation is based upon the properties of the first kind Tschebyscheff polynomial. This method is a compromise between the uniform and the binomial arrays, because the beamwidth is narrower than the binomial array and the side lobe is lower than the uniform one. Some extended investigations have been cited in[2-7]. However, because of the uniform minor lobe distribution, the loss of beam efficiency occurs in the Dolph-Tschebyscheff array. To solve this problem, Phongcharoeñpanich *et.al.* proposed alternative methods to control the amplitude current excitation coefficient by using Legendre [8] and the second kind Tschebyscheff[9] polynomials. Additionally, for a radar system, it is desirable for the array to have a tapered minor lobe because interfering or spurious signals would be decreased further when they enter through the tapered minor lobes. Therefore, the significant contributions from interfering signals would be through the pattern in the vicinity of the main lobe.

Taylor introduced another method to produce the tapered minor lobes which is referred to as the Taylor (one-parameter) method, but this method is more applicable to the continuous line source distribution. For a discrete source with large spacing between the elements, the continuous line source distribution is not accurately approximated. To apply the one parameter Taylor method to the discrete array, it is found that for the specified side lobe level in the design, the array pattern possesses the lower nearest minor lobe.

In this paper, the one-parameter Taylor method will be modified for the design of the discrete array which provides the tapered minor lobes for the specified side lobe level. The array characteristics of this modified one-parameter Taylor method, the first and second kind Tschebyscheff arrays and Legendre will be compared. The characteristics are normalized amplitude current excitation coefficient, the nearest to the furthest minor lobe ratio, beam efficiency, beamwidth and directivity. The advantages and disadvantages of each method are summarized.

2. Array factor and array design procedure

2.1 Array factor consideration

Let us assume that there is a linear array of isotropic elements. The elements are aligned symetrically with the center of the array and are equidistant. When the number of the elements is even, an array factor (AF) can be written as[10]

$$AF_{2N}(\theta) = \sum_{n=1}^{N} I_n cos[(2n-1)\frac{\pi d}{\lambda} cos\theta]. \qquad (1a)$$

An array factor of the odd number of the elements can be expressed as

$$AF_{2N+1}(\theta) = \sum_{n=1}^{N+1} I_n cos[2(n-1)\frac{\pi d}{\lambda} cos\theta], \qquad (1b)$$

where I_n is the amplitude current excitation coefficient, $2N$, $2N+1$ is the number of even and odd elements, respectively, d is the spacing between each elements, λ is the wavelength of the operating frequency and θ is the angle between the field direction to the z-axis.

To synthesize the array pattern, the summation of the cosine term for the case of even and odd elements will be expanded. The order of harmonic cosine term is equal to the total number of the element minus one and the argument of the cosine term is the positive integer times of the fundamental frequency that can be written in the form

$m=k$;

$$\cos(mu) = \cos^k(u) - (\frac{k}{2})\cos^{k-2}(u)\sin^2(u) + (\frac{k}{4})\cos^{k-4}(u) \times \sin^4(u) - ... - (\frac{k}{k-2})\cos^2(u)\sin^{k-2}(u) + \sin^k(u) \qquad (2)$$

where $(\frac{k}{n}) = \frac{k!}{n!(k-n)!}$ and

$sin^2(u) = 1 - cos^2(u)$.

2.2 Orthogonal polynomial array design procedure

In this section, the orthogonal polynomial array design procedure will be summarized. Assume that the number of elements, the spacing between the elements in term of wavelength and the ratio of major to the first minor lobe intensity ratio are known. To obtain the array factor the following step can be applied.

1. From the known number of elements, we can select the array factor from (1a) or (1b) which corresponds to the even or odd number of elements.
2. Select the appropriate cosine term function from (2) and substitute in the expanded array factor.
3. Find the order of the orthogonal polynomial by subtracting one from the total number of elements. Equating this orthogonal polynomial with the major to the first minor lobe intensity ratio and then solve for the root of this polynomial $x = x_m$ (the point that maximum main lobe occurs). The side lobe of the array pattern can be formed from -1 to the null point nearest to $+1$ ($x = x_n$) region and the main lobe from x_n to x_m region.
4. Normalize x, to ensure that the magnitude of cosine term is not more than unity, by dividing by x_m.
5. Equating the expanded array factor to the orthogonal polynomial, the amplitude current excitation coefficient I_n will be obtained.

After the current excitation coefficients are known, by using (1a) or (1b) we can write the complete expression of the array factor.

2.3 One parameter Taylor method design procedure

One-parameter Taylor method was first introduced by Taylor in his unpublished classic memorandum and the details have been widely described by many authors[10-14]. In practice, one parameter Taylor method is more applicable to the continuous line source distribution. However, for the large spacing between the elements, the continuous line source distribution is not appropriately applied. Balanis[10] describes the application of the one-parameter Taylor method to the discrete antenna array by means of the source distribution expression. In this fashion, it is found that for a certain specified side lobe level, the array pattern gives the first side lobe less than that certain value by about *2* dB. Therefore, the authors proposed the novel by expression [11] for the weighting parameter calculation to achieve the pattern with the specified side lobe level.

In the design procedure, the weighting parameter B will be calculated for the specified side lobe level. The expression for the relationship between B-parameter and the side lobe level R_n is obtained by using the least square polynomial regression of the third order curve fitting as [11]

$$B = 0.0000769287R_n^3 - 0.00575009R_n^2 + 0.1834R_n - 1.33104 \quad (3)$$

where B is the weighting parameter to be determined, R_n is the specified side lobe level (dB) in the design and the accuracy of the calculation is quite sensitive to the value of the constant coefficient of the polynomial, so it should neither be rounded nor truncated. The amplitude current excitation coefficients can be calculated from the source distribution which is given as

$$I_n = I_0(\pi B\sqrt{1-\xi^2}), \quad (4)$$

where I_n denotes the amplitude current excitation coefficient, $I_0\ (x)$ is the modified Bessel function of the first kind of order zero which is related to the ordinary Bessel function of the first kind of order zero $(J_0(x))$ as

$$I_0(x) = J_0(jx) \quad (5)$$

and ξ is the normalized distance along the overall source which is defined as

$$\xi = \frac{z'}{l/2} \quad (6)$$

where z' is the dimension along the array, with the origin at the array's center and l is the array's total length. The constrained value of ξ is between *-1* and *1*.

3. Example of each method

To get insight into the design procedure, a demonstration of a *10* element broadside linear array with the major to the first minor lobe intensity ratio of *25.00* dB (which equals *17.79* in dimensionless) with half wavelength of antenna spacing is illustrated. The apparent array factor is

$$AF_{10}(\theta) = \sum_{n=1}^{N=5} I_n cos[(2n-1)\frac{\pi d}{\lambda}cos\theta] \quad (7)$$

It can be expanded in the form

$$AF_{10}(\theta) = I_1cos(\frac{\pi d}{\lambda}cos\theta) + I_2cos(\frac{3\pi d}{\lambda}cos\theta) + I_3cos(\frac{5\pi d}{\lambda}cos\theta) + I_4cos(\frac{7\pi d}{\lambda}cos\theta) + I_5cos(\frac{9\pi d}{\lambda}cos\theta). \quad (8)$$

3.1 Orthogonal polynomial array method

To determine the maximum value of the main lobe, the maximum value of minor lobe (x_n) and the major to the first minor lobe intensity ratio (R_n) are multiplied. The value of x_m of Legendre, the first and the second kind Tschebyscheff array can be solved by determining the root of the following characteristic equation

$$k_9x^9 + k_7x^7 + k_5x^5 + k_3x^3 + k_1x + k_0 = 0. \quad (9)$$

where x_n, R_n, k_9, k_7, k_5, k_3, k_1, k_0 and x_m in the case of this demonstation are shown as

$f_n(x)$	$P_n(x)$	$T_n(x)$	$U_n(x)$
x_n	0.41	1.00	2.25
R_n	17.79	17.79	17.79
k_9	94.96	256.00	512.00
k_7	-201.09	-576.00	-1024.00
k_5	140.77	432.00	672.00

k_3	-36.09	-120.00	-160.00
k_1	2.46	9.00	10.00
k_0	-7.29	-17.79	-40.03
x_m	1.07	1.08	1.06

where $f_n(x)$, $P_n(x)$, $T_n(x)$, $U_n(x)$, denote the general expression of the orthogonal polynomial, Legendre function and the first and the second kind Tschebyscheff function, respectively. This value of x_m is substituted to normalize in the cosine term so that

$$\cos\left(\frac{\pi d}{\lambda}\cos\theta\right) = \frac{x}{x_m}. \qquad (10)$$

By following step 5 in the orthogonal polynomial array design procedure, the normalized amplitude current excitation coefficients I_n are derived as follows

$f_n(x)$	$P_n(x)$	$T_n(x)$	$U_n(x)$
I_1	3.15	2.53	3.67
I_2	2.83	2.28	3.29
I_3	2.26	1.83	2.62
I_4	1.58	1.28	1.81
I_5	1.00	1.00	1.00

3.2 One-parameter Taylor method

For the specified side lobe level of *25.00* dB, by using (3), the weighting parameter B can be determined and is equal to *0.86*. By substituting this parameter in (4) for each element, the amplitude current excitation will be determined as follows

Element number	ξ	I_n
#1	0.11	3.84
#2	0.33	3.44
#3	0.56	2.74
#4	0.78	1.87
#5	1.00	1.00

3.3 Array pattern analysis

After normalizing the array factor by the minimum amplitude, the *10* element Legendre array pattern is plotted to compare with the two kinds Tschebyscheff and one-parameter Taylor arrays simultaneously. These patterns are shown in fig.1.

To consider the beam efficiency, it is defined as the ratio of the power transmitted (received) within the main beam to the power transmitted (received) by the antenna. For the broadside linear array, beam efficiency can be formulated as[10]

$$Beam\ Efficiency(BE) = \frac{\int_{\theta_1}^{\frac{\pi}{2}} |AF(\theta)|^2 \sin\theta d\theta}{\int_{0}^{\frac{\pi}{2}} |AF(\theta)|^2 \sin\theta d\theta}, \qquad (11)$$

where θ_1 is the half angle of the cone where the first null occurs[10].

In order to compare the tapered minor lobe distribution characteristics, the nearest to the furthest minor lobe ratio is defined as the ratio of the level of the nearest minor lobe to the furthest minor lobe when the nearest and the furthest minor lobes are referred to with respect to the main lobe position.

Another important antenna characteristic is the directivity which is defined as the ratio of the maximum radiation intensity of the antenna to the radiation intensity of isotropic source. Determination of the broadside linear array directivity is carried out by using [10]

$$Directivity(D_0) = \frac{2\left|AF\left(\frac{\pi}{2}\right)\right|^2}{\int_{0}^{\pi} |AF(\theta)|^2 \sin\theta d\theta}. \qquad (12)$$

In the *10* element linear array, with *25.00* dB side lobe level of $\lambda/2$ spacing, the first null angle(θ_1) ,beam efficiency(*BE*), half power beamwidth(*HPBW*), first null beamwidth (*FNBW*), the nearest to the furthest minor lobe ratio(*N/F*) and directivity(D_o)are shown as

$f_n(x)$	$P_n(x)$	$T_n(x)$	$U_n(x)$	$O_n(x)$
$\theta_1(rad)$	1.29	1.30	1.28	1.27
BE(%)	99.41	98.95	99.57	99.60
HPBW(rad)	0.22	0.22	0.22	0.22
FNBW(rad)	0.56	0.54	0.58	0.60
N/F(dB)	4.00	0.00	7.00	8.00
$D_o(dB_i)$	9.46	9.57	9.38	9.35

where $O_n(x)$ denotes the one-parameter Taylor method.

4. Characteristics comparison

Some characteristics such as normalized amplitude current excitation coefficients, the nearest to the furthest minor lobe ratio, beam efficiency, beamwidth and directivity of Legendre, the first, and the second kind Tschebyscheff and one-parameter Taylor arrays are compared in this section. The observation of the behavior and the advantages and disadvantages of each method are also discussed.

4.1 Normalized Amplitude current excitation coefficient

Following the design procedure, for the side lobe level between 20 to 40 dB, the normalized amplitude current excitation coefficient versus the side lobe level is illustrated in fig 2. From this graph, it is found that the maximum to minimum current ratio increase, respectively is as follows: the first kind Tschebyscheff, Legendre, the second kind Tschebyscheff and one-parameter Taylor arrays. From this viewpoint of the current ratio, the feeder structure for the case of the first kind Tschebyscheff array is the easiest to fabricate whereas the one-parameter Taylor array is the most difficult.

4.2 The nearest to the furthest minor lobe ratio

The nearest to the furthest minor lobe ratio is defined to compare the tapered minor lobes characteristics. The nearest to the furthest minor lobe ratio of 10 element with $\lambda/2$ spacing for the side lobe level between 20 to 40 dB are shown in fig.3. From fig.3, the nearest to the furthest minor lobe ratio at the side lobe level equal to 20 dB, the one-parameter Taylor array possesses an identical ratio level to the second kind Tschebyscheff array, however, for the side lobe level 20 to 30 dB, the ratio level of one-parameter Taylor increases very rapidly but the second kind Tschebyscheff array still gives the fixed ratio. At the side lobe level up to 30 dB, the one parameter Taylor pattern possesses the constant ratio at the same as the other arrays when their ratios are constant for all of the side lobe levels. Except for the side lobe level less than 30 dB, the ratio difference between the current and the next of each following pairs, the one-parameter Taylor, the second kind Tschebyscheff, Legendre and the first kind Tschebyscheff arrays are about 3-4 dB.

4.3 Side lobe level and beam efficiency

From the array pattern comparison in fig 1, it is found that the far-out minor lobes of the one-parameter Taylor array is the lowest whereas the first kind Tschebyscheff array keeps the uniform amplitude of minor lobe. Therefore, the beam efficiency of the one-parameter Taylor array is the highest. Fig.5 shows the beam efficiency of the four arrays, simultaneously. For the high side lobe level, the differences in beam efficiency are small and that beam efficiency characteristic is also flat. However, for the low side lobe level, the differences of each array are more pronounced.

4.4 Beamwidth and directivity

In radar applications, high azimuth resolution is desirable. Therefore, the azimuthal beamwidth of the antenna must be narrowed. For this requirement, these four arrays are suitable because of their narrow beamwidth property. From the array pattern comparison in fig.1, it is obvious that the half power beamwidth of four arrays are almost identical and the first null beamwidth is slightly different. In the 10 element broadside linear array design, the directivity is of the order of 8-10 dB_i. The comparison of directivity between Legendre, two kinds Tschebyscheff and one-parameter Taylor array is illustrated in fig.6. For a given side lobe level, the directivity of the first kind Tschebyscheff array is the highest whereas the one-parameter Taylor array is the lowest. The directivity of Legendre and two kinds Tschebyscheff arrays are very close, only *0.1* dB smaller for all of the side lobe level. The one-parameter Taylor method gives very close directivity to the second kind Tschebyscheff array for the small side lobe level but for the high side lobe level, the difference is very large.

5. Conclusion

The discrete antenna array pattern synthesis which provides the tapered minor lobes can be formed from the Legendre, the second kind Tschebyscheff and the modified conventional one-parameter Taylor arrays to keep away from the uniform amplitude minor lobe distribution. From the characteristics comparison as mentioned in the previous section, it can be concluded that for the low side lobe level, the second kind Tschebyscheff and modified conventional one-parameter Taylor arrays are more suitable because of the high beam efficiency and the the nearest to the furthest minor lobe ratio properties while sacrificing only some small maximum to minimum current ratio and directivity. On the opposite side, for the high side lobe level, the most appropriated arrays are the first kind Tschebyscheff and Legendre arrays due to their low maximum to minimum current ratio and high directivity properties.

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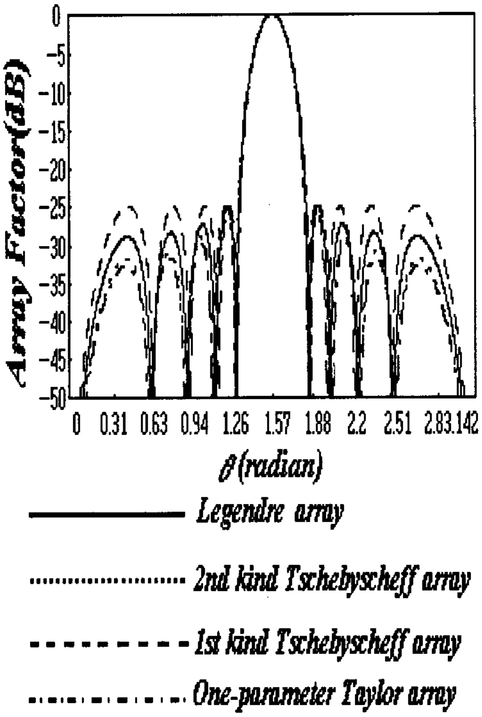


Fig.1 Array pattern of *10* element broadside linear array with SLL *25.00* dB and $\lambda/2$ spacing, of Legendre, the first and the second kind Tschebyscheff and one-parameter Taylor array

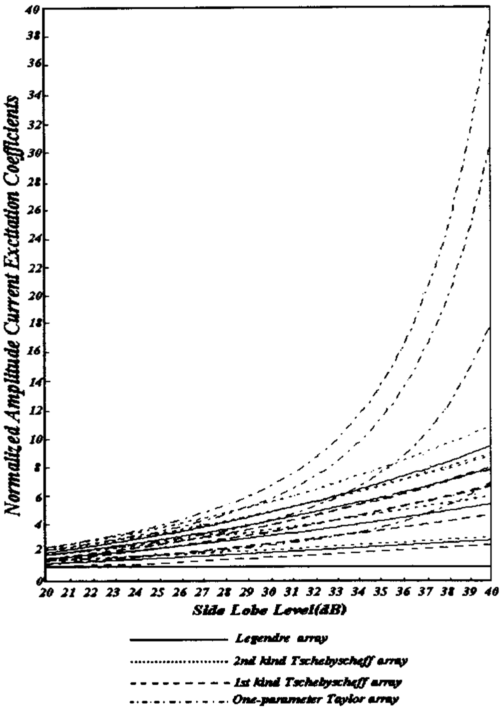


Fig.2 Normalized amplitude current excitation coefficients of *10* element Legendre, the first, and the second kind Tschebyscheff and one-parameter Taylor arrayss

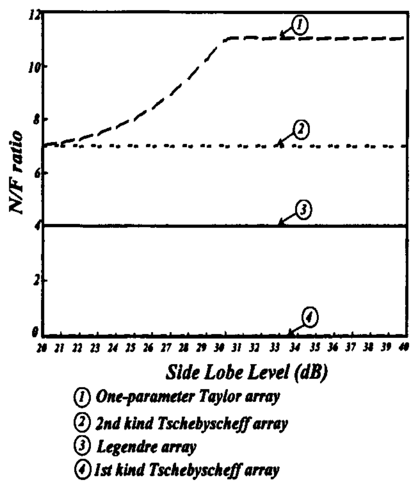
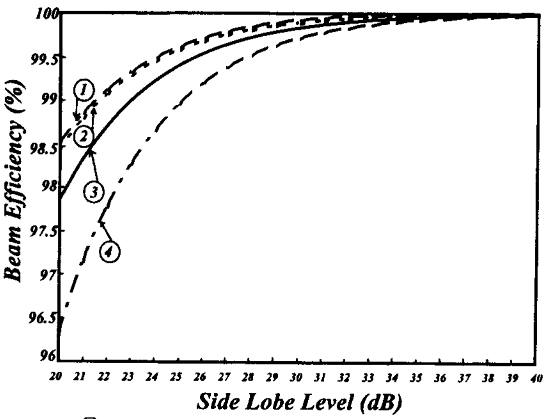


Fig.3 The nearest to the furthest minor lobe ratio of a *10* element Legendre, the first and the second kinds Tschebyscheff and one-parameter Taylor arrays



(1) One-parameter Taylor array
(2) 2nd kind Tschebyscheff array
(3) Legendre array
(4) 1st kind Tschebyscheff array

Fig.4 Beam efficiency of a *10* element Legendre, the first, and the second kind Tschebyscheff and one-parameter Taylor arrays

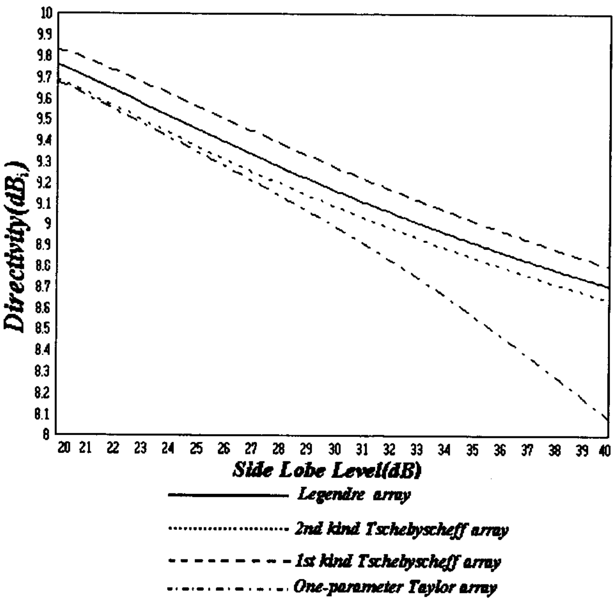


Fig.5 Directivity of a *10* element Legendre, the first and the second kind Tschebyscheff and one-parameter Taylor arrays