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Numerical Solutions of Fractional Covid-19 Model Using Spectral Collocation Method

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ABSTRACT

In this work, the mathematical model is considered on COVID-19 which makes the lives of people in the world into a hell. This present model has four components that are expressed as susceptible, exposed, infected and recovered (SEIR). Spectral collocation method (SCM) is presented here for numerical simulations because it is one of the important numerical technique having high rate convergence. Also, convergence analysis of the above method is presented here briefly. There is detailed description about the comparision of the rate of increasing of COVID-19 of India, Srilanka, Pakistan, Bangladesh respectively. If the four components are considered as zero initially, the effect of population to increase the disease is presented here.

Keywords: Convergence analysis; COVID-19; Mathematical modeling; Spectral collocation method

1. Introduction

The fractional calculus (FC) is an important part of calculus with the applications of fractional order integration and differentiation. The use of classical calculus and fractional calculus was initiated almost simultaneously. The idea of FC was first introduced by Leibniz and L'Hospital in 1695 as a semi derivative. Then FC was developed by many famous mathematicians like Euler, Laplace, Riemann, Letnikov and many others. Now a days, we can say that there is no such disciplines in science, engineering, finance without the concept of FC. There are so many efficient methods in FC to express non- linear phenomena in the fields of physics and engineering [1]. Also, FC is used to express real life problems which have huge applications in so many field like economics [2], biology [3], digital circuits [4], cryptography [5] etc. Some recent applications of FC are described in

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[6, 7].

The disease caused by Coronavirus is called COVID-19. Now, CO stands for the first two letters of Corona, VI is for first two letters of Virus and D comes for disease. Here, 19 means 2019, as this disease was first seen in Wuhan in December 2019. It is pendemic disease which is announced by WHO [8]. Now, it has spread quickly across the world. As there is still now no vaccine for it, social distancing is the most important way for controlling this disease [9]. There are so many countries those are badly affected by Coronavirus around the world [10, 11]. Some literatures are published on the epidemic characteristics of COVID-19 based on Wuhan, China [12, 13]. Special attention and care should be taken for the patients who has diabetes and COVID-19 [14].

Respiratory illness is caused by this disease with fever and cough. Difficulty in breathing is another most important symptoms in severe cases. We can keep safe ourself by washing our hands, not to touch our face, and keeping away from people who are already effected. Older people having high blood pressure, heart problems or diabetes are more likely to suffer serious illness.

This disease spreads basically through contact with an infected person when they sneeze or cough. The spreading rate is also increases due to touch a surface or object where this virus is already exist. Then if any people touches their eyes, nose, or mouth, got infected.

Still now there is some research articles to find the characteristics of infected patients with COVID-19. Around 41 cases were announced by Huang C in Wuhan in the Lancet on Jan 24, 2020. Yueling Ma et al. described the effects of humidity and temperature on the death of COVID-19 collecting the data from 20 January, 2020 to 29 February, 2020 in Wuhan, China. In that work, author has applied Generalized additive model to get the results in [15]. Li Q has also reported 425 cases in Wuhan on Jan 29,2020. The clinical characteristics of death cases with COVID-19 was presented by Xun Li et al. which helps to find critically ill patients early in [16]. Chen N presented characteristics of around 99 cases in Wuhan on Jan 30, 21,020. There are some reseraChers who has implemented a conceptual model for the coronavirus disease 2019 (COVID-19) in Wuhan, China [17]. Also Transmission network model is developed by TianMu Chen et al. in [18]. The details about COVID - 19 is described in [19–25]. There is a brief description about the effects of this disease in [26–33]. Some important results about this model are presented in [34–36]. This model simulate transmission from the bats infection to the human infection. In this work, we have considered COVID-19 model [37] as Caputo sense like

$$D_t^{\alpha} S(t) = a_1 - k_1 I(t) S(t)(1 + \alpha I(t)) - d_1 S(t). \quad (1.1)$$

$$D_t^{\alpha} E(t) = k_1 I(t) S(t)(1 + \alpha I(t)) - (d_1 + \gamma_1) E(t). \quad (1.2)$$

$$D_t^{\alpha} I(t) = b_1 + \gamma_1 E(t) - (d_1 + \beta_1 + \mu_1) I(t). \quad (1.3)$$

$$D_t^{\alpha} R(t) = \beta_1 I(t) - d_1 R(t). \quad (1.4)$$

The parameters which are used in the above COVID-19 model are described in Table1 in detail. In this case, the Caputo fractional operator is used due to it's good data fitting properties. Also, It is easy to use.

Spectral method is one of the high convergent numerical method which is applied mainly in applied mathematics. The basis function is applied due to it's orthogonality properties for approximating the equations to get the solutions of the differential equations. The spectral-collocation method using Jacobi polynomial is presened by Yanping Chen et al. [38] with it's convergence analysis. Also, solution of stochastic delay differential equations using spectral collocation method is described in [39] with the interpolation of Chebyshev polynomials. This method is presented to find the solutions of linear and non-linear fractional-order differential equations in [40]. In the present study, for approximating variable order fractional partial differential operator, authors has utilized Lagrange interpolation. Also a spectral pelanty method is implemented to explain the convergency of this method. Spectral method is developed to get solution of convection-reaction-diffusion equations having time fractional with the help of finite difference method [41]. Also, in this work stability and convergence analysis of the method are described briefly to get accurate and efficient results. The solution of nonlinear fractional differential equations is reported in [42] using spectral method.

The present work is sorted out as: In Section 2, the defination of fractional operator in Caputo sense is described. Then we have discussed about Chebyshev polynomial of 1st kind with it's shifted forms in Section 3. There are some statements on convergence analysis of the described method in Section 4. In Section 5, detail about procedure of the method is presented. Also numerical simulations are illustrated briefly in Section 6. The conclusion statements are in Section 7.

2. Definitions and Preliminaries

Definition 2.1 ([43]). The fractional differential operator D^{α} of order α in Caputo sense is expressed by

$$D^{\alpha} f(\eta) = \frac{1}{\Gamma(m-\alpha)} \int_0^{\eta} \frac{f^{(m)}(t)}{(\eta - t)^{\alpha-m+1}} dt, \tag{2.1}$$

where $m-1 < \alpha < m$, $m \in \mathbb{N}$, $\alpha > 0$, $\eta > 0$.

Caputo fractional derivative operator satisfies the linearity property like:

$$D^{\alpha}(\lambda f(\eta)+\mu g(\eta)) = \lambda D^{\alpha} f(\eta)+\mu D^{\alpha} g(\eta).$$

and it confirms the following property [44]

$$\begin{aligned} D^{\alpha} C &= 0, \text{where C is a constant,} \\ D^{\alpha}\eta^{n} &= \begin{cases} 0, & n < \lceil\alpha\rceil; \\ \frac{\Gamma(n+1)}{\Gamma(n+1-\alpha)}\eta^{n-\alpha}, & n \geq \lceil\alpha\rceil. \end{cases} \end{aligned}$$

Where the smallest integer greater than or equal to α is denoted by $\lceil\alpha\rceil$ and $n \in \mathbb{N}_0$, with $\mathbb{N}_0 = 0, 1, 2, \cdots$.

3. Chebyshev Spectral Method

The Chebyshev polynomials (CP) of first kind are explained on the interval $[-1, 1]$. This kind of CPs are expressed using the below recurrence relation [45] as

$$\zeta_{k+1}(\eta) = 2\eta\zeta_k(\eta) - \zeta_{k-1}(\eta), \quad k = 1, 2, 3, ...,$$

where $\zeta_0(\eta) = 1$ and $\zeta_1(\eta) = \eta$. Now, we want to use these polynomials on the interval $[0, L]$ as shifted CPs. The transformation $\eta = \frac{2t}{L} - 1$ is taken for defining those kind of polynomial in the interval $[0, L]$. These kind of polynomials are denoted by $\zeta_k(\frac{2t}{L} - 1)$ with the recurrence relation as

$$\zeta^*_{k+1}(\eta) = 2(\frac{2t}{L} - 1)\zeta^*_k(\eta) - \zeta^*_{k-1}(\eta), \quad k = 1, 2, 3, ..., \tag{3.1}$$

where $\zeta_0^*(\eta) = 1$, $\zeta_1^* = \frac{2t}{L} - 1$. The analytic form of shifted CPs $\zeta_n^*(t)$ of degree n is defined by

$$\zeta_n^*(t) = n\sum_{i=0}^{n}(-1)^{n-i}2^{2i}\frac{\Gamma(n+i)}{\Gamma(2i+1)\Gamma(n-i+1)L^i}t^i. \tag{3.2}$$

With $\zeta_n^*(0) = (-1)^n$, $\zeta_n^*(1) = 1$. Now, the function $S(t)$ is defined in $[0, L]$ and a square integrable function on that interval. So, $S(t)$ can be expressed in terms of shifted CPs of first kind as:

$$S(t) = \sum_{i=0}^{\infty} d_i\zeta_i^*(t). \tag{3.3}$$

Now, for numerical approximation, only the first $(m + 1)$ terms of shifted CPs are considered. Hence, we get

$$S_m(t) = \sum_{i=0}^{m} d_i\zeta_i^*(t). \tag{3.4}$$

$$D^\alpha S_m(t) = \sum_{i=0}^{m} d_i D^\alpha(\zeta_i^*(t)). \tag{3.5}$$

So, for $i = 1, 2, ..., m$,

$$\begin{aligned} D^\alpha(\zeta_i^*(t)) &= i\sum_{k=\lceil\alpha\rceil}^{i}(-1)^{i-k}2^{2k}\frac{\Gamma(k+i)}{(2k)!(i-k)!L^i}D^\alpha t^k, \\ &= i\sum_{k=\lceil\alpha\rceil}^{i}(-1)^{i-k}2^{2k}\frac{\Gamma(k+i)t^{k-\alpha}}{(i-k)!(k-\alpha)!(2k)!L^i}. \end{aligned} \tag{3.6}$$

Then

$$D^\alpha S_m(t) = \sum_{i=0}^{m} d_i\Xi_{i,k}\,, \tag{3.7}$$

where $\Xi_{i,k}$ is presented by

$$\Xi_{i,k} = i\sum_{k=\lceil\alpha\rceil}^{i}\frac{(-1)^{i-k}2^{2k}\Gamma(k+i)t^{k-\alpha}}{(i-k)!(k-\alpha)!(2k)!L^i}. \tag{3.8}$$

4. Convergence analysis

Theorem 4.1 ([46]). *Suppose that the function $S(t)$ is so constrained that $S''(t) \in L_2[0, L]$ and $|S''(t)| \leq \epsilon$, where ϵ is a constant. Then the series (3.7) of the shifted Chebyshev expansion is uniformly convergent and:*

$$|d_l| < \frac{\epsilon}{l^2}, \quad (l \in 1, 2...) \tag{4.1}$$

Theorem 4.2 ([46]). *Suppose that $S(t) \in C^m[0, 1]$. Then the error in approximating the function $S(t)$ by $S_m(t)$ bu using the formula (3.7) can be bounded by:*

$$\begin{aligned} &||S(t) - S_m(t)|| \leq \frac{\rho\Delta^{(m+1)}}{(m+1)!}, \\ &where \quad \rho = max_{t\in[0,1]}S^{(m+1)}(t), \\ &(\Delta = max\{t, t - t_0\}). \end{aligned} \tag{4.2}$$

5. SCM for Solving Fractional Order COVID-19 Model

Now, SCM with the help of shifted CPs will be used for solving the systems of fractional order differential equations (1.1) to (1.4) with the given initial conditions. For applying this technique, we have to approximate $S(t)$, $E(t)$, $I(t)$, and $R(t)$. So, we have

$$S_m(t) = \sum_{i=0}^{m} d_i\zeta_i^*(t), \tag{5.1}$$

$$E_m(t) = \sum_{i=0}^{m} e_i\zeta_i^*(t), \tag{5.2}$$

$$I_m(t) = \sum_{i=0}^{m} f_i\zeta_i^*(t), \tag{5.3}$$

$$R_m(t) = \sum_{i=0}^{m} g_i\zeta_i^*(t). \tag{5.4}$$

Now, there are $m+1-\lceil\alpha\rceil$ collocation points t_p. Using the collocation points, the above system of equations can be approximated as:

$$\sum_{i=0}^{m}\sum_{k=\lceil\alpha\rceil}^{i} d_i\Xi_{i,k}t_p^{k-\alpha} = a - k\sum_{i=0}^{m} f_i\zeta_i^*(t_p) \sum_{i=0}^{m} d_i\zeta_i^*(t_p)(1+ \alpha\sum_{i=0}^{m} f_i\zeta_i^*(t_p))- d_0\sum_{i=0}^{m} d_i\zeta_i^*(t_p), \quad (5.5)$$

$$\sum_{i=0}^{m}\sum_{k=\lceil\alpha\rceil}^{i} e_i\Xi_{i,k}t_p^{k-\alpha} = k\sum_{i=0}^{m} f_i\zeta_i^*(t_p) \sum_{i=0}^{m} d_i\zeta_i^*(t_p)(1+ \alpha\sum_{i=0}^{m} f_i\zeta_i^*(t_p))- (d_0+\gamma)\sum_{i=0}^{m} e_i\zeta_i^*(t_p), \quad (5.6)$$

$$\sum_{i=0}^{m}\sum_{k=\lceil\alpha\rceil}^{i} f_i\Xi_{i,k}t_p^{k-\alpha} = b + \gamma\sum_{i=0}^{m} e_i\zeta_i^*(t_p)- (d_0+\mu+\beta) \sum_{i=0}^{m} f_i\zeta_i^*(t_p). \quad (5.7)$$

$$\sum_{i=0}^{m}\sum_{k=\lceil\alpha\rceil}^{i} g_i\Xi_{i,k}t_p^{k-\alpha} = \beta\sum_{i=0}^{m} f_i\zeta_i^*(t_p)- d_0\sum_{i=0}^{m} g_i\zeta_i^*(t_p). \quad (5.8)$$

In this case, the roots of the shifted CPs $T_m^*(t)$ are taken as collocation points. Now, we get utilizing the initial conditions

$$\sum_{i=0}^{m}(-1)^i d_i = \delta_1. \quad (5.9)$$

$$\sum_{i=0}^{m}(-1)^i e_i = \delta_2. \quad (5.10)$$

$$\sum_{i=0}^{m}(-1)^i f_i = \delta_3. \quad (5.11)$$

$$\sum_{i=0}^{m}(-1)^i g_i = \delta_4. \quad (5.12)$$

So, there are total $4m + 4$ no of non-linear system of algebraic equations where the degree of shifted Chebyshev polynomials is denoted by m. Newton's iteration method is used for solving those kind of system of euations to get the values of unknowns d_i, e_i, f_i and g_i, $i = 0, 1, 2, ..., m$.

6. Numerical Results and Discussion

In this section, we shall discuss the numerical simulation of the given COVID 19 model. Here, spectral collocation method is used for approximating the solution. From Figs. 1-4, we have plotted susceptible components, exposed components, infected components, recovered components for different values of $\alpha = 1, 0.9, 0.8, 0.7$ respectively with the given initial values. Also, the behaviour of different components for different values of $\alpha = 1, 0.8, 0.6, 0.4$ are presented from From Figs. 5-8.

Now, the values in Table1 which is used here are taken from [37]. The data from Table 2 from WHO [47] express the values of different components in different time for India, Pakistan, Bangladesh, Srilanka. It shows that susceptibility is reducing which produced the increasing of exposures. So, the infection class is increasing.

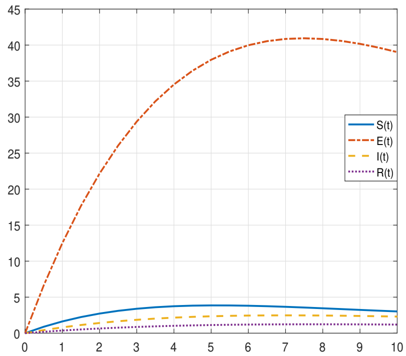

Fig. 1. Plots of S(t), E(t), I(t) and R(t) of the COVID-19 model vs. time for $\alpha = 1$.

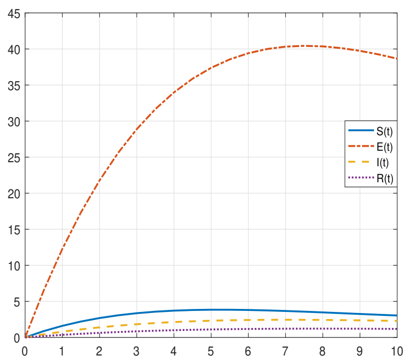

Fig. 2. Plots of S(t), E(t), I(t) and R(t) of the COVID-19 model vs. time for $\alpha = 0.9$.

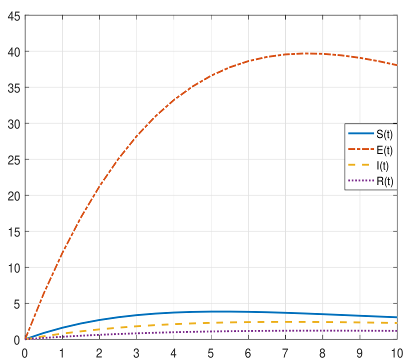

Fig. 3. Plots of S(t), E(t), I(t) and R(t) of the COVID-19 model vs. time for $\alpha = 0.8$.

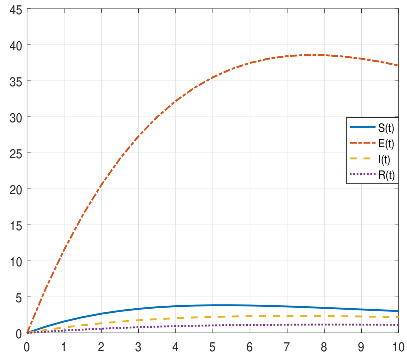

Fig. 4. Plots of S(t), E(t), I(t) and R(t) of the COVID-19 model vs. time for $\alpha = 0.7$.

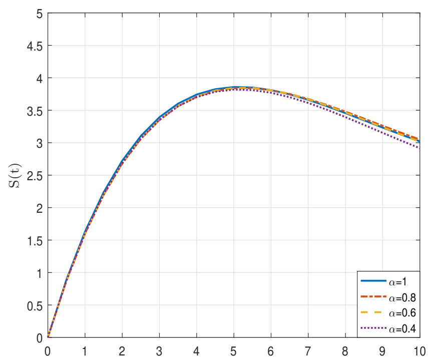

Fig. 5. Plots of S(t) of the COVID-19 model for different values of α.

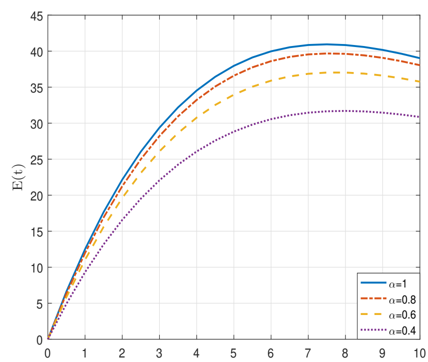

Fig. 6. Plots of E(t) of the COVID-19 model for different values of α.

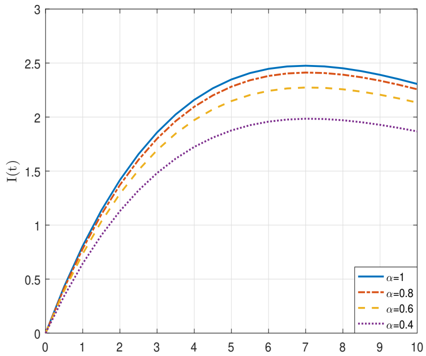

Fig. 7. Plots of I(t) of the COVID-19 model for different values of α.

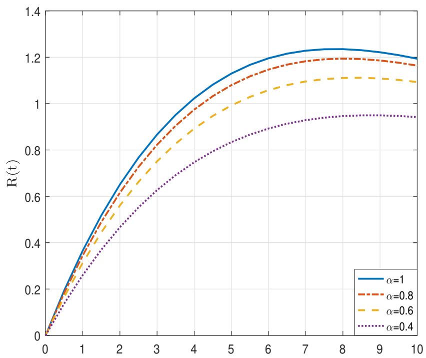

Fig. 8. Plots of R(t) of the COVID-19 model for different values of α.

Also, the cure and the recovery class also are increasing because of more death cases.

In Figs. 9-12, the results for the four different countries are replicated. These show that the precipitability ratio of India is faster than the other countries due to large papulation as in 9, Pakistan is in second and so on. Here, the susceptible compartment is decreasing with the increasing of exposing. Hence, the rate of infection is increasing because of large papulation of India. Srilanka has less infection flow than Pakistan, Bangladesh as shown in Figs. 10-12 respectively. Also susceptible, exposed, infected and recovered class for different countries are plotted from Figs. 13-16.

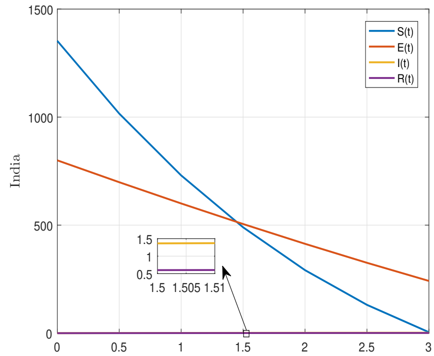

Fig. 9. Plots of S(t), E(t), I(t) and R(t) for the given initial values for India.

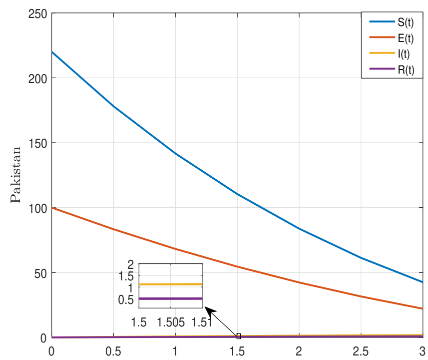

Fig. 10. Plots of S(t), E(t), I(t) and R(t) for the given initial values for Pakistan.

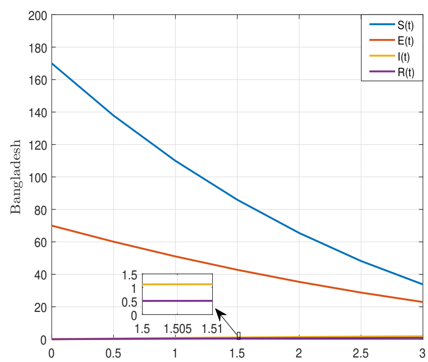

Fig. 11. Plots of S(t), E(t), I(t) and R(t) for the given initial values for Bangladesh.

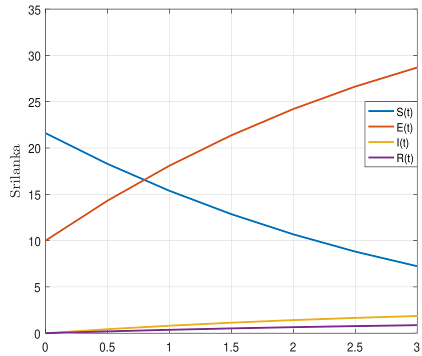

Fig. 12. Plots of S(t), E(t), I(t) and R(t) for the given initial values for Srilanka.

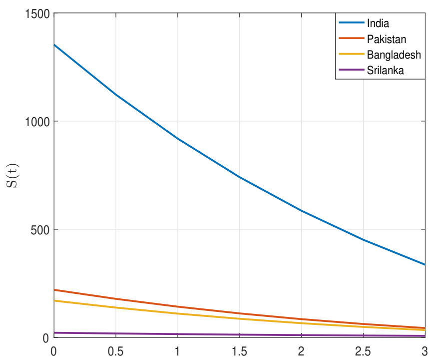

Fig. 13. Plots of S(t) of the COVID-19 model for the given different initial values.

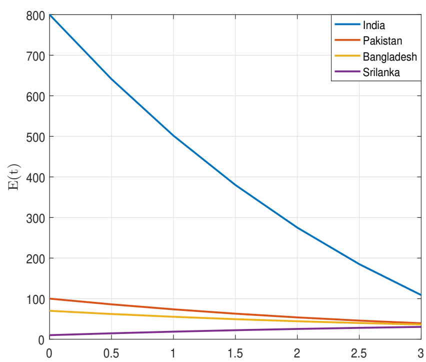

Fig. 14. Plots of E(t) of the COVID-19 model for the given different initial values.

Table 1. Explanation of the parameters with numerical values.

Parameters	Physical interpretation	Values
a_1	Negative cases population	0.73 $Millons$
d_1	Natural death	0.02
μ_1	Death due to Covid	0.0009
b_1	Population with positive cases	0.06003
α	Individuals lose immunity	0.00009
k_1	Constant of proportionality	0.098601
γ_1	The rate of infection	0.00007
β_1	The rate of recovered	0.01

Table 2. Explanation of the parameters with numerical values.

Parameters	Numerical Values
$S_0(t)$	$1353, 220, 170, 21.6\ Ms$
$E_0(t)$	$800, 100, 70, 10\ Ms$
$I_0(t)$	$0.027977, 0.013328,$ $0.005149, 0.000523\ Ms$
$R_0(t)$	$0.007407, 0.003310,$ $0.000267, 0.000127\ Ms$

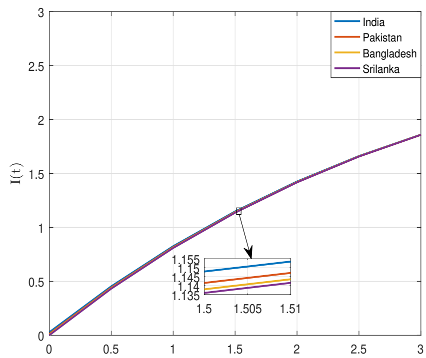

Fig. 15. Plots of I(t) of the COVID-19 model for the given different initial values.

7. Conclusion

COVID-19 model having four components is described here. Also, the solutions are presented using one of the well known numerical technique spectral collocation method. Further, we have simulated

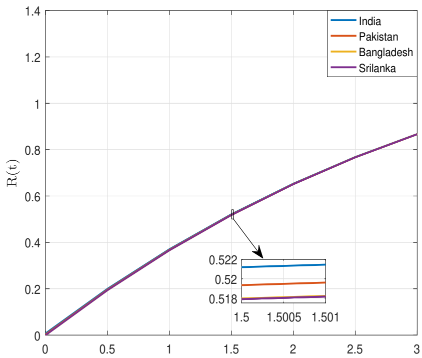

Fig. 16. Plots of R(t) of the COVID-19 model for the given different initial values.

the results for the given different initial conditions with diffrent values of fractional order α. Also, from the graphical representations, it can be said that how the rate of COVID-19 is increasing of India, Srilanka, Pakistan, Bangladesh respectively. At last, we have compared the results with the real data for four different countries. It is presented that the infection spread rates are different for these four countries. We also concluded that spreading rate of India and Pakistan is faster in comparision with other two countries, Bangladesh and Srilanka because of having huge population. In future, We can use this method to solve COVID-19 models having five or six components to get better results.

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