Modeling of Stiffness of Aggregate Phase for Shrinkage Restraint in Hardened Concrete

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Abstract

This study aims to propose a two-dimensional constitutive model for computing the stiffness of the aggregate phase of concrete. The model is used in the prediction of shrinkage of concrete taking into account the restraint from aggregate. The aggregate particles are considered to be uniformly distributed in the concrete and in contact with one another at various contact angles. It is assumed that particle contact angles vary from 0 to $\pi/2$ with most at $\pi/4$. The relationship between strain of the aggregate particle system and the local strain in the directions normal and parallel to the contact plane was derived, from which the latter could be transformed to be stresses at contact using the local constitutive model. A nonlinear hardening relationship for normal component was assumed whereas the Coulomb's friction law was considered for the parallel direction. The integral of the resulting contact forces from all contact angles considering the density function of the contact angles which were transformed into global coordinate system was then conducted to obtain the two-dimensional stress and strain relationship of the aggregate particulate system. Stiffness of the aggregate particle system was then derived from the ratio between stress and strain. Verification tests on fine and coarse aggregates were conducted on mortar and no-fine concrete specimens, respectively. The verification indicates that the model was effective for deriving the stiffness of the aggregate phase.

1. Introduction

In simulating the shrinkage of concrete, a two-phase material model, taking into account the restraint shrinkage due to aggregate particle interaction, has been proposed by the author [1]. In the proposed model, the stiffness of the aggregate phase has to be obtained as one of the phase properties. There are many well-studied constitutive models for predicting deformational behavior of the aggregate phase. One of these is the deformational model for the solid phase of fresh concrete under compression, proposed by Tangtermsirikul, S. and Maekawa, K. [2] This can be modified to be applied in this study. The deformational behavior of aggregate particle system can be predicted using the idea from the two-dimensional constitutive model for solid phase under compression. This paper proposes a part of the model which is applicable to fine aggregate and coarse aggregate as single materials. Another part, which is not described in this paper, concerns the mixture concept for combining the behavior of fine and coarse aggregates to obtain the stiffness of the total aggregate.

2. General Concept of the Model

Aggregate is considered to be composed of particles which are contacting one another. Each contact has its own contact angle (θ) and the density distribution of the contact angle is assumed to be $\Omega(\theta)$. The $\Omega(\theta)$ can be simply explained as to represent the ratio of the numbers of contact which have angles θ to the total numbers of contact. The force system contains normal force which is caused by the deformation normal to the contact plane and friction force which is due to the deformation parallel to the contact plane. Stress-strain relation, that is applied for relating the deformation normal to the contact plane to the corresponding stress, is assumed. Friction is treated as dry Coulomb's friction. Contact area increases as the deformation progresses and is affected by particle shape, size and grading of the aggregate. Re-arrangement of particles is also a significant factor especially for low friction particles.

2.1 Probability Density Function for Contact Angle

Particles are considered to have a density function for contact angle as in Fig.1. Li and Maekawa [3] proposed a different function for effective contact area to model the shear transfer across crack. Here it is reasonable to assume that there are negligible contact angles which are normal and parallel to the principle strain direction (θ equals and π/2), but most contact angles are nearly or just π/4. Then the function for contact angle is assumed as

$$\Omega(\theta) = \sin 2\theta \qquad (1)$$

where $\int_0^{\frac{\pi}{2}} \Omega(\theta)d\theta = 1.0$

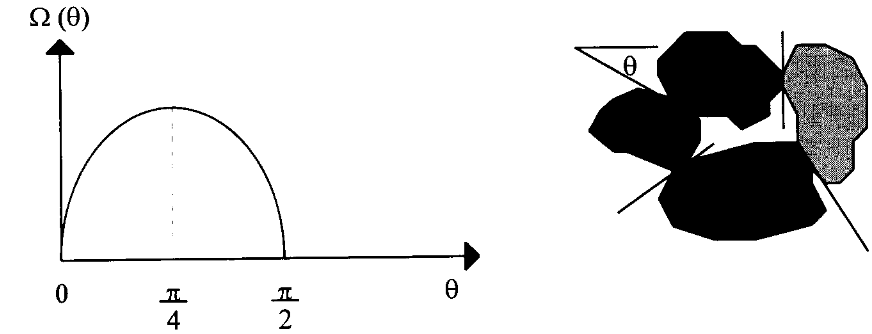

Fig. 1 Density Function of Contact Angle

2.2 Deformation at a Contact

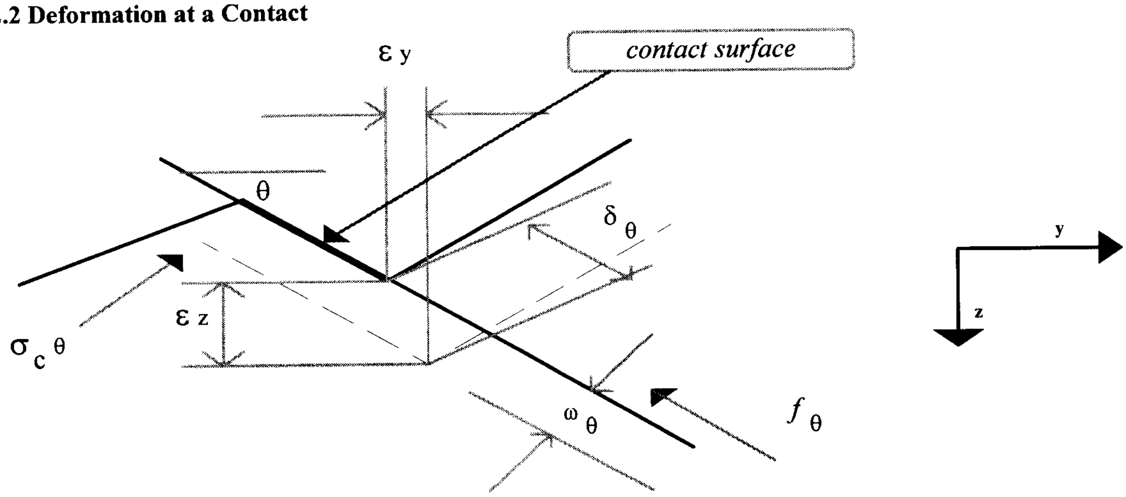

Fig. 2: The 2-Dimensional Displacement Compatibility of a Contact at Contact Angle θ Showing Initial as well as Deformed Configurations of a Contact

From the geometry in Fig. 2., considering a unit volume, the deformation, ω_θ and δ_θ, can be related to strains (ε_y and ε_z) by coordinate transformation. Then

$$\omega_\theta = \varepsilon_z \cdot \cos\theta + \varepsilon_y \cdot \sin\theta \qquad (2)$$

$$\delta_\theta = \varepsilon_z \cdot \sin\theta + \varepsilon_y \cdot \cos\theta \qquad (3)$$

As the co-ordinate axis coincides with the principal strain directions, the shear strain, ε_{xy}, equals zero.

2.3 Constitutive Relation for Normal Direction

In this study, the stress-strain relationship for relating the normal stress ($\sigma_{c\theta}$) to its corresponding deformation (ω_θ) is considered to be nonlinear. The monotonic local stress-strain relation of a contact is assumed to be

$$\sigma_{c\theta} = E_{c'}.(\omega_\theta)^{0.15} \qquad (4)$$

where $E_{c'}$ is stiffness of the stress-strain relationship normal to contact plane.

2.4 Stress in Direction Parallel to Contact Plane

To simplify the problem, the frictional stress (f_θ) is assumed to be constant independent of slip (δ) as in the following expression

$$f_\theta = \mu \cdot \sigma_{c\theta} \qquad (5)$$

where μ is coefficient of the physical friction between grain.

2.5 Equilibrium Equations

The local force system acting on the contact at angle θ can be transformed to be the forces ($F_{z\theta}$, $F_{y\theta}$) in the global coordinate system as

$$F_{z\theta} = (\sigma_{c\theta} \cdot \cos\theta + f_\theta \cdot \sin\theta) \cdot A_{c\theta} \qquad (6)$$

$$F_{y\theta} = (\sigma_{c\theta} \cdot \sin\theta - f_\theta \cdot \cos\theta) \cdot A_{c\theta} \qquad (7)$$

where $A_{c\theta}$ is contact area.

Equilibrium is satisfied by integrating the multiplication product of forces with the density function of the contact angle in the global coordinate over contact angles from θ = 0 to π /2 and equate the integral to the external forces as

$$\sigma_z \cdot A_z = \int_0^{\frac{\pi}{2}} \Omega(\theta) \cdot F_{z\theta} \cdot d\theta \qquad (8)$$

$$\sigma_y \cdot A_y = \int_0^{\frac{\pi}{2}} \Omega(\theta) \cdot F_{y\theta} \cdot d\theta \qquad (9)$$

where σ_y and σ_z are the stresses in y and z directions in global coordinate system, respectively; and A_y and A_z are the area normal to y and z directions in global coordinate system, respectively.

Substituting equations (6) and (7) into equations (8) and (9), since $A_y = A_z = 1$, the principal stresses can be obtained as

$$\sigma_z = \int_0^{\frac{\pi}{2}} \Omega(\theta) \cdot (\sigma_{c\theta} \cdot \cos\theta + f_\theta \cdot \sin\theta) \cdot A_{c\theta} \cdot d\theta \qquad (10)$$

$$\sigma_y = \int_0^{\frac{\pi}{2}} \Omega(\theta) \cdot (\sigma_{c\theta} \cdot \sin\theta - f_\theta \cdot \cos\theta) \cdot A_{c\theta} \cdot d\theta \qquad (11)$$

By introducing the function for contact area ($A_{c\theta}$), equations (2), (3), (4), (5), (10) and (11) can be solved simultaneously. Subsequently, the two-dimensional stress-strain relationship of the single materials can be obtained as

$$E_{az} = \frac{\sigma_z}{\varepsilon_z} = \frac{\int_0^{\frac{\pi}{2}} \Omega(\theta)(\sigma_{c\theta} \cdot \cos\theta + f_\theta \cdot \sin\theta) A_{c\theta} \cdot d\theta}{\varepsilon_z} \qquad (12)$$

$$E_{ay} = \frac{\sigma_y}{\varepsilon_y} = \frac{\int_0^{\frac{\pi}{2}} \Omega(\theta)(\sigma_{c\theta} \cdot \sin\theta - f_\theta \cdot \cos\theta) A_{c\theta} \cdot d\theta}{\varepsilon_y} \quad (13)$$

where E_{ay} and E_{az} are the aggregate stiffnesses in y and z directions of the global coordinate, respectively.

2.6 Contact Area

The factors affecting contact area of the particles are size, shape, gradation and re-arrangement of the particles. An important phenomenon is the increase of the contact area as the deformation progresses. By assuming that the contact area of a contact angle θ increases along with the amount of slip in that contact (δ_θ). The contact area at a contact angle θ can be expressed as

$$A_{c\theta} = (A_{co} + \int dA_{c\theta}).\phi \quad (14)$$

where A_{co} is initial contact area.

As in the geometry in Fig. 2 and considering a unit volume so that a unit width can be applied, the summation of the increase of contact area can be derived from

$$\int dA_{c\theta} = 1.\delta_\theta \quad (15)$$

2.7 Initial Contact Area

Since the level of stress in aggregate mixture is low, the initial contact area (A_{co}) plays an important role and is taken into account regarding the conditon of aggregate from no-contact to closely compacted. The initial contact area can be assumed to be expressed by a non linear function of the total surface area of aggregate (consider a unit volume of concrete). Let α be defined as a ratio between surface area of aggregate in a unit volume of concrete to surface area of the same type of aggregate which is densely compacted in the same unit volume. So

$$\alpha = S/S_{max} \quad (16)$$

where S is the surface area of aggregate in a unit volume of concrete and S_{max} is the surface area of the same type of aggregate which is densely compacted in the same unit volume. The surface area, S, is considered to be a function of aggregate volume ratio (n_a) that is defined as

$$n_a = Va/Vc \quad (17)$$

where Va and Vc are volumes of aggregate and concrete, respectively.

The relationship of initial contact area of the aggregate and the ratio α is given in Fig. 3 and Fig. 4 for fine aggregate and coarse aggregate, respectively.

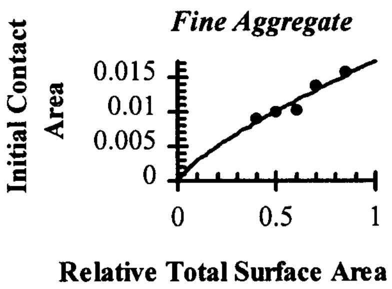

Fig. 3 Relationship between Initial Contact Area of Fine Aggregate and Relative Total Surface Area

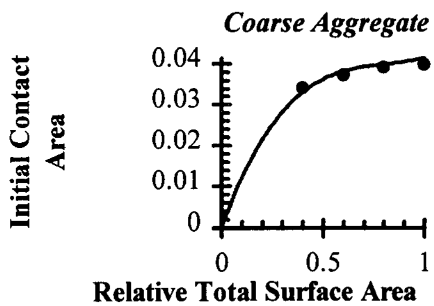

Fig. 4 Relationship between Initial Contact Area of Coarse Aggregate and Relative Total Surface Area

3. Verification of the Stiffness Model

The stiffness model was verified using the results from the shrinkage test of mortar and no-fine concrete. The author's derived model for simulating shrinkage of concrete taking into account the aggregate restraint [1] was utilized to compute the stiffness of the sand and coarse aggregate used as the test results. Fig.5, Fig. 6 and Fig.7 show the comparison between the test and analytical results of stiffness of sand whereas those of the coarse aggregates are demonstrated in Fig.8, Fig.9 and Fig.10. The stiffness of aggregate particle system increases with the aggregate volume ratio, n_a, and the shrinkage strain of the concrete which is equal to the strain of the aggregate phase according to the assumption of the model for simulating

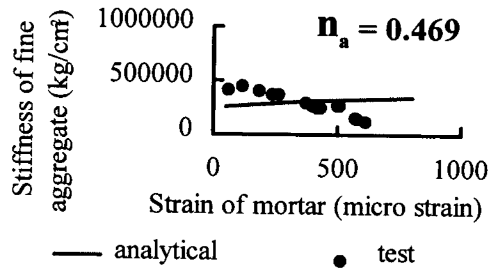

Fig. 5: Test and Analytical Results of Stiffness of Fine Aggregate (n_a = 0.469)

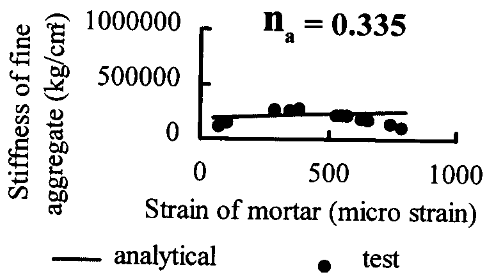

Fig. 6: Test and Analytical Results of Stiffness of Fine Aggregate (n_a = 0.335)

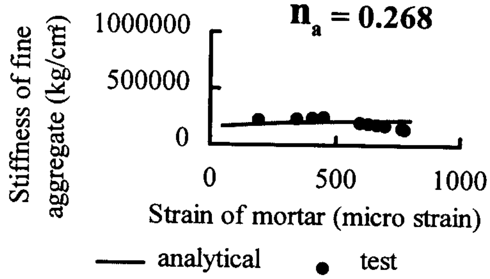

Fig. 7: Test and Analytical Results of Stiffness of Fine Aggregate (n_a = 0.268)

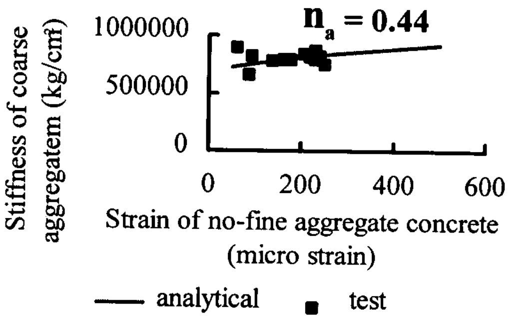

Fig. 8: Test and Analytical Results of Stiffness of Coarse Aggregate (n_a = 0.44)

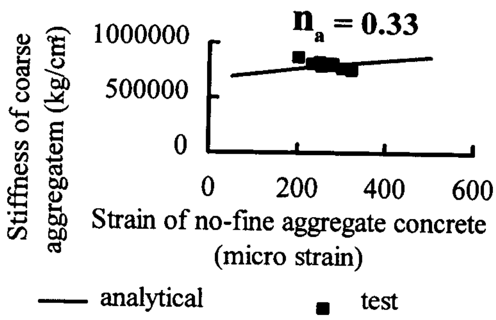

Fig. 9: Test and Analytical Results of Stiffness of Coarse Aggregate (n_a = 0.33)

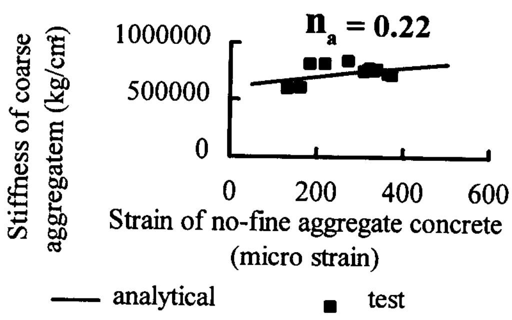

Fig. 10: Test and Analytical Results of Stiffness of Coarse Aggregate (n_a = 0.22)

shrinkage [1]. After comparing results, the proposed aggregate stiffness model based on particle contact density concept is proved to be satisfactory for simulating the stiffness of aggregate phase in concrete.

4. Conclusion

A mathematical model for simulating stiffness of aggregate phase was derived based on the particle contact density concept. The aggregate phase was considered to consist of particles contacting among one another with various contact angles. A contact density function, $\Omega(\theta) = \sin 2\theta$, was assumed for the problem. Strain of the aggregate particle system can be transformed to be local contact strain and converted to be local contact stresses using assumed constitutive laws. The nonlinear hardening stress-strain behavior was assumed for the normal strain direction and friction law was applied to the direction along the contact plane. The functions for initial contact area were assumed for fine aggregate and coarse aggregate. The contact area increased when strain of the aggregate phase increased. By integrating the local contact stress from contact angle 0 to $\pi/2$, the stress of the aggregate particle system was obtained. Then stiffness of the system was derived by dividing the stress with the strain of the aggregate particle system. Shrinkage tests were conducted on mortar and no-fine concrete specimens to verify the stiffness model for fine aggregate and coarse aggregate. It was found from the comparison between the test and analytical results that the model was effective for simulating the stiffness of fine aggregate and coarse aggregate in this study.

5. References

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[3] Li, B.L. and Maekawa, K. (1987), Modeling of Stress Transfer Across Crack Plane of Concrete under Cyclic Loading, Proceedings of the 42th Annual Conference of JSCE, pp.140-141.