

Fuzzy Sliding Mode Controller Design

Siripun Thongchai* and Panarit Sethakul*

Abstract

This paper describes *fuzzy sliding-mode control* (FSMC), a fuzzy logic control based upon *variable structure techniques*. A variable structure approach is designed with guaranteed stability properties. A ball and beam system is used here as an example. The results are compared with a direct fuzzy control. The simulation results indicate that the fuzzy sliding-mode controller performs better than the direct fuzzy controller.

1. Introduction

Fuzzy logic control is derived from the *fuzzy logic* and *fuzzy set* theory that were introduced in 1965 by Professor Lotfi A. Zadeh of the University of California at Berkeley [13]. Fuzzy logic control can be applied in many disciplines such as economics, data analysis, engineering and other areas that involve a high level of uncertainty, complexity, or nonlinearity. In engineering, engineers can use the fundamentals of fuzzy logic and fuzzy set theory to create the pattern and the rules, then design the fuzzy controllers. Finally, the output responses of many systems can be improved by using a fuzzy controller [8-10].

This paper presents a fuzzy control which is based upon *variable structures standpoint techniques* (VSS) called *fuzzy sliding-mode control* (FSMC). FSMC can be applied to several nonlinear systems such as a ball and beam system, a robot manipulator, etc [5, 9]. The fuzzy controller and the FSMC are used to control the position of a steel ball on a pair

of conducting beams. The position of the ball will effect the input voltage and current of a DC motor. The ball and beam system was analyzed to obtain all parameters, which were used to design the fuzzy controller.

2. Fuzzy Control

Fuzzy control is an area of research under fuzzy system theory. Fuzzy system theory is the idea of set membership and logic that has its origins in ancient Greek philosophy, and applications at the leading edge of Artificial Intelligence. This theory involves classical sets, fuzzy sets, and set of membership functions [5, 11].

The first step in fuzzy controller¹ design is to know and define the linguistic values for all linguistic variables. Therefore, we need to define the membership functions for the linguistic variables. There are many ways to describe the membership functions. The important property of membership functions is the shape [6]. Fuzzy and fuzzy sliding mode controllers can be described as the following.

2.1 Fuzzy Controller

The fuzzy controller has three parts. The inputs of the controller will be an error value between reference input and feedback of the system.

¹Sometime a fuzzy controller is called a “fuzzy logic controller” (FLC) or even a “fuzzy linguistic controller”

* Department of Teacher Training in Electrical Engineering, Faculty of Technical Education, King’s Mongkut Institute of Technology North Bangkok.

1. *Fuzzification* of Input Variables. The input values will be classified by the input membership functions. The fuzzy controller receives input information, then, applies it to the rule bases.

2. *Rule Inference* (Inference Engine). The *Fuzzy Rule-Based System* converts input information into output membership function.

3. *Defuzzification* of Output Variable. There are many methods which can be used to convert the conclusions of the inference mechanism into actual input for the process or for the plant. *Center of Gravity* (COG) defuzzification method (also called *Centroid* method or *Center of Area*) is the most prevent and physically appealing of all the defuzzification methods (equation (1)). *Center-Average* defuzzification method is given by equation (2). *Weighted average* method is the same as Center-Average defuzzification method.

$$COG : y_q^{crisp} = \frac{\sum_{i=1}^R c_q^i \int_{y_p} \mu_{A_q^i}(y_q) dy_q}{\sum_{i=1}^R \int_{y_p} \mu_{A_q^i}(y_q) dy_q} \quad (1)$$

$$CA : y_q^{crisp} = \frac{\sum_{i=1}^R c_q^i \sup_{y_p} \mu_{A_q^i}(y_q)}{\sum_{i=1}^R \sup_{y_p} \mu_{A_q^i}(y_q)} \quad (2)$$

where R is the total number of control rules, c_q^i denotes the center value of y_q which is the output region of rule i , and y_q is the current degree of membership function $\mu_{A_q^i}$ for sensory reading input A at the rule i .

2.2 Fuzzy Sliding Mode Controller

This method uses a procedure of designing fuzzy controllers, which is based upon variable structures techniques. We can use the fuzzy equivalents of the *sliding-mode* controller, *saturating* controller, and *tanh* controller. The methods based upon *variables structures* techniques, which guarantee the stability of each controller [1]. By using a sliding surface, the order of the rule base is reduced to size $r \times m$

where r is the number of inputs and m is the number of fuzzification levels. This combination makes the proposed design procedure able to generate simple controllers with guaranteed stability properties. Consider from equation (39), the design steps will be shown as the following.

Step 1: Define an error between the desired states (X_R) and the actual states (X): $E = X_R - X$ and a sliding surface

$$S = CE \quad (3)$$

Where C is an arbitrary ($r \times n$) matrix chosen such that $S = 0$ defines a stable dynamic system of reduced order.

Step 2: Define a positive definite function of S , $K(S)$ such that

$$-S^T(CAX + CBK) < 0 \quad (4)$$

Step 3: Define a fuzzy rule base for mapping S to $K(S)$. The fuzzy logic controller can generate any function with an arbitrary degree of accuracy given enough fuzzification levels. Therefore, each element of $K(S)$ can be approximated using m fuzzification levels where m is free to be selected by the designer. Since $K(S)$ has r elements (one per input), rm fuzzy rules result.

Step 4: Define U to be the defuzzified value of (S) .

Several options for choosing $K(S)$ result in different types of fuzzy controllers. For example, assuming that CB is bounded from below so that $CB > I > 0$ then stability is assured by choosing $K(S)$ as

$$K(S) = \gamma I \cdot \text{sign}(S) \quad (5)$$

where I is an ($r \times r$) identity matrix and γ is a positive constant chosen so that $\gamma \epsilon$ larger than the largest element of CAX

$$\gamma \epsilon > \max(|CAX|) \quad (6)$$

This results in a fuzzy version of a *sliding model controller*, which is called *fuzzy sliding mode control* (FSMC). Alternatively, choosing $K(S)$ as

$$K(S) = \gamma \text{sat}(\beta S) \quad (7)$$

where $\text{sat}(x)$ is the element-by-element saturation function

$$\text{sat}(x) = \begin{cases} 1, & x > 1 \\ x, & -1 < x < 1 \\ -1, & x < -1 \end{cases} \quad (8)$$

and γ and β are positive scalars chosen such that

$$\gamma, \beta = \begin{cases} \gamma \varepsilon > \max(|\mathbf{CAx}|), & |\beta S| > 1 \\ (\mathbf{A} - \gamma \beta \mathbf{BC}) < 0, & |\beta S| < 1 \end{cases} \quad (9)$$

This results in a fuzzy version of a *saturating controller*, which is called *fuzzy saturating controller* (FSC). Alternatively, choosing $K(S)$ as

$$K(S) = \gamma \cdot \tanh(\beta S) \quad (10)$$

where γ and β are as defined in (9) results in the fuzzy version of the *tanh controller*.

Step 5: Replace all parameters in the control system and get the results from the simulation.

3. Ball and Beam System

The schematic diagram for the ball and beam system is shown in Figure 1. It is necessary to analyze the relation between the position of the ball, $x(t)$ and the angle of the beam, $\alpha(t)$ [3].

From Figure 1a), the ball is moving on the beam and the equation of the relation between mg (mass and gravitational force) and the direction of motion is obtained by using the law of motion, which is given in equation (11).

$$\frac{d^2x(t)}{dt^2} = \ddot{x} = gm \sin\alpha(t) \quad (11)$$

where $\alpha(t)$ is the angle of the beam shaft. The distance traveled by the ball is $x(t)$. From the ball on the beam in Figure 1c) and d), the equation of $x(t)$ is

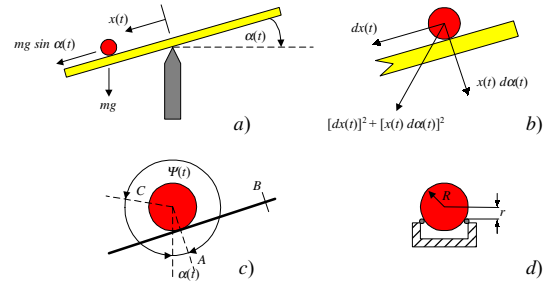


Figure 1 Ball and Beam System.

$$x = r \Psi(t) \quad (12)$$

where r is the radius of the ball, $\Psi(t)$ is the rotational angle of the ball with respect to the shaft and r . The rolling distance A and B is related to the peripheral distance A and C as follows, $AB = AC$, the distance $x(t)$ equals equation (12).

From Figure 1c) and d), we have the total angle of the ball, $\theta(t)$, equals to the summation of $\Psi(t)$ and $\alpha(t)$ in equation (13). Then, the rotational velocity of the ball is given by equation (14).

$$\theta(t) = \Psi(t) + \alpha(t) = \frac{x(t)}{r} + \alpha(t) \quad (13)$$

$$\omega = \frac{d\Psi(t)}{dt} + \frac{d\alpha(t)}{dt} + \frac{\dot{x}(t)}{r} + \dot{\alpha}(t) \quad (14)$$

The translational velocity of the ball, v , is found from Figure 1 b).

$$v = \sqrt{\dot{x}^2 + (x\dot{\alpha})^2} \quad (15)$$

Using the equations for rotational and translational velocities in the Lagrangian form yields equation (16)

$$L(q_i, \dot{q}_i) = U - T \quad (16)$$

Where U is the kinetic co-energy, T is the potential energy of the system.

The kinetic co-energy is composed of the translational, rotational kinetic co-energy of the ball, and the kinetic energy of the beam. Therefore, the kinetic co-energy, U , is given by equation (17)

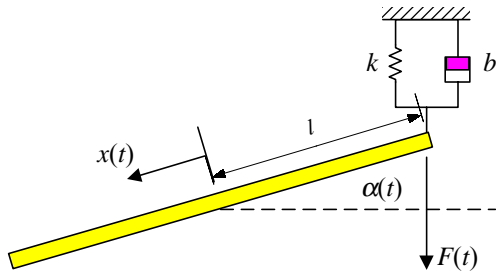


Figure 2 The potential energy of the system.

$$U = \frac{1}{2}mv^2 + \frac{1}{2}I_b\omega^2 + \frac{1}{2}I_a(\dot{\alpha})^2 \quad (17)$$

Where v is the translational velocity of the ball, ω is the angular velocity of the ball, I_a is the beam inertia, and I_b is the ball inertia.

The potential energy of the system is found by using Figure 2. It equals the energy stored in the spring.

If only small angular excursions are considered and the spring is linear with stiffness k , then we have

$$T = \frac{1}{2}k(l\alpha)^2 \quad (18)$$

Therefore, from equation (14) to equation (18), the Lagrangian's equation is given by

$$L = \frac{1}{2}[mv^2 + I_b\omega^2 + I_a(\dot{\alpha})^2] - \frac{1}{2}k(l\alpha)^2 \quad (19)$$

$$\frac{1}{2} m \dot{x}^2 - x^2 + \frac{1}{2} I_b \frac{\dot{x}^2}{r^2} - \frac{1}{2} I_a \dot{\alpha}^2 - \frac{1}{2} k l^2 \alpha^2 \quad (20)$$

In addition, if we need to consider the energy dissipation in the system, then we will have the system co-content, J , in this system. Therefore, we will have;

$$J = \frac{1}{2}b(l\dot{\alpha})^2 \quad (21)$$

From the Lagrangian, the equations of motion of dynamic system are obtained

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i \quad (22)$$

Then, we have the equations of motion for the ball and beam as follow :

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} + \frac{\partial J}{\partial \dot{x}} = F \quad (23)$$

and

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{\alpha}} - \frac{\partial L}{\partial \alpha} + \frac{\partial J}{\partial \dot{\alpha}} = \tau \quad (24)$$

where F is the generalized force, τ is the generalized torque.

In this case, F is equal to $mg \sin \alpha(t)$, because the component of gravity is performed in the x direction. The generalized torque, τ , is composed of the components torque contributed by the external forces acting about the pivot. One component is due to the input force and another opposing component stems from the gravitational force on the ball. Then, the generalized torque is obtained

$$\tau = \cos \alpha(t) (mgx - F(t) l) \quad (25)$$

The equations of motion of the system can be formulated by substituting the appropriate quantities into equations (23) and (24). Differentiating the result, then we have

$$m \frac{I_b}{r^2} \ddot{x} + \frac{I_b}{r} \ddot{\alpha} - mx \ddot{\alpha} = mg \sin \alpha \quad (26)$$

$$(mx^2 + I_b + I_a) \ddot{\alpha} + (2mx\dot{\alpha} + bl^2) \dot{\alpha} + k l^2 \alpha + \frac{I_b}{r} \ddot{x} - xmg \cos \alpha = -F(t)l \quad (27)$$

Equations (26) and (27) are the equations of motion of the ball and beam. They constitute a nonlinear coupled set of differential equations which are referred from the Figure 1.

1. The moment of inertia of the ball I_b and its mass m are small and can be regarded as not having any effect on beam behavior.

2. $\ddot{\alpha}$ and $\dot{\alpha}$ are small and have a negligible effect in equation (26)

To linearize the equation (26) and (27), we assume that the control system, for control disturbances, will act to return the ball to rest with minimal shaft movement [7]. Therefore, the shaft

angle and its derivatives are assumed to be small, and the dynamic equation becomes,

$$\left(m + \frac{I_b}{r^2}\right) \ddot{x} = mg\alpha \quad (28)$$

$$(I_a \ddot{\alpha} + (b l^2) \dot{\alpha} + k l^2 \alpha = -F(t)l \quad (29)$$

Now, the plant model is found by taking the Laplace transform of the preceding into equation (28), then we have

$$\begin{aligned} \left(m + \frac{I_b}{r^2}\right) s^2 X(s) &= mg\alpha(s) \\ \frac{X(s)}{\alpha(s)} &= \frac{mg}{\left(m + \frac{I_b}{r^2}\right) s^2} \end{aligned} \quad (30)$$

We can simplify the transfer function in equation (30) by substituting the moment of inertia of a sphere of radius R , that is

$$I_b = \frac{2}{5} m R^2 \quad (31)$$

$$G(s) = \frac{g}{\left(1 + \frac{2R^2}{5r^2}\right) s^2} \quad (32)$$

The equation (32) can be used as the model of the ball and beam dynamics [7].

Next, the motor beam dynamics are derived. It means that we have to find the transfer function of the motor, which has the relation between output angle and input voltage. The input voltage $V_a(s)$ will change the output speed $\Omega_m(s)$. This relationship is given by equation (33) [12]. The results of the transfer function of the DC motor which is a relationship between output angle and input voltage is shown in equation (35)

$$\frac{m s}{V_a s} = \frac{K_{mt}}{L_a J s^2 \frac{R_a J}{R_a} L_a s} \quad (33)$$

$$\omega_m(t) = \frac{d\theta(t)}{dt} \Rightarrow \Omega_m(t) = s\Theta(s) \quad (34)$$

$$\frac{s}{V_a s} = \frac{K_{mt}}{s \frac{L_a J s^2}{R_a} \frac{R_a J}{K_{me} K_{mt}} L_a s} \quad (35)$$

In this case we may also have the state space modeling as follow:

$$\begin{aligned} \text{let, } x_1 &= x, x_2 = \dot{x}, x_3 = \alpha, \text{ and } x_4 = \dot{\alpha} \\ \dot{x}(t) &= [\dot{x}_1 \quad \dot{x}_2 \quad \dot{x}_3 \quad \dot{x}_4]^T \end{aligned} \quad (36)$$

Now, we have

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{m x_1 x_4^2 - g \sin x_3}{m + \frac{I_b}{r^2}} \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{-2m x_1 x_2 x_4 - m g x_1 \cos x_3 + u}{\Phi + m x_1^2} \\ u &= \frac{-2m x_1 x_2 x_4 - m g x_1 \cos x_3 + u}{\Phi + m x_1^2} \end{aligned}$$

and $y = x_1$

Example of a Ball and Beam System

Let the dynamics of the ball and beam system are given by²

$$\begin{aligned} \left(m + \frac{I_b}{r^2}\right) \ddot{x} &= x\alpha^2 - g \sin \alpha \\ (\Phi + m x^2) \ddot{\alpha} &= -2m \dot{x} \alpha - m g x \cos \alpha + u \end{aligned}$$

where the parameters are $g = 9.81$, $m = 0.01679$

$g = 9.81$: Gravitational force

$m = 0.01679$: Mass (kg)

$r = \frac{2.54}{1000}$: Level radius = 2.54 cm.

$R = 3.5878 \times 10^{-2}$: (radius)

$x = \frac{50}{100} = 0.5\text{m.}, |x| < \frac{50}{100}$

$I_b = \left(\frac{2}{5}\right) m R^2 = 8.645 \times 10^{-6}$: Ball inertia

$\Phi = 0.0079$: Rotational inertia of the beam (kg m²)

Now, we can write in the state-space model as follow :

²From Glower and Jeffrey [1]

$$\dot{x}(t) \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{x\ddot{\alpha}^2 - g \sin \alpha}{\left(m + \frac{I_b}{r^2}\right)} \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \frac{(-2mx_1x_2x_4 - mgx_1 \cos x_3 + u)}{(\Phi + mx_1^2)} \end{cases} \quad (37)$$

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-g}{\left(m + \frac{I_b}{r^2}\right)} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-2m}{\Phi} & 0 & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{\Phi} \end{bmatrix} u$$

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -7.223 & 0 \\ 0 & 0 & 0 & 1 \\ -20.828 & 0 & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 126.58 \end{bmatrix} u \quad (38)$$

$y = x_1$

3.1 Fuzzy Control Design

As we know that the ball and beam system cannot be stabilized by a PID controller. Therefore, fuzzy controllers which utilize only the state error and its derivative will not work. Further, the system is fourth-order, resulting in extensions of fuzzy controllers which fuzzify all of the states becoming unmanageable. However, a direct fuzzy logic and a fuzzy sliding mode controllers can be used. Consider the state-space of the system:

$$\dot{x}(t) = \mathbf{A}x(t) + \mathbf{B}u(t) \quad (39)$$

$$y(t) = \mathbf{C}^T x(t) \quad (40)$$

From the equation (36), the system has four state variables (inputs). We also know that the system needs torque to control the angle of the beam. The control system is designed as a multi-input and single-output fuzzy system. The inputs of the controller are x_1 , x_2 , x_3 , and x_4 . The output of the controller is u . However, the number of inputs can be changed by the methods of fuzzy controller design.

3.1.1 Direct Fuzzy Controller

Step 1: Construct the membership function for inputs x_1 , x_2 , x_3 , and x_4 . Consider the input x_1 , which has three membership functions. In this case, we define the membership functions as the following values: *positive* (P), *zero* (Z), and *negative* (N). The inputs x_1 , x_2 , x_3 , and x_4 are also defined the same as the input x_1 . Therefore, the membership function for inputs x_1 , x_2 , x_3 , and x_4 are shown as follows.

input x_1 has P, Z, and N x
input x_2 has P, Z, and N \bar{x}
input x_3 has P, Z, and N and
input x_4 has P, Z, and N \square

Step 2: Construct the membership function for the output. Start with nine membership functions for output u , which can be defined as *positive-max* (mf1), *positive-big* (PP), *positive* (P), *positive-small* (PS), *zero* (Z), *negative-small* (NS), *negative* (N), *negative-big* (NN), and *negative-max* (mf9).

Step 3: The fuzzy controller has four inputs and one output. In this step, the truth table is used to analyze the output. The results of a truth table are the control actions. These results are used as the rules of fuzzy controller. The number of fuzzy rules is equal to $3 \times 3 \times 3 \times 3 = 81$. All rules involve the inputs x_1 , x_2 , x_3 , and x_4 .

Step 4: From the physical system shows in Figure 1, set the regions of the inputs and output as follows.

Input x_1 is the position of the ball, x , then we have, $-\frac{1}{2} < x < \frac{1}{2}$, (m) where l : is the range of the beam (m)

Input x_2 is the derivative of distance, \dot{x} , then we have, $-\frac{1}{2} < \dot{x} < \frac{1}{2}$, $\left(\frac{\text{m}}{\text{sec}}\right)$.

Input x_3 is the angle of the beam shaft, α , then we have, $-\alpha_{\max} < \alpha < \alpha_{\max}$ (red)

Input x_4 is the derivative of the angle, $\dot{\alpha}$, then we have, $-\dot{\alpha}_{\max} < \dot{\alpha} < \dot{\alpha}_{\max}$, (red/sec)

All inputs can be adjusted by using gains g_1 , g_2 , g_3 and g_4 . The gains g_1 , g_2 , g_3 and g_4 can be found by using Nonlinear Control Design or using Linear

Quadratic Regulator design (LQR). For LQR, the gains g_1, g_2, g_3 and g_4 are based on the linearized model.

Output u is the input of ball and beam system, which generated torque. Let us assume that the system needs input $-10 < u < 10$, $\left(\frac{\text{Nm}}{\text{Amp}} \frac{\text{V}}{\text{red/sec}}\right)$. The fuzzy inference system for this method is shown in Figure 3.

Step 5: Replace all parameters in the ball and beam system and simulate the control system to obtain the results.

3.1.2 Fuzzy Sliding Mode Controller

Consider the dynamic equation of the ball and beam from equation (37), a fuzzy sliding mode controller can be designed as the following steps.

Step 1: Defines a sliding surface $S = 0$ and a stable dynamic system. Now, using a linearized model of the ball and beam as equation (38) and using LQR techniques, thus the feedback gain K is found.

$$K = [-1.1782 \quad -1.5405 \quad 4.8977 \quad 1.0380]$$

Since C is an arbitrary $(r \times n)$ matrix chosen such that $S = 0$, then we may have

$$S = K(X_R - X) = K \begin{bmatrix} x_{ref} - x \\ \dot{x}_{ref} - \dot{x} \\ \alpha_{ref} - \alpha \\ \dot{\alpha}_{ref} - \dot{\alpha} \end{bmatrix}$$

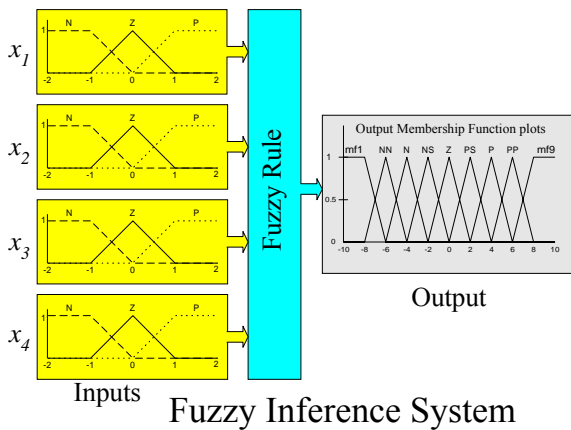


Figure 3 The fuzzy inference system.

We know that the dynamics associated with $S = 0$ for this system linearized about $X = 0$.

Step 2: Define a positive definite function of S , $K(S)$ such $-S^T(CAX + CBK) < 0$ as shown in equation (4). Now, a FSMC is to be designed, $K(S)$ is chosen according to equation (5) where γ is chosen to satisfy (6). Substituting in the dynamics of the ball and beam system results in choosing γ , we have

$$S = K(X_R - X) = -KX(CAX + CBK) < 0$$

$$CA = \begin{bmatrix} -1.1782 \\ -1.5405 \\ 4.8977 \\ 1.038 \end{bmatrix}^T \begin{bmatrix} x_2 \\ (x_1 x_4^2 B - gB \sin x_3) \\ x_4 \\ \frac{(-2mx_1 x_2 x_4 - mgx_1 \cos x_3 + u)}{(\Phi + mx_1^2)} \end{bmatrix}$$

$$= -1.1782x_2 - 1.5405(x_1 x_4^2 B - gB \sin x_3) + 4.8977x_4 + 1.038 \left(\frac{(-2mx_1 x_2 x_4 - mgx_1 \cos x_3 + u)}{(\Phi + mx_1^2)} \right) < 0$$

Now, the γ is depended on the upper limits of the physical system and the limits are $x = 1, \dot{x} = 1, \alpha = 1$, and $\dot{\alpha} = 1$. Then, we have

$$\begin{aligned} &|-1.1782x_2| < 1.1782 \\ &\left| \frac{-1.5405}{1.4} ((0.73705) - 9.18(0.73705) \sin 1) \right| < 5.8838 \\ &|4.8977x_4| < 4.8977 \\ &1.038 \left(\frac{u}{(\Phi + mx_1^2)} \right) > 42.041|u| \\ &(1.1782 + 5.8838 + 4.8977 + 5.1531) < 42.041|u|\gamma \\ &\frac{17.113}{42.041} = 0.40706 < \gamma|u| \end{aligned}$$

Now, if $|u| = 1$, then $\gamma > 0.40706$ will be satisfied. Hence, for a FSMC, $K(S)$ is selected as

$$K(S) = 0.40706 \text{ sign}(S) \quad (41)$$

a fuzzy saturating controller, $K(S)$ is selected as

$$K(S) = 0.40706 \cdot \text{sat} \left(\frac{S}{0.40706} \right) \quad (42)$$

a fuzzy tanh controller, $K(S)$ is selected as

$$K(S) = 0.40706 \cdot \tanh\left(\frac{S}{0.40706}\right) \quad (43)$$

Step 3: Define a fuzzy rule base for mapping S to $K(S)$. The fuzzy logic controller can generate any function with an arbitrary degree of accuracy given enough fuzzification levels. The fuzzy rule base can be presented as Table 1 and Table 2, where S^* denotes a particular value for S , and $\mu_{PL}(S^*)$ denotes the degree of membership of S^* in fuzzy category PL .

The results of each controllers can be shown as the following.

$$S : \begin{cases} \mu_{PL}(S^*) = 1, & S^* > 0 \\ \mu_{PO}(S^*) = 1, & S^* = 0 \\ \mu_{NL}(S^*) = 1, & S^* < 0 \end{cases} \quad (44)$$

$$K : \begin{cases} \mu_{PL}(K^*) = 1, & K^* > 0.407 \\ \mu_{PO}(K^*) = 1, & K^* = 0 \\ \mu_{NL}(K^*) = 1, & K^* < -0.407 \end{cases} \quad (45)$$

$$K : \begin{cases} \mu_{PB}(K^*) = 1, & K^* = 0.407 \\ \mu_{PL}(K^*) = 1, & K^* = 0.3515 \\ \mu_{PM}(K^*) = 1, & K^* = 0.296 \\ \mu_{PS}(K^*) = 1, & K^* = 0.1665 \\ \mu_{ZO}(K^*) = 1, & K^* = 0 \\ \mu_{NS}(K^*) = 1, & K^* = -0.1665 \\ \mu_{NM}(K^*) = 1, & K^* = -0.296 \\ \mu_{NL}(K^*) = 1, & K^* = -0.3515 \\ \mu_{NB}(K^*) = 1, & K^* = -0.407 \end{cases} \quad (46)$$

Now, the fuzzy inference system³ for this method can be shown as Figure 4.

Table 1 The fuzzy rule base for implementing an FSMC as well as an FSC

K	S
	NL ZO PL
	NL ZO PL

³The relation of input and output membership function

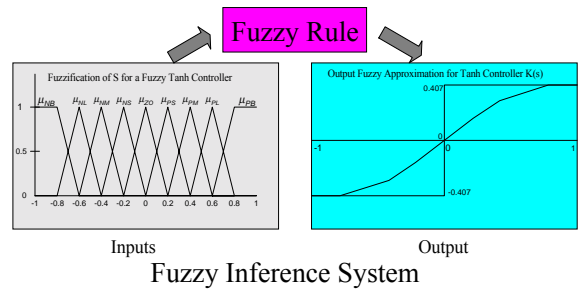


Figure 4 Fuzzy controller.

Table 2 The fuzzy rule base for implementing a Tanh-type controller

K	S
	NB NL NM NS ZO PS PM PL PB
	NB NL NM NS ZO PS PM PL PB

Step 4: Define U to be the defuzzified value of (S) using equation (2).

4. Implementation

4.1 Direct Fuzzy Controller

The gains g_1 , g_2 , g_3 , and g_4 can be found by using nonlinear control and linear quadratic regulator (LQR) design technique. The results can be shown as Table 3.

Table 3 Using nonlinear control and LQR technique to find gains g_1 , g_2 , g_3 , and g_4

Gains	Nonlinear Control	LQR
g_1	-1.2731	-1.1780
g_2	-1.5576	-1.5407
g_3	5.4775	4.8957
g_4	0.9064	1.0380

The outputs of the system are the ball position (x) and the beam angle (α). The simulation results are shown in Figures 5 and 6. The results are somewhat similar.

4.2 Fuzzy Sliding Mode Controller

Now, a fuzzy sliding mode controller was used, the number of rules should be reduced. In addition,

the system is stable. The matrix K was found by using LQR technique, which is the same as Table 3.

The outputs of the system are the ball position (x) and the beam angle (α). The simulation results of both controllers are shown as Figure 7 for saturating controller and Figure 8 for tanh controller.

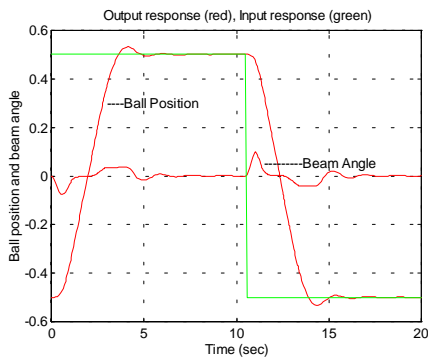


Figure 5 The output responses of direct fuzzy control system with nonlinear control design.

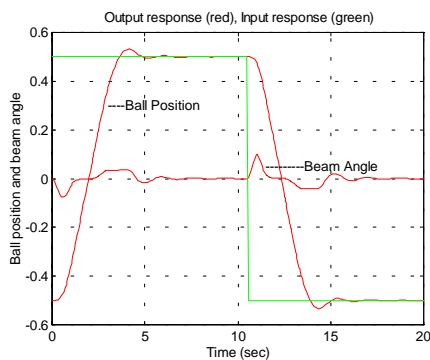


Figure 6 The output responses of direct fuzzy control system with LQR design the gains g_1, g_2, g_3 and g_4 are based on the linerrized model.

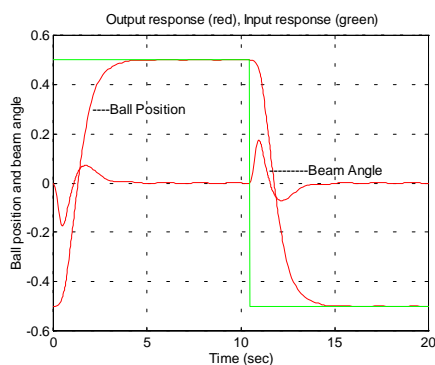


Figure 7 The output responses of FSMC with saturating controller.

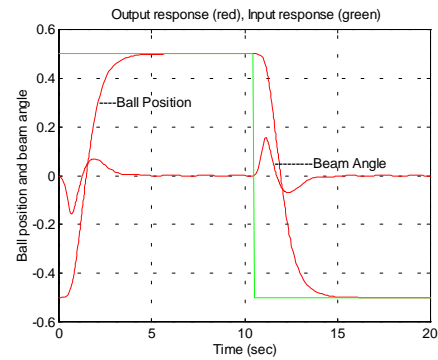


Figure 8 The output responses of FSMC with tanh controller.

5. Conclusions

Two fuzzy controllers were presented. The first was the direct fuzzy logic controller, which can be directly used to control a system. However, the feedback matrix gain was needed to optimize the system. Both nonlinear and LQR techniques were applied. The results showed that the direct fuzzy logic controller is simple and can be directly applied to control a ball and beam system.

The fuzzy logic controller of the ball and beam system has four inputs and one output, which needed 81 rules for the fuzzy rule base. Each input has three membership functions. It was shown that the ball could be moved on the beam, as needed. The ball position also had an overshoot. The second controller is the fuzzy sliding mode controller using fuzzy saturating and fuzzy tanh controllers. The resulting rule base for fuzzy saturating controller presented in Table 1 and the fuzzy tanh controller presented in Table 2. The number of fuzzy rules was reduced from 81 rules to 3 and 7 rules.

6. Acknowledgement

The authors would like to thank Professor Jacob S. Glower for his help and for providing correct data.

Proof : The proof that each of these fuzzy controllers guarantees stability is as follows. First, define a Lyapunov function as

$$\dot{v} = \frac{1}{2} S^T \dot{S} \quad (47)$$

From Lyapunov's second method, stability is assured provided that

$$\dot{v} = S^T \dot{S} < 0 \quad (48)$$

Now, substituting for S and assuming that X_r is constant (implying that the set-point is constant) results in

$$-S^T (CAX + CBK) < 0 \quad (49)$$

which is the stability condition given in Step 2 equation (4)

If $K(S)$ is chosen to be a sign (\cdot) function and CB is bounded from below by ϵ , then the equation (4) becomes

$$-S^T (CAX + \gamma CB \text{sign}(S)) < -S^T (CAX + \gamma \epsilon \cdot \text{sign}(S)) < 0$$

$$\gamma CB < \gamma \epsilon$$

Equation (6) assures that this inequality holds, if $K(S)$ is a saturating function as equation (7), then the additional requirement of

$$-S^T (CAX + \gamma \epsilon \cdot \text{sign}(S)) < 0$$

is required when $|\beta S| < 1$. Substituting for S and letting $X_r = 0$ arbitrarily, gives⁴

$$S^T CAX \quad \text{sign}(S) \quad 0$$

$$\max |CAX|$$

$$K S \quad \text{sat} \quad S$$

⁴This method will thus illustrate how a fuzzy controller can be defined with significantly fewer rules than 81

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