Research Article

The Development of Mathematical Model for a University Course Timetabling Problem

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Abstract

This research is to study the problem of classroom timetable generating due to an increase amount of student. The generator of classroom timetable needs to arrange it properly according to the limited resources , i.e., a number of classrooms, loading capacity of classroom, periods, and number of teachers. Importantly, the generating of this timetable construction has to be considered together-with the proportion between capacity of classroom and number of students; otherwise it will increase expense and extra-period requiring. An inappropriate assignment between courses to classrooms will require more classroom utilization. The expense will increase since fee is charged and forwarded to the faculty of Engineering for classroom utilization. In addition inappropriate assignment will also cause requiring extra periods between 3:45 and 6:25 PM. This occurs often at present and leads to ineffective learning.

Consequently, the generating of timetable needs to be concerned about expense and time. This research is to study a course timetabling problem of Silpakorn University by building up mathematical model to find out an optimal solution. The main objectives are 1) the lowest expense generating; and 2) the least extra-period spending. The programs used for the mathematical solution in this research are IBM ILOG CPLEX 12.2. The result was found that solving the classroom timetable problem using a mathematical model could reduce the costs of 8,115 baht / week and the extra-periods required could be reduced from 10 periods to 6 periods.

Keywords: Classroom Timetable; Mathematical Model; Integer Linear Programming

Introduction

Course timetabling is a multi-dimensional assignment problem which is the assignment of courses to faculty members and then, the assignment of these courses to classroom and time slots.

Problem encountered in academic departments such as school, college and university. Nowadays, the number of students has increased substantially. The compulsory and elective course may be in the same and/or different faculties. The timetabling problem has become much more complex. This problem is solved based on many restrictions such as the period time, the available number of classrooms, capacity of the classrooms or number of seats in each, amount of the registered student and other restrictions from faculty members.

Because of the reasons mentioned, a scheduler, a human decision-maker, consumes time to solve the problem. Furthermore, the assignment of the students to classrooms requires concerning of the suitable number of students for the classroom since it may increase cost and waste opportunity for other appropriate classroom. Some courses may be shifted to the extra-periods, such as 3:45 PM – 6:25 PM, because of inefficient assignment. This affects student's preference. Furthermore, generating of timetable needs to be concerned about time and expense.

This research is to study a course timetabling problem of Silpakorn University by building up mathematical model to find out an optimal solution with the multi objectives in order to the lowest expense generating; and the least extra-period spending.

Theory and Literature Review

Integer Linear Programming (ILP)

Optimisation problem is the problem involving one or more decisions with restrictions. A goal or an objective is considered. The objective is represented by an objective function which identifies the function of the decision variables. The decision maker may want to either maximize or minimize the objective e.g. minimizing cost and maximize profit. The constraint is represented in a mathematical model, e.g.

$f(x_1,x_2,..,x_n) \leq b$, $f(x_1,x_2,..,x_n) \geq b$ (Ragsdale et al., 2004).

Linear programming (LP) involves an optimisation problem with linear objective functions and linear constraints. LP model has three basic components (Hamdy A. Taha., 2003)

1) Objective of goal that is aimed to optimise

2) Constraints or restrictions that are needed to satisfy, for example a limited amount of raw materials or labors.

3) Decision variables or the solutions, the non-negativity restrictions accounting for this requirement.

Integer linear programming (ILP) is linear programming in which some or all the variables are restricted to integer value.

Assignment Problem

An assignment problem is a special case of a transportation model in which the workers represent the sources and the jobs represent the destinations (Ragsdale et al., 2004). e.g. the resource allocation of labors, equipments or machine to workplaces.

The course time tabling problem is an assignment of courses to classrooms and time slots with restrictions in order to minimizing cost or maximizing utilization.

Table 1 shows literature review about course timetabling problem.

Table 1 Literature review

Reference	Institution(student, courses, faculty, section, rooms)	Problem Description	Algorithm
Nakasuwan et al., 1999	Thammasat University	Schedule courses and assign them to room	ILP
Oladokun and Badmus, 2008	University of Ibadan	- Multi-objective Optimisation - Schedule courses and assign them to room subject to several side constraints	ILP
Wasfy and Aloul, 2007	American University of Sharjah	Schedule class	ILP
Daskalaki et al., 2004	University of Patras	Schedule courses and assign them to room	ILP
Gunawan et al., 2007	University of Singapore	Satisfy teacher preferences	hybrid algorithm
Dammak et al., 2006	University of Sharjah	Problem of assigning exams to timeslots	Heuristic

Research Methodology

1. Studying the course timetabling in practical application

The relevant persons, scheduler and faculty member, were interviewed. It was found that firstly the courses are assigned in period slots. Then, each course will be assigned in a time slots and classrooms. The data about courses, lecturers, groups of student are required e.g. the number of course, faculty member, classrooms and classroom capacity etc.

2. Developing mathematical model with multi objectives minimizing classroom cost and minimizing study in the extra-period.

3. Testing the numerical model by using a case problem of Industrial Engineering department, Silpakorn university

4. Conclusion

The results of the test were analyzed.

Problem Statement

This problem concerns a course timetabling problem which is a multi-dimension assignment problem. These are an assignment of courses, students, faculty members, class rooms, and period times. Courses are subjects taught one time a week. This problem assumes that each course has one section and each course is lectured by one faculty member. A faculty member can teach more than one course. Each course is designed for a group of students such as a section of 1st years student, 2nd years student, 3rd years student and 4th years student. The number of the students in a course are varied which depend on the number of registered students. There are *k* periods in a day which consists of general period and extra-period. There are *j* class rooms. The capacity of each class room is known. The cost of each class room depends on its capacity. Cost of classroom means money charge by the Faculty of Engineering due to classroom utilization.

Table 2-5 provides the example of data applied in the problem.

Table 2 Classroom capacity and classroom cost

Room (j)	Classroom capacity Fj	Cost Cj (baht/ a time slot)
515	60	75
513	100	135
135	80	270
136	80	270
137	80	270
138	80	270
139	80	270
140-1	120	360
142-3	120	360
144-6	180	540

Table 3 Example data of the course data

Course (i)	Faculty member (r)	Group of students (s)	No. registered student (N_i)
614442	A	IE 4th year	80
614352	B	IE 3nd year	80
619352	C	IML 3nd year	85
619353	D	IML 3nd year	90

Table 4 Example data of the faculty member (R_{ir})

Faculty member (r)	Course (i)
A	614442, 614432, 614346, 614341, 614343
B	614352, 614351, 614212
C	614330, 614331
D	619353, 619492

Table 5 Example data of group of students (S_{is})

Group of students (s)	Course(i)
IE 4th year	614442, 614351, 614362, 614413, 614442
IML 3nd year	614331, 614101, 614322, 614341, 614344, 619352, 619353, 619311

We want to solve a course timetabling problem where all courses must be assigned to class rooms.

Each class room in a period can be used for only one course and the faculty member who teaches the course cannot lecture another course at the same time. At a particular period time, each group of students can register only one course. The number of students in each class rooms is limited by class room capacity. The objectives are to minimize cost and extra-period studying. The problem is formulated in a mathematical model.

The mathematical model presented below determines which course $\{i=1,\ldots,I\}$ and class room $\{j=1,\ldots,J\}$ have to be assigned to a period $\{k=1,\ldots,K\}$. The model uses a binary decision variable (x_{ijk}),

The integer linear programming problem for this problem will be defined using the following notations:

Indices

r = Faculty member , r = 1, 2, …, R
s = Group of students , s = 1, 2, … S
i = Course , i = 1, 2, …, I
j = Class room , j = 1, 2, …, J
k = Period , k = 1, 2, …,K

Parameters

F_j = capacity of class room j
N_i = the registered student of course i
A_{ir} = the course i taught by faculty member r
B_{is} = the course i studied by a group of students
T_k = weight of the extra-period
C_j = Classroom Cost j

Decision Variables

$$X_{ijk}=\begin{cases}1 & \text{if course i taught in classroom j in period k}\\ 0 & \text{otherwise}\end{cases}$$

2) Objective Function

Minimizing Classroom cost

$$f_1(x) = ((\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} C_j X_{ijk}) - Tar1)/Tar1 \quad (1)$$

Minimizing study in the extra-period

$$f_2(x) = ((\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} T_k X_{ijk}) - Tar2)/Tar2 \quad (2)$$

This problem is multi objective problem which is designed to minimize classroom cost and the extra-period of studying. Function (1) represents a formulation to transform the cost in a proportion. Tar1 is a target value which derived from the optimal solution when only one objective function, minimizing cost is considered. And Tar2 is the optimal solution obtained from minimizing the extra-period study. In Function (2), the weight of each period, T_k is added to obtain the solution. The extra-period will have higher weight level than the general period in order to minimize selecting the extra-period.

Formulation (3) shows the multi objective problem which is linear function to minimize the weighted percentage deviation from the goals' target value.

$$\text{Min} \quad \omega_1 f_1(x) + \omega_2 f_2(x) \quad (3)$$

The weights assigned to ω_1 and ω_2 to set the priority of each objective. This problem sets $\omega_1 + \omega_2 = 1$

3) Constrained

$$\sum_{i=1}^{I} X_{ijk} \le 1 \qquad ; \forall j, \forall k \quad (4)$$

Constraints (4) force all classrooms in each period to be assigned to at least one task.

$$\sum_{j=1}^{J}\sum_{k=1}^{K} X_{ijk} = 1 \qquad ; \forall i \quad (5)$$

Constraints (5) represent that all courses have to be assigned in the timetable.

$$N_i X_{ijk} \le F_j \qquad ; \forall i, \forall j, \forall k \quad (6)$$

Constraints (6) limit the capacity. The number of students in each class rooms has to less than classroom capacity.

$$\sum_{i=1}^{I}\sum_{j=1}^{J} X_{ijk}A_{ir} \leq 1 \quad ; \forall k, \forall r \qquad (7)$$

Constraint (7) represents that the faculty member who teaches a course cannot lecture another course at the same time.

$$\sum_{i=1}^{I}\sum_{j=1}^{J} X_{ijk}B_{is} \leq 1 \quad ; \forall k, \forall s \qquad (8)$$

Constraint (8) force that each group of student can register a course at a period time.

$$x_{ijk}=\{0,1\} \quad ; \forall i, \forall j, \forall k \qquad (9)$$

Case Study

A course timetabling problem of Industrial Engineering Department, Silpakorn University is a case problem. There are j = 10 classrooms available to be assigned. Two classrooms are located inside the Industrial Engineering Department, and the other 8 classrooms are in the central area of the Faculty of Engineering. Scheduler has to assign forty eight courses (i) to classrooms (j) and time slots (k). Faculty member teaches from Monday to Friday, 9:25 AM - 6:25 PM. There are three time slots 9:25 AM - 12:05 PM, 1:00 PM – 3:40 PM and 3:45 PM – 6:25 PM. This is a timetabling problem for 1st year student to 4th year Industrial Engineering (IE) students and Industrial Management and Logistics (IML) students in year 2010.

Table 2 shows the cost and capacity of the available classrooms. List of courses, the faculty members, groups of students and the number of registered students in each group are given as shown in Table 3. Table 4 presents the courses (i) which are taught by each lecturer. Table 5 shows the courses that each group of students will study. Two objectives are to minimize cost and extra-period studying. The priority of each objective is equal, which means that the issues of cost and extra-period are equally important, so the weight 0.5 is assigned to ω_1 and ω_2.

Computational Result and Conclusion

To test the improvement of the optimal solution obtained from the mathematical model by using IBM ILOG CPLEX 12.2 software, it is compared to the solution which is from the practical application. We test the problem size (i × j × k) of 8,640 variables. Table 6 - Table 11 presents examples of the solution. The course 614323, 614437, 614445, 614395, 619352 and 619313 are assigned in extra period of classroom no. 515 and 513. The total cost of the proposed method is compared to the solution obtained in the practical application. It was found that the proposed method can improve the cost 8,115 Baht/ week and the extra-periods were reduced by 4 periods as shown in Table 7.

Table 6 Timetable of classroom no. 515

	9:25 AM - 12:05 PM	1:00 PM - 3:40 PM	3:45 PM - 6:25 PM
Mon	614202	619316	614323
Tue	614101	614413	-
Wed	614301	619492	614432
Thu	614101	614432	614445
Fri	619314	614362	-
Sat	-	-	614395

Table 7 Timetable of classroom no. 513

	9:25 AM - 12:05 PM	1:00 PM - 3:40 PM	3:45 PM - 6:25 PM
Mon	614331	614231	619352
Tue	614454	614344	619313
Wed	619353	619311	-
Thu	614203	614322	-
Fri	614101	614341	-
Sat	-	-	-

Table 8 Timetable of classroom no. 135

	9:25 AM - 12:05 PM	1:00 PM - 3:40 PM	3:45 PM - 6:25 PM
Mon	614394	614343	-
Tue	614330	614352	-
Wed	-	614442	-
Thu	614351	614351	-
Fri	614452	614322	-
Sat	-	-	-

Table 9 Timetable of classroom no. 142-3

	9:25 AM - 12:05 PM	1:00 PM - 3:40 PM	3:45 PM - 6:25 PM
Mon	-	614211	-
Tue	614211	614232	-
Wed	614101	614212	-
Thu	614213	619254	-
Fri	614291	614232	-
Sat	-	-	-

Table 10 Timetable of classroom no. 144-6

	9:25 AM - 12:05 PM	1:00 PM - 3:40 PM	3:45 PM - 6:25 PM
Mon	619211	-	-
Tue	-	619251	-
Wed	-	614111	-
Thu	-	-	-
Fri	-	619253	-
Sat	-	-	-

Table 11 The comparison of the quality of the solution

	Total Cost (baht)	No. extra period
The practical method	18,615	10
The proposed method	10,500	6

Conclusion and Suggestion

We have described a model that can be used to schedule courses in universities. The of timetable achieved by use the model comparing to the solution in the practical application.

This study assumes one to one assignment between a course and faculty member. In further study, the constraint of a course having more than one faculty member will be considered for more practical approach.

Problem concerns multi objectives which are to minimize cost and extra-period studying. A mathematical model of the timetabling problem is proposed. There is an improvement

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