Design and Implementation of IIR Multiple Notch Filter with Modified Pole-Zero Placement Algorithm

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บทคัดย่อ

บทความนี้นำเสนอการออกแบบและสร้างตัวกรองเชิงเลข IIR แบบ Notch ชนิดหลายความถี่หยุด โดยจะทำการปรับปรุงวิธีการวาง pole-zero ด้วยเทคนิคการประมาณค่า Least square เพื่อหาค่าอัตราการขยายในช่วงความถี่ผ่านและตำแหน่งของ pole ที่เหมาะสมในวงกลมรัศมี 1 หน่วยบนระนาบ Z ซึ่งค่าอัตราการขยายและตำแหน่งของ pole นี้จะใช้ควบคุมขนาดของตัวกรอง การออกแบบและจำลองการทำงานใช้โปรแกรม MATLAB ส่วนการสร้างจริงทำบนบอร์ดประมวลผลสัญญาณเชิงเลข TMS320C31

Abstract:

This paper presents a design and construction of IIR multiple notch filter by modifying the pole-zero placement with least square estimation to find the appropriate gain and pole positions for the filter within a unit circle in z-plane. The appropriated gain and pole position will render the controllable unit gain of filter magnitude. Algorithm design and system simulation are performed on MATLAB while the actual implementation is done on the TMS320C31 digital signal processing board.

Keywords: IIR, notch filter, least square, overdetermine.

1. Introduction

Notch filters can be constructed as either FIR or IIR notch filters. However, FIR notch filters cannot have very narrow bandwidth due to the fact that FIR filters are non-recursive filters whose poles are at the origin. On the other hand, IIR filters are recursive filters which have poles on a unit circle instead on the origin. The pole positions of IIR filters have enable the filters to be constructed as band-pass filters and notch filters with very narrow bandwidths.

There are several methods to implement IIR notch filters such as

1. Implementing notch filters from all-pass filters [1],
2. Transforming analog notch filter into digital IIR notch filters [2], and
3. Pole-zero placements on the z-planes [2]

Of all 3 methods, the pole-zero placements has easy implementation steps but the pole-zero placements has limitations on asymmetric pass-band gain as well as the inability to control the size of pass-band gain according to the specifications[3].

Therefore, this paper is introducing the improved algorithm for pole-zero placements to design IIR notch filter by applying least square approximation technique [4] to solve the asymmetric and uncontrollable pass-band gain. The design of new algorithm will go according to the following steps: designing a single notch filter by pole-zero placements, applying the least square approximation techniques to the old algorithm to get symmetrical and controllable pass-band gain, and cascading a single notch filters to create a multiple notch filter.

The applications of IIR multiple notch filters with very narrow bandwidths are for eliminating noises at the specified frequencies such as interfering noises from the AC power lines in medical instrument as well as other type of instruments for measurements.

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2. Single Notch Filter

2.1 Previous Design

The frequency response specification of ideal single notch filter is given by

$$H(e^{j\omega}) = \begin{cases} 0 & \omega_0 \\ 1 & otherwise \end{cases} \qquad (1)$$

ω_0 is a cutoff frequency of the notch filter.
The transfer function of a single notch filter will be as shown as follows:

$$H(z) = b_0 \frac{1 - 2\cos\omega_0 z^{-1} + z^{-2}}{1 - 2r\cos\omega_0 z^{-1} + r^2 z^{-2}} \qquad (2)$$

b_0 is filter gain.

ω_0 is pole angle on Z plane or cutoff frequency or notch frequency.

r is distance between pole and the origin.

The pole-zero placements which poles and zeros are at the same angles will be shown on Z-plane in Figure 1.

If $\omega_0 = 0.3\pi$ and r = 0.6, 0.7, 0.8, 0.9, 0.95 the magnitude response will be as shown in Figure 2.

Figure 2 has shown that the radius r has controlled the bandwidth of notch filter. If r is closer to the circumference or r is closer to 1, the bandwidth will become narrower, and the bandwidth size can be calculated by using geometrical properties [5]. Even though the notch frequency in Figure 2 is exactly at 0.3π, the pass-band gain is asymmetric since the gain at dc is not equal to the gain at nyquist frequency(π radian). Such inequality is due to the inappropriate values of pole positions which results in the asymmetric gain. Therefore, it is necessary to find the new pole positions to render the pass-band gain symmetric.

2.2 Propose Design

According to the previous method for the notch filter implementation by pole-zero placements, it has shown that the poles were not at the appropriated positions. Therefore, it becomes necessary to find the method to get the new pole positions. However, the zero position will not be changed since the center frequency of notch filter was defined from the positions of zeros. Therefore, it is necessary to find the new pole positions according to the pole in Figure 3 which has the transfer function of the notch filter in Eq. (1) changed to become a modified transfer functions in Eqs. (3) and (4).

$$\hat{H}(z) = b_0 \left[\frac{1 - 2\cos\omega_0 z^{-1} + z^{-2}}{1 - 2r\cos(\omega_0 + \phi) z^{-1} + r^2 z^{-2}} \right] \qquad (3)$$

$$\hat{H}(z) = b_0 \left[\frac{1 - 2\cos\omega_0 e^{-j\omega} + e^{-2jw}}{1 - 2r\cos(\omega_0 + \phi) e^{-j\omega} + r^2 e^{-2j\omega}} \right] \qquad (4)$$

$\hat{H}(z)$ is a transfer function of notch filter with modified pole positions.

ϕ is a modified pole angle.

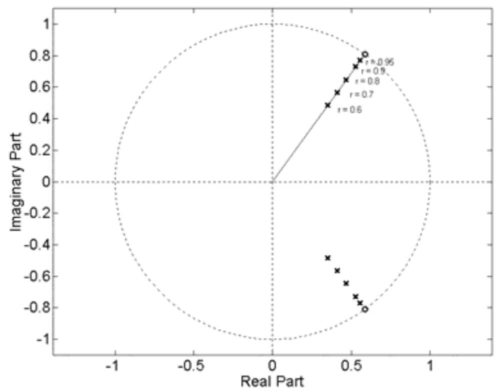


Figure 1 Traditional pole-zero plot for a single notch filter from the previous design.

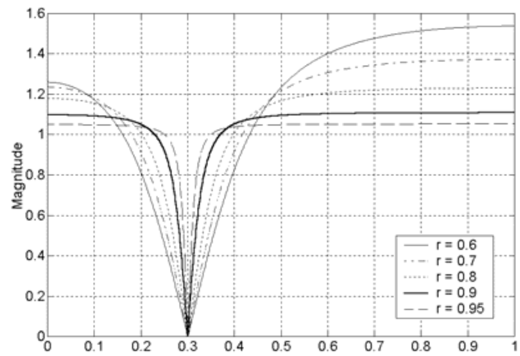


Figure 2 Magnitude response of a single notch filter using the previous design.

To make a symmetric pass-band gain, and frequency response of a modified IIR notch filter fitted into Eq. (4), the gain at dc and nyquist frequency must be equal according to Eq. (5) to Eq. (7).

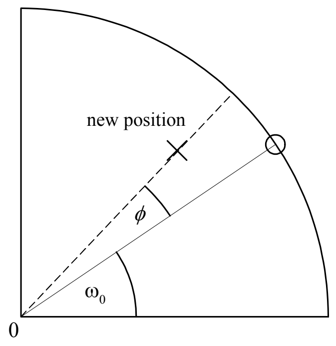


Figure 3 Modified pole positions according to the new methods.

$$\hat{H}(0) = b_0\left[\frac{1-2\cos\omega_0 e^{-j0}+e^{-2j0}}{1-2r\cos(\omega_0+\phi)e^{-j0}+r^2e^{-2j0}}\right] \quad (5)$$

$$\hat{H}(\pi) = b_0\left[\frac{1-2\cos\omega_0 e^{-j\pi}+e^{-2j\pi}}{1-2r\cos(\omega_0+\phi)e^{-j\pi}+r^2e^{-2j\pi}}\right] \quad (6)$$

$$\hat{H}(0) = \hat{H}(\pi) \quad (7)$$

$$b_0\left[\frac{1-2\cos\omega_0 e^{-j0}+e^{-2j0}}{1-2r\cos(\omega_0+\phi)e^{-j0}+r^2e^{-2j0}}\right] = b_0\left[\frac{1-2\cos\omega_0 e^{-j\pi}+e^{-2j\pi}}{1-2r\cos(\omega_0+\phi)e^{-j\pi}+r^2e^{-2j\pi}}\right] \quad (8)$$

Substitute $e^{j\omega} = \cos\omega + j\sin\omega$ into (8) to become (9).

$$\frac{1+\cos\omega_0}{1+2r\cos(\omega_0+\phi)+r^2} = \frac{1-\cos\omega_0}{1-2r\cos(\omega_0+\phi)+r^2} \quad (9)$$

From Eq. (9), the solution for ϕ will come out in Eq. (10).

$$2r\cos(\omega_0+\phi) = \cos\omega_0 + r^2\cos\omega_0$$

$$\phi = \cos^{-1}\left[\frac{1+r^2}{2r}\cos\omega_0\right] - \omega_0 \quad (10)$$

Defining $\hat{\omega}_0 = \omega_0 + \phi$

$\hat{\omega}_0$ is an improved pole angle. Therefore, the transfer function of digital notch filter can be written down as shown in Eq. (11).

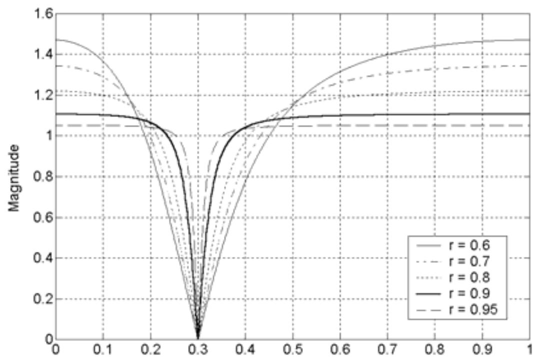


Figure 4 Magnitude response of notch filter after changing pole positions.

$$\hat{H}(z) = b_0\left[\frac{1-2\cos\omega_0 z^{-1}+z^{-2}}{1-2r\cos\hat{\omega}_0 z^{-1}+r^2z^{-2}}\right] \quad (11)$$

Assigning r = 0.6, 0.7, 0.8, 0.9 and 0.95, ω0 = 0.3π and b0 =1 into Eq. (11) and the magnitude response of IIR filter will results in the magnitude response in Figure 4.

Figure 4 has shown that the changes of pole positions will render the magnitude response of the notch filters symmetric with the equal gain at the frequency of zero and π radian. However, the pass-band gains haven't been controlled according to the specification yet. The radius r still controls the changes of gains. Therefore, it is necessary to find suitable values of b_0 to control and stabilize the pass-band gain by defining that the gain of the transfer function in Eq. (11) at dc, and nyquist frequency to be equal as shown in Eqs. (12) and (13) respectively.

$$\hat{H}(0) = b_0\left[\frac{1-2\cos\omega_0 e^{-j0}+e^{-2j0}}{1-2r\cos\hat{\omega}_0 e^{-j0}+r^2e^{-2j0}}\right] \quad (12)$$

$$\hat{H}(\pi) = b_0\left[\frac{1-2\cos\omega_0 e^{-j\pi}+e^{-2j\pi}}{1-2r\cos\hat{\omega}_0 e^{-j\pi}+r^2e^{-2j\pi}}\right] \quad (13)$$

Defining the gain at dc and nyquist frequencies are always equal to k and the result will show as:

$$b_0\left[\frac{1-2\cos\omega_0 e^{-j0}+e^{-2j0}}{1-2r\cos\hat{\omega}_0 e^{-j0}+r^2e^{-2j0}}\right] = k \quad (14)$$

$$b_0\left[\frac{1-2\cos\omega_0 e^{-j\pi}+e^{-2j\pi}}{1-2r\cos\hat{\omega}_0 e^{-j\pi}+r^2e^{-2j\pi}}\right]=k \tag{15}$$

Defining a_1 and a_2 as

$$a_1=\left[\frac{1-2\cos\omega_0 e^{-j0}+e^{-2j0}}{1-2r\cos\hat{\omega}_0 e^{-j0}+r^2e^{-2j0}}\right]$$ and

$$a_2=\left[\frac{1-2\cos\omega_0 e^{-j\pi}+e^{-2j\pi}}{1-2r\cos\hat{\omega}_0 e^{-j\pi}+r^2e^{-2j\pi}}\right]$$ and then

substitute a_1 and a_2 into Eqs. (14) and (15) to get the expression in (16).

$$\left.\begin{aligned} a_1b_0 &= k \\ a_2b_0 &= k \end{aligned}\right\} \tag{16}$$

a_1, a_2 and k are constant.

Defining vector $\mathbf{A}=\begin{bmatrix}a_1\\a_2\end{bmatrix}$, $\mathbf{x}=\begin{bmatrix}b_0\\b_0\end{bmatrix}$ and $\mathbf{b}=\begin{bmatrix}k\\k\end{bmatrix}$ to substitute into Eq. (16) as a matrix as shown in Eq. (17).

$$\mathbf{Ax}=\mathbf{b} \tag{17}$$

Eq. (16) has more equations than variables which can be considered as an overdetermined system, thus it is necessary to estimate the value of b_0 by applying the least square estimation to minimize the errors and $b_0$can be found from Eq. (18).

$$\mathbf{x}=(\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{b} \tag{18}$$

3. Multiple Notch Filter

The specification for frequency response specification for the ideal multiple notch filter is given by the following expression:

$$H(e^{j\omega})=\begin{cases}0 & \omega=\omega_{0k} \quad k=1,2,\ldots,M\\ 1 & otherwise\end{cases} \tag{19}$$

ω_{0k} is the Notch frequency while

$$\omega_{01}<\omega_{02}<\ldots<\omega_{0M}$$

The transfer function of Eq. (19) can be expressed in z-domain in Eq. (20)

$$H(z)=\frac{\prod_{k=1}^{M}b_{0k}\left[1-2\cos(\omega_{0k})z^{-1}+z^{-2}\right]}{\prod_{k=1}^{M}\left[1-2r\cos(\omega_{0k})z^{-1}+r^2z^{-2}\right]} \tag{20}$$

After the modification of the old algorithm by applying Eqs. (10) and (11), the new transfer function will be come as shown in Eq. (26), and the calculation by using Eqs. (16), (17) and (23) will result in the new value of ω0k as shown in Eq. (27).

$$\hat{H}(z)=\frac{\prod_{k=1}^{M}b_{0k}\left[1-2\cos(\hat{\omega}_{0k})z^{-1}+z^{-2}\right]}{\prod_{k=1}^{M}\left[1-2r\cos(\hat{\omega}_{0k})z^{-1}+r^2z^{-2}\right]} \tag{21}$$

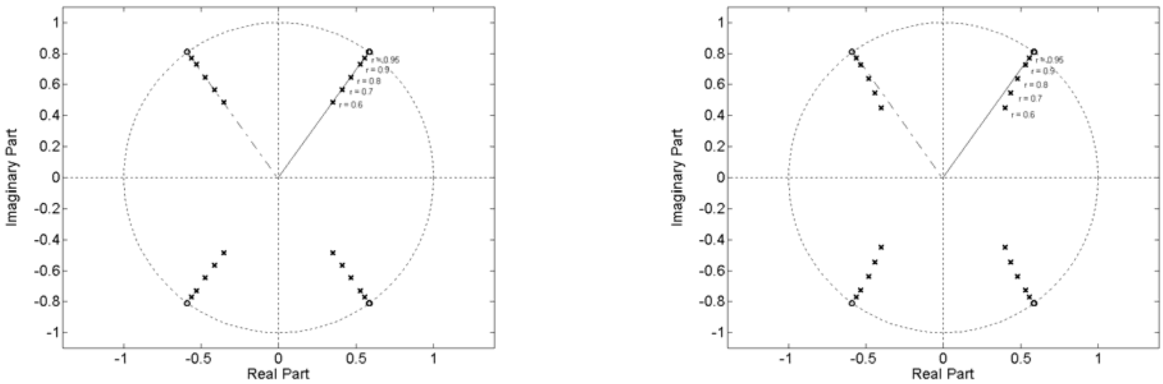
$$\hat{\omega}_{0k}=\omega_{0k}+\phi_k \tag{22}$$

4. Design Examples and Results

The design examples have specified the notch frequency at 0.3π, 0.7π r = 0.6, 0.7, 0.8, 0.9 and 0.95 for both previous design and propose design. The previous design will use those specifications on Eq. (20) while the propose design will use those specifications on Eqs. (10), (16) ~ (18), (20) and (21). After that, implementing the system by writing MATLAB codes to simulate the system by using pole-zero b_{0k} and ϕ_k to write down the pole-zero positions and the magnitude response shown in Figures. 5 and 6 respectively. The magnitude response for the simulated MATLAB program and actual magnitude response implemented on TMS320C31 DSP board at sampling frequency of 800 Hz and notch frequency at 50 Hz and 250 Hz will be show in Figure. 7

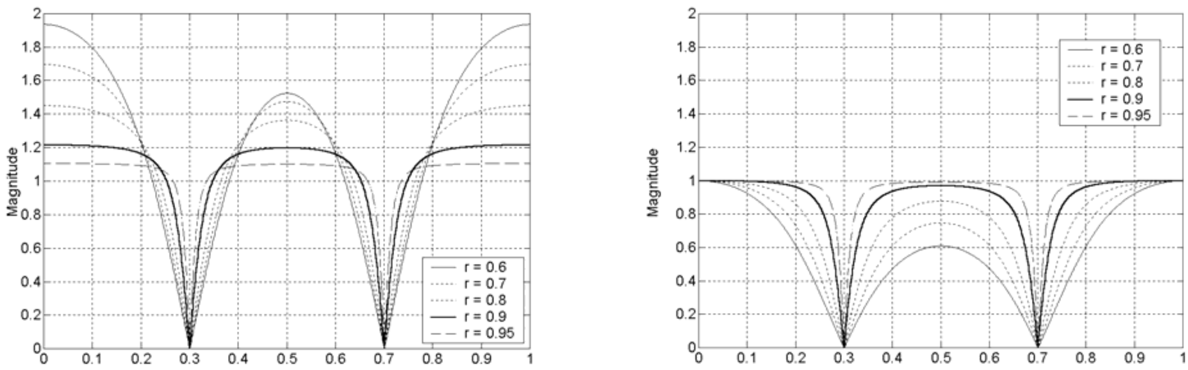
5. Conclusion

Experiment results have shown that the new design of IIR notch filter by applying modified pole-zero placement algorithms resulted in symmetric and controllable pass band gains according to the specifications, thus better results comparing to the previous design despite of more complicated design procedures than the previous design. Furthermore, the filters could eliminate AC noise and the harmonic noises according with very good results.



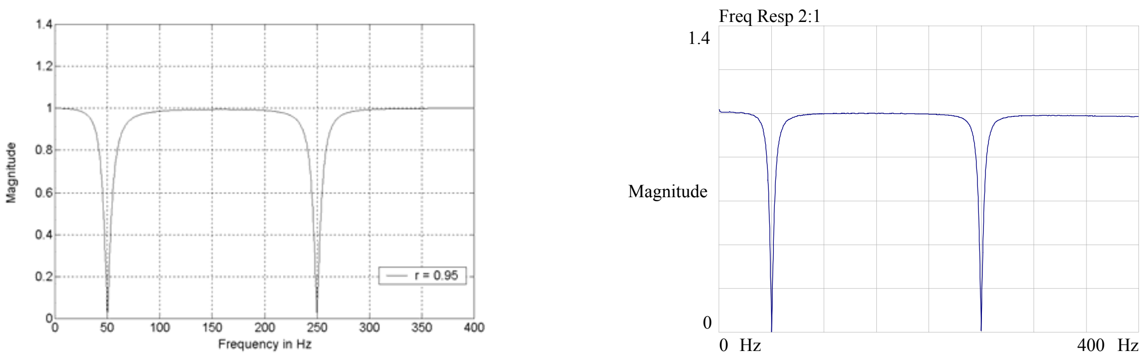
(a) Previous design. (b) Propose design.

Figure 5 Pole-zero plot at ω01 = 0.3π, ω02 = 0.7π r = 0.6, 0.7, 0.8, 0.9 and 0.95.



(a) Previous Design. (b) Propose Design.

Figure 6 Magnitude response at ω01 = 0.3π, ω02 = 0.7π r = 0.6, 0.7, 0.8, 0.9 and 0.95.



(a) Magnitude response from MATLAB simulation. (b) Actual results from dynamic signal analyzer.

Figure 7 Magnitude response at the notch frequencies of 50Hz and 250Hz at r = 0.99.

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