

# Mathematical Model of an Activated Carbon-Ethanol Refrigerator

J. Tiansuwan, J. Hirunlabh

King Mongkut's Institute of Technology Thonburi,  
Bangkok 10140, Thailand

T. Kiatsiriroat

Department of Mechanical Engineering, Chiang Mai University  
Chiang Mai 50200, Thailand

## Abstract

A mathematical model of an activated carbon-ethanol intermittent refrigerator has been developed. The adsorber is heated and cooled by hot and cold oil with prescribed temperatures and the condenser is controlled with cooled water. The temperature of the working pair in the adsorber, the amount of ethanol leaving and reabsorbed and the temperature in the refrigerated box could be estimated. The simulated results agree well with those of the experiment.

## Nomenclatures

A = Area, m<sup>2</sup>  
 $c_c$  = Specific heat of activated carbon, kJ/kg-K  
 $c_e$  = Specific heat of ethanol, kJ/kg-K  
 $D$  = Coefficient of affinity in Dubinin equation  
 $h$  = Heat transfer coefficient, kW/m<sup>2</sup>-K  
 $k$  = Equivalent conductivity of activated carbon, kW/m<sup>2</sup>-K  
 $L_e$  = Latent heat of ethanol, kJ/kg  
 $L_{ice}$  = Latent heat of fusion of ice, kJ/kg  
 $m_c$  = Mass of activated carbon, kg  
 $m_{ice}$  = Mass of ice, kg  
 $m_w$  = Mass of water, kg  
 $n$  = Exponent in Dubinin equation  
 $P$  = Pressure, bar  
 $P_s(T)$  = Saturated pressure of ethanol at temperature (T), bar  
 $q_g$  = Heat rate per unit volume, kW/m<sup>3</sup>  
 $q_{st}$  = Heat of sorption, kJ/kg<sub>et</sub>-K  
 $R_u$  = Universal gas constant, kJ/k mol-K  
 $r$  = Radius, m  
 $T$  = Temperature, K  
 $t$  = Time, sec  
 $U$  = Overall heat transfer coefficient, kW/m<sup>2</sup>-K

$V$  = Total volume of adsorbed, m<sup>3</sup>  
 $W$  = Volume of ethanol adsorbed in activated carbon, l/kg carbon  
 $x$  = Concentration of ethanol in activated carbon, kg ethanol/kg carbon

## Greek symbols

$\rho_c$  = density of activated carbon, kg/m<sup>3</sup>  
 $\rho_e$  = density of ethanol, kg/m<sup>3</sup>

## Introduction

Adsorption refrigeration is similar in certain respects to absorption refrigeration. The previous case uses porous solid to adsorb refrigerant whereas the latter case uses liquid absorbent. The intermittent adsorption refrigeration cycle is shown in Fig. 1.

During the desorption-condensation process, the working pair of the adsorbent and the adsorbate in tank A is heated and the vapor of the adsorbate leaves tank A and condenses in tank B when the tank is cooled down. During adsorption evaporation, the working pair in tank

A is cooled and the pressure in the tank decreases then the refrigerant in tank B can evaporate and be adsorbed back to the tank A. The refrigerant can extract heat from the tank's surrounding thus space cooling is obtained.

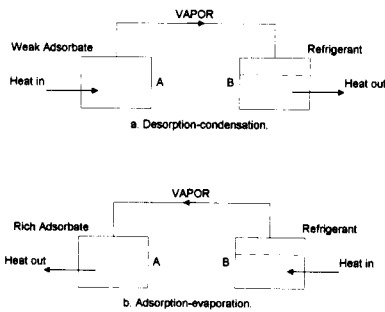


Fig. 1. The intermittent refrigeration cycle.

Different pairs of the working substances have been used for solar-operated refrigerators [1-7]. Tiansuwan et al [8] has also studied the potential of using the local activated carbon-ethanol pair for refrigeration and found that the theoretical COP could be up to 0.6 and the suitable size of the local activated carbon should be 2-3 mm (6x12 mesh size).

In this paper, the performance of an activated carbon-ethanol refrigerator is studied. A mathematical model to predict the system performance has also been developed.

**The Refrigeration System**

Fig. 2 describes the activated carbon-ethanol refrigeration system. The refrigeration unit consists of an adsorber, a condenser and an evaporator. The adsorber as shown in Fig. 3 has 18 black-painted copper tubes of 50 mm. OD with 1.5 mm wall thickness and 1.2 m length. Inside each tube there is a 28.6 mm. OD coaxial tube with 8 mm holes drilled thoroughly. The inner tube is wrapped with stainless steel net and is used for the passage of ethanol to and from the activated carbon contained between the tubes. The adsorber contains about 14.9 kg of carbon.

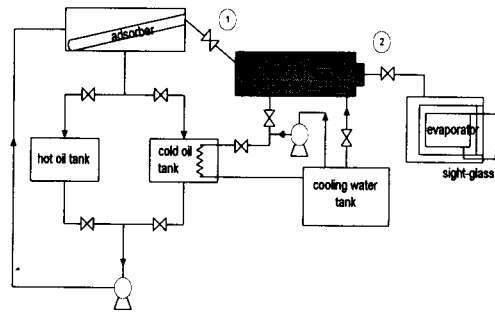


Fig. 2. The refrigeration system.

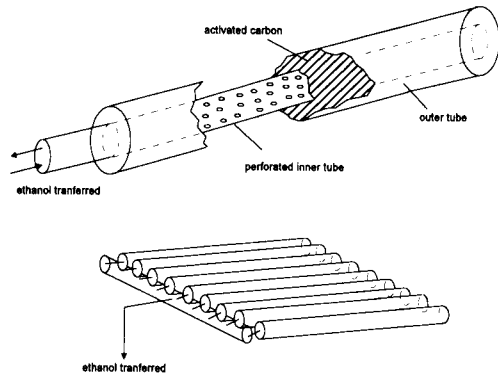


Fig. 3 A schematic sketch of the adsorber.

The condenser is a set of copper tubes with aluminum fins immersed in a water tank.

The evaporator is a set of vertical finned tubes kept in the refrigerated box. The cooled box contains a set of trays filled with water. The amount of water is varied and its temperature is monitored to study the cooling capability of the systems.

Hot oil from a heated tank is used to heat the adsorber in which the carbon and the ethanol are contained. The ethanol evaporates and condenses in the evaporation coil. To generate the cooling effect, the adsorber is cooled down by cold oil in another tank, then the pressure inside the adsorber decreases and the ethanol in the evaporator can reevaporate and be adsorbed in the adsorber, and the temperature in the refrigerated box can be reduced.

**Analysis and Computation**

The energy analysis at each adsorber tube has been carried out with the following assumptions:

- Each adsorber tube is identical.
- Temperature variation exists in the radial direction only.
- The ethanol vapor pressure is uniform along the tube.
- High heat transfer coefficient between oil and the adsorber tube is obtained, thus the adsorber tube surface has the same temperature as that of the oil.

**Adsorber-Condenser**

The carbon-ethanol in the adsorber tube is divided into 4 concentric cylindrical shells and located point 1 to 5 at the boundaries between each shell and the next as shown in Fig. 4.

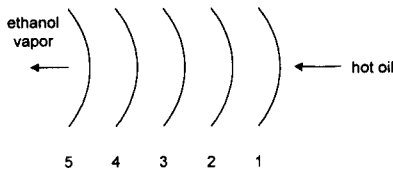


Fig. 4 The concentric cylindrical shells of the carbon-ethanol in the adsorber tube.

During the desorption process the energy balance around the  $m^{th}$  point could be written as

$$kA_{m+1/2} \left( \frac{T_{m-1} - T_m}{\Delta r} \right) + kA_{m+1/2} \left( \frac{T_{m+1} - T_m}{\Delta r} \right) + q_{g,m} v_m = \frac{pc}{\Delta t} V_m (T_m - T'_m) ; m = 2, 3, 4 \tag{1 a}$$

The energy balance at point 5 is

$$kA_{m+1/2} \left( \frac{T_4 - T_5}{\Delta r} \right) + q_{g,5} \frac{V_5}{2} = \frac{pc}{\Delta t} \frac{V_5}{2} (T_5 - T'_5) \tag{1 b}$$

and at point 1

$$T_1 = T_{oil} \tag{1 c}$$

$T'_m$  is the temperature of shell  $m$  after  $\Delta t$  has elapsed.

$pc$  is the density-specific heat capacity products of the carbon-ethanol in adsorber tube which is calculated from

$$pc = p_c (c_c + xc_e) \tag{2}$$

$p_c$ ,  $c_c$  and  $c_e$  are the carbon density, carbon heat capacity and liquid heat capacity, respectively.  $x$  is the mass concentration of the ethanol in the activated carbon which can be calculated from Dubinin equation [9] as

$$q_{g,m} = q_{st} \frac{\partial}{\partial T} x(P, T) P_c$$

$$x = p_e W_o \exp \left[ -D (T \ln(P_s / P))^n \right] \tag{3}$$

In this study, with the 6x12 mesh size activated carbon  $W_o = 0.431267$  lit/kg carbon,  $D = -1.34872 \times 10^{-5}$  and  $n = 1.63$  [8].  $P_s$  is the saturated pressure of the ethanol at temperature  $T$ .

$q_{g,m}$  is the heat rate per unit volume at the  $m^{th}$  shell due to heat of sorption of the ethanol and can be estimated from

$$q_{g,m} = q_{st} \frac{\partial}{\partial T} x(P, T) P_c \tag{4}$$

$q_{st}$  is the heat of sorption which can be evaluated by Clausius-Clapeyron equation [10] as

$$\frac{q_{st}}{RT} = \frac{d \ln P}{dT} | x \tag{5}$$

$P$  is the saturated pressure of ethanol at the temperature  $T$ . The temperature  $T$  is the temperature of the ethanol leaving the condenser and of that leaving the evaporator in the cases of desorption and adsorption processes, respectively.

(1 c)

**Refrigerated Box**

For cooling purpose, the adsorber is cooled down by cooled oil at ambient temperature, therefore, the pressure in the system is reduced until the ethanol in the evaporator coil evaporates and is re-adsorbed in the adsorber and lower temperature of air in the cooled box is obtained.

In the cooled box, there is a set of trays filled with water. The energy balance at the cooled box is

**Evaporating coil :**

$$(x - x')m_c L_e + U_w \Delta t (T_w - T_{ev}) - U_a \Delta t (T_a - T_{ev}) = m_{ev} c_{ev} (T_{ev}' - T_{ev}) + m_e c_e (T_{ev}' - T_{ev}) \quad (6)$$

**The water tray :**

$$m_w \Delta t (T_w - T_{ev}') = m_w c_w (T_w' - T_w) + m_{ice} L_{ice} \quad (7)$$

From the experiment, the  $U_w$  and  $U_a$  are 0.5 and 4.5 W/K, respectively.  $T_{ev}'$  and  $T_w'$  are the temperature of evaporator and water in the tray after  $\Delta t$  has elapsed respectively.

With the values of : the hot oil and cold oil temperatures, the condensing temperature and the temperature of carbon-ethanol in the adsorber-the amount of ethanol desorbed and adsorbed can be evaluated. The steps in this calculation are shown in Fig. 5.

**Results and Discussion**

During the desorption-condensation process the working pair in the adsorber tank is heated and its temperature increases. The ethanol vaporises and condenses in the refrigerated box. When the temperature reaches about 120 °C, the adsorber tank is cooled down then the adsorption-evaporation process starts. The temperature decreases and creates a refrigeration effect in the refrigerated box.

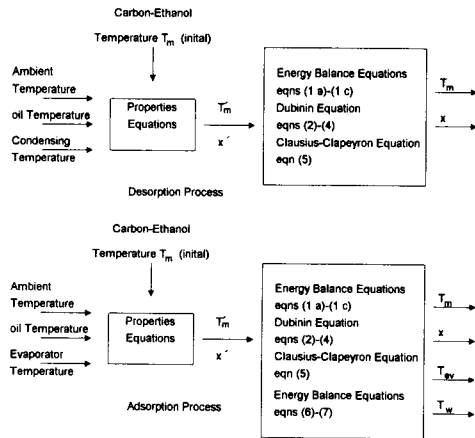


Fig. 5 steps of the calculation

Fig. 6 shows the experimental results of the temperature variation of the activated carbon-ethanol during the desorption-condensation and the adsorption-evaporation processes, the temperature of the cooling water at the condenser is at 20 °C. The mathematical model as described above is also used to predict the average temperature. The results agree well with those of the experiments.

Fig. 7 shows the simulated amounts of the liquid ethanol during desorption and adsorption processes and these are compared with those of the experimental results. The amount of ethanol could be read directly from a scale at the evaporator. During the desorption process, part of the liquid ethanol is accumulated in the condenser coil thus the experimental data is lower than those of the simulated values. During the adsorption, the simulated results agree well with the experimental results.

Fig. 8 shows the time history of the water in the refrigerated box during the adsorption-evaporation process, heat is extracted from the refrigerant thus the water temperature in the box decreases with time. Firstly when the ethanol is re-absorbed, the temperature drops slightly and reduces quickly when a high amount of ethanol is re-evaporated. When the adsorption capability of the adsorber is low, the temperature decreases only slightly again. The results from the model also agree well with the experiments.

**Conclusion**

There is a high potential of using activated carbon-ethanol for cooling purpose. The mathematical model developed can be used to estimate the parameters such as the amount of liquid ethanol during the desorption-condensation and the adsorption-evaporation processes, including the temperature in the refrigerated box, with good agreement with those of the experiments.

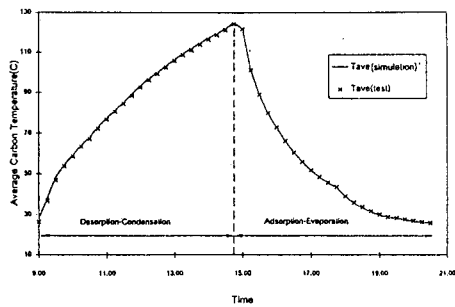


Fig. 6 The temperature of the carbon-ethanol pair in the adsorber during the processes of desorption-condensation and adsorption-evaporation. The condenser is controlled at 20°C

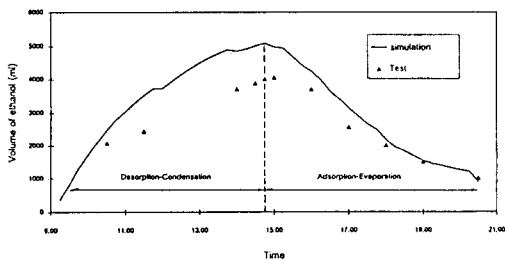


Fig. 7 The volume of liquid ethanol in the evaporator during the processes of desorption condensation and adsorption-evaporation. The condenser is controlled at 20°C

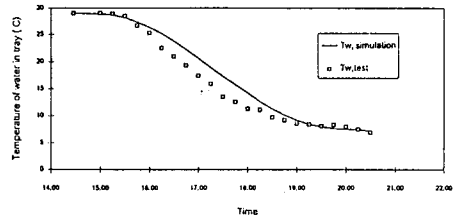


Fig.8 The temperature history of the water in the refrigerated box during the adsorption evaporation process.

**References**

- [1] Meunier, F. and Guilleminot, J.J. (1979), On the Use of a Zeolite 13x-H<sub>2</sub>O Intermittent Cycle for Application to Solar Climatization of Building, **Sun II**, Pergamon Press.
- [2] Grenier, Ph., Pons, M. and Diby, A. (1985), Modelization of a Solar Refrigerator Working with a Solid Adsorbent, **ISES Congress Intersol 85**.
- [3] Pone, M. and Grenier, Ph. (1986), Solar Ice Maker Working with Activated Carbon-Methanol Adsorbent-adsorbate Pair, **ISES Congress Intersol 85**.
- [4] Passos, E.F., Escobedo, J.F. and Meunier, F. (1989), Simulation of an Intermittent Adsorptive Solar Cooling System, **Solar Energy**, vol. 42, no.2, pp. 103-111.
- [5] Exell, R.H.B., Bhattacharya, S.C. and Upadhyaya, Y.R. (1987), Research and Development of Solar Powered Desiccant Refrigerator for Cold-Storage Applications, Grant no. DPE-5542-G-SS-405700, Asian Institute of Technology, Bangkok.
- [6] Critoph, R.E and Vogel, R. (1986), Possible Adsorption Pairs for Use in Solar Cooling, **Int. Ambient Energy**, vol. 7, no. 4, pp. 183-190.
- [7] Kiatsiroat, T., Chatthanapornyothin, C. and Wongsirodkul, S. (1992), Performance Study on a Solar Refrigeration System Working with Activated Carbon-Methanol Pair, **Research and Development**

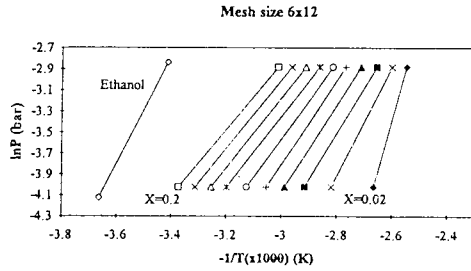
**Journal**, The Engineering Institute of Thailand, vol. 3, no. 2, pp. 61-69.

- [8] Tiansuwan, J., Kiatsiroat, T. and Hirunlabh, J. (1995), Activated Carbon-Ethanol : An Alternative Working Substance for Adsorption Cooling System, **J. Energy Heat and Mass Transfer**, vol. 17 pp. 65-73.
- [9] Dubinin, M.M., and Astakhov, V.A. (1970) *Adv. Chem. Ser.*, vol. 102, pp. 69.

**Appendix :** Properties of activated carbon and ethanol

- Activated Carbon size 6x12
- $W_o = 0.431267 \text{ K/kg carbon}$
- $D = -1.34872 \times 10^{-5}$
- $n = 1.63$

- Heat of Sorption ( $q_{st}$ )
- $q_{st} = (R/M_{mol}).\text{slope}(x)$
- $R = \text{Universal gas constant}$
- $= 8314.4 \text{ J/(k mol-K)}$
- $M_{mol} = \text{Molecular weight}$
- $= 46.07 \text{ kg/k mol of ethanol}$
- $\text{slope}(X) = \text{slope of constant X line}$   
P v.s. (-1/T) diagram



- Saturation vapor pressure of ethanol (in bar) at  $253 \text{ K} < T < 351.3 \text{ K}$   
 $\ln p_s = 12.68486796 - (3991.86/T) - (162933.878/T^2)$
- at  $351.3 \text{ K} < T < 403 \text{ K}$   
 $\ln p_s = 8.62004456 - (1225.519/T) - (633237.051/T^2)$
- Specific heat of Ethanol ( $c_e$ ) (kJ/kg-K)  
 $c_e = 1.913217 - .006937 (T) + .00002962 (T^2)$
- Density of Ethanol  $\rho_e$  (kg/m<sup>3</sup>)  
 $\rho_e = 1059.12687 - .878024 (T) - .0001801 (T^2)$
- Heat of vaporization (L) (kJ/kg)  
 $L = 916.056713 + 1.3933002 (T) - .004654 (T^2)$