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Original research article

Using Adaptive Integral Gain for Overshoot Reduction in PID Control Systems

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ABSTRACT

This paper examines intrinsic characteristics of the integral control of a PID controller, and points out one that can cause excessive overshoot when the reference signal changes abruptly. Reasons for not totally relying on the derivative control to suppress overshoot are discussed, and then augmentation of an adaptive integral gain to an existing PID controller is proposed. The associated smooth adaptive law for the gain is presented, with the correspondingly allowable upper and lower bounds that guarantee input-to-state stability for the system of interest. Effectiveness, simplicity, and desirable properties of the proposed adaptive integral gain are clearly shown in two design examples. Numerical simulations show that maximum overshoot can be reduced by approximately 50%, without upsetting rise time.

Keywords: Adaptivel; Integrator; Overshoot; Stability; PID

1. Introduction

For decades, PID controllers and their variants have been successfully applied to many types of systems found in various industries [1-5]. In simple applications of these controllers, parameters of the systems of interest are treated as constants. This approach facilitates the controller designs, in which controller parameters are tuned and fixed at appropriate values. However, it is certain that system parameters can change as the control systems operates over time. Depending on characteristics of these parameter variations, performance of the control systems could be degraded in various aspects. These include excessive rise time, overshoot, and steady state error.

In a PID controller, the integral control is usually responsible for eliminating or reducing steady-state error due to disturbances and parameter variations [6]. This property is extremely useful in important practices such as precision

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machining, and robotics [7]. However, it is a widely known fact that using the integral control reduces relative stability of the control system [4]. Because of this, it commonly appears in some situations such as when reference signal changes abruptly that outputs of the control system exhibit large overshoots. The latter undesirable characteristic could be well suppressed by the derivative component of the controller. However, it appears in real applications that the derivative signal is very sensitive to noises and measurement errors. Accordingly, using a large derivative gain to suppress excessive overshoot could mean introducing large noises into the control system and degrading its performance. A large derivative gain could also increase rise time accordingly. These intrinsic properties of the derivative component prohibit its excessive uses in practices.

A solution for this problem is to introduce adaptive capabilities to the controllers so that they could enhance compensation for disturbances and parameter variations in the systems. Depending on the system of interest, an adaptive PID controller could vary a combination of its principal parameters, which can be the proportional gain, the integral gain, the derivative gain, and others [5], [8-9]. Some adaptive PID control systems can incorporate reference model [10], or feedforward control [11] in their designs. When intended ranges of controller parameter variations can be estimated, it is generally most desirable to assert a type of stability for the resulting control systems. Only after stability is guaranteed do we seek further to improve performance. The latter is usually represented in terms of bounds on state vectors in many researches. See [6], [12-13] and references therein. While such bounds could be drawn naturally from the influential and powerful theorem of Lyapunov stability, the results are usually associated with significant conservatism. Accordingly, they are rarely good indicators for important transient response characteristics such as rise time and overshoot. It is then difficult to see how the resulting adaptive PID controllers could make these transient characteristics better than those corresponding to static PID controllers.

When compared to the above existing techniques, ours is original in the sense that it could be employed to reduce overshoot in existing PID control systems without increasing rise time when reference signals change abruptly. It does so by augmenting an adaptive law to the integral gain of the controllers. Because the derivative signal is not involved, our technique is insensitive to noises and measurement error. It also yields desirable smooth control law. In effect, however, our technique transforms static PID controllers to be the corresponding adaptive controllers. Stability of the resulting control systems can be guaranteed for a determined range of the adaptive integral gain by using an existing robust stability theorem. The flexibility of beginning from existing controllers allows us to preserve their desirable properties, which may not simple to obtain. This could reduce design time and effort significantly. Our adaptive scheme is derived from an observation in time domain. It is then simple to see how it could reduce overshoot without increasing rise time. The adaptive law requires very little computational resource, making it practical to implement in real time.

2. Adaptive Law for the Integral Gain

Consider a dynamic system whose error dynamics can be reasonably modeled as:

$$\dot{x} = Ax + Bu + w(t) \tag{2.1}$$

where $x \in \Re^n$ is the state vector, the system matrix $A \in \Re^{n \times n}$ is known, the input matrix $B \in \Re^{n \times m}$ is known, $u \in \Re^m$ is the control input vector, and $w(t) \in \Re^n$ is the bounded time-varying perturbation vector. The control vector u is computed from the control law $u = -Kx$, in which $K \in \Re^{m \times n}$. We denote an element of K by $k_{ij} \in \Re \ \forall i \ \forall j$. Notice that

$$u_i = \sum_{j=1}^{n} u_{ij} = -\sum_{i=j}^{n} k_{ij} x_j \qquad (2.2)$$

where u_i is the i-th component of u, and u_{ij} is the j-th component of u_i. To facilitate our discussion, the model is arranged so that $x_1 = \int_0^t e(t)dt$, $x_2 = e(t)$, and $x_3 = \dot{e}(t)$, in which $e(t) = r(t) - y(t)$ is the error vector, r(t) is the reference vector, and y(t) is the output vector. Using this arrangement, we have $u_i = u_{i1} + u_{i2}$ for PI control, and $u_i = u_{i1} + u_{i2} + u_{i3}$ for PID control. Note that the number of states n need not be limited by 3. For clarity in some parts of our discussion, we call $u_{i1} = u_{iI}$ the integral control, $u_{i2} = u_{iP}$ the proportional control, $u_{i3} = u_{iD}$ the derivative control, $k_{i1} = k_{iI}$ the integral gain, $k_{i2} = k_{iP}$ the proportional gain, and $k_{i3} = k_{iD}$ the derivative gain. We are concerned with the assumption that a constant gain matrix $K = K_c$ is already obtained by a designer such that the resulting control system is stable, and is associated with satisfactory response characteristics other than having excessive overshoot. The designer then wants to reduce overshoot without using large derivative gains because this can adversely increase rise time and introduce large noise into the system. Note that we limit our discussion for the cases in which the integral gain k_{iI} and the proportional gain k_{iP} have the same sign.

In a PI control system, it is known that the integral control is primarily employed to reduce or to eliminate steady-state error due to unknown disturbances. However, it can simultaneously reduce relative stability of the resulting control system, causing excessive overshoot when the reference signal changes abruptly. A known effective solution for this is to incorporate the derivative control to obtain the well-known PID control law. While the derivative control can decrease or even eliminate overshoot, it can simultaneously increase rise time significantly. For some demanding applications, such as PCB fabrications in which a large number of parts must be placed at various positions very rapidly, it is most preferable that the motion control system should be associated with both small rise time and small overshoot when reference position changes abruptly. Discussion of important characteristics of PID control systems and their variants can be found in the literature, in particular, see [4].

Generally, a sole objective of a control system is convergence of e(t) to the origin. The proportional control is then regarded as the principal control component in the sense that it is the only control component that depends explicitly on e(t). On the other hand, the integral control is regarded as supplementary component because we do not require $\int_0^t e(t)dt$ to converge to the origin. By definition, the state variable $\int_0^t e(t)dt$ depends on a long chain of historical values of e(t). It rarely gives any information of the present state of e(t). Accordingly, the corresponding integral control component does not contribute to convergence of e(t) at all times. It is precisely this property that causes the undesirable overshoot mentioned previously. To see this, consider the PID position control system for a Maxon DC motor discussed in Section 4. This is a single-input single-output system, for which we define shaft angular displacement $\theta_M(t)$ as output y(t), and r(t) as reference signal. The control system is designed in the manner that positive control voltage drives y(t) in the positive direction (CCW), while negative control voltage does the opposite. With e(t) = r(t) – y(t), it follows that positive control voltage decreases e(t) and vice versa. For a step reference signal, we employ numerical simulations to obtain e(t) as shown in Fig. 1. Shown in the same figure are the control components u_I, and u_P. The control component u_D is not shown because its magnitude is negligible when compared to those of u_I, and u_P. Starting from t = 0 with e(0) = 10, u_P is positive while u_I increases from zero to positive values. The signs of these control components are both positive during the first two seconds, and their combined effort drives e(t) to decrease rapidly. After t = 2 s, the sign of e(t) becomes negative, and so does that of u_P. Accordingly, u_P now drives e(t) to increase back to zero. However, u_I remains positive, and continues to drive e(t) to decrease further to more negative values. Clearly, this undesirable characteristic of u_I can cause excessive overshoot.

To reduce the above undesirable overshoot due to the integral control, we propose an adaptation law for the integral gain. According to the above observation in time domain, we see that overshoot can be decreased by decreasing the magnitude of the integral gain when the signs of $x_1 = \int_0^t e(t)dt$ and $x_2 = e(t)$ are opposite. On the other hand, the integral gain should be restored to its nominal value when the signs are the same. Noticing this, we write an adaptive integral gain k_I as:

$$k_I = k_{Ia} + k_{Ic} \qquad (2.3)$$

where k_{Ia} is the adaptive part of the integral gain of interest, and k_{Ic} is the constant part of the integral gain whose value is available by our assumption.

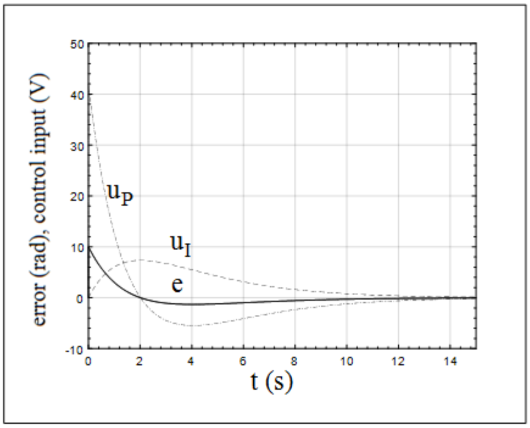

Fig. 1. Integral control is slow to react to error, causing overshoot.

Now, a smooth adaptation law that encapsulates the above required characteristics is given by:

$$\dot{k}_{Ia} = \beta k_{Ic} \tanh(x_1 x_2) \qquad (2.4)$$

where $\beta \in \Re^+$ is the parameter that governs adaptation rate. Figure 2 depicts the adaption law. Note that $\dot{k}_I = \dot{k}_{Ia} + \dot{k}_{Ic} = \dot{k}_{Ia}$. The feedback gain matrix K can now be written as:

$$K = K_a + K_c \qquad (2.5)$$

where K_a is the adaptive part of the gain matrix, and K_c is the known constant gain matrix.

For multiple-input multiple-output (MIMO) systems, tuning controller parameters is usually much more difficult than that for single-input single-output (SISO) systems because of multiple coupling effects among the inputs and outputs. However, our adaptation law remains convenient to apply in the sense that there is no need to apply it to all the integral gains simultaneously. Indeed, it is typical to find that an output exhibits excessive overshoot while others do not. In this case, we can apply the adaptation law to the integral gain that causes excessive overshoot of the output of interest only.

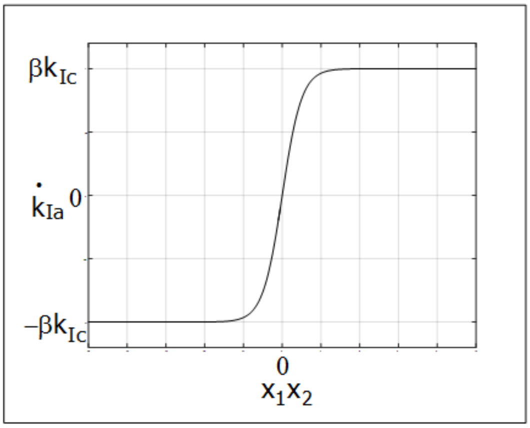

Fig. 2. Adaptation law as a function of $x_1 x_2$, assuming $k_{Ic} \in \Re^+$

Simplicity of the above adaptation law allows its real-time implementation. In addition, it yields a smooth control signal. The latter property is very desirable for motion control applications in which mechanical wears should be minimized. Clearly, the adaption law introduces nonlinear dynamics to the controller, and the resulting control system is now nonlinear. It is well known that stability of the system can no longer be guaranteed by merely confining all the eigenvalues of $A - BK$ in the LHP at all time. We now need an additional criterion that can be employed to guarantee stability of the system. This is addressed in the next section.

3. Stability

When the adaptive integral gain is incorporated, the resulting control law becomes nonlinear. In general, global stability of our now nonlinear control system cannot be addressed by using the concept of poles. We then require a special tool that allows us to guarantee this mandatory property. For this, we substitute $K = K_a + K_c$ in Eq. (2.1) and write:

$$\dot{x} = [A - BK_c]x - BK_a x + w(t) \quad (3.1)$$

The system can be written as a linear time varying uncertain system:

$$\dot{x} = \bar{A}x + \sum_{j=1}^{N}[h_j(t)E_j]x + w(t) \quad (3.2)$$

where $\bar{A} = A - BK_c$ is strictly Hurwitz by our assumption that a stabilizing K_c is known, $E_j \in \Re^{n \times n}$ is known, and $h_j(t) \in \Re$ is a time-varying function representing variation of an adaptive integral gain k_{Ia}. Note that $h_j(t)$ is associated with known strict upper bound $h_{uj} > h_j$ and strict lower bound $h_{lj} < h_j$. When the system is written in the form of Eq. (3.2), there are robust stability analysis (RSA) theorems that can be employed to assert exponential stability of the system of interest when the perturbation vector w(t) does not present. However, it can often be found that allowable bounds on $h_j(t)$ resulting from an RSA theorem are too conservative, and do not allow satisfactory suppression of overshoot. Here, we employ the RSA theorem in [14-15] because it is convenient to apply and is usually able to yield sufficiently large allowable bounds that lead to clear suppression of overshoot in our investigation. Because of its importance in our discussion, the theorem is reproduced for convenience of the reader in the following:

Theorem 1 [14-15] If the dynamical system in Eq. (3.2) is uniformly globally Lipschitz with $\bar{A}$ being Hurwitz, w(t) = 0, and $\max(\lambda(Z)) < 0$, then the equilibrium point at the origin is uniformly globally exponentially stable. The matrix $Z = Z^T \in \Re^{n \times n}$ is obtained by:

1) Specified Q > 0 and $\bar{A}$ to compute P from the Lyapunov equation $-Q = (1/2)[P\bar{A} + \bar{A}^T P]$
2) Compute $\bar{A}_l = \bar{A} + \sum_{j=1}^{N} h_{lj} E_j$ and $\Phi = P\bar{A}_l + \bar{A}_l^T P$.
3) Compute $\Psi_j = [PE_j + E_j^T P] = \Psi_j^T \;\; \forall j$.
4) Compute $\Lambda_{\Psi_j} = T_{\Psi_j}^T \Psi_j T_{\Psi_j} = \mathrm{diag}[\lambda_{\Psi_{j1}} \; \cdots \; \lambda_{\Psi_{jn}}]$ $\forall j$, where $T_{\Psi_j} = [v_{\Psi_{j1}} \mid \cdots \mid v_{\Psi_{jn}}]$, $\{v_{\Psi_{j1}}, \; ..., v_{\Psi_{jn}}\}$ is the set of n orthogonal unit (orthonormal) eigenvectors of Ψ_j , and $\{\lambda_{\Psi_{j1}}, \; ..., \lambda_{\Psi_{jn}}\}$ is the corresponding set of n real eigenvalues of Ψ_j .
5) Set all negative elements of Λ_{Ψ_j} to zero to get $\Lambda_{\Psi_j}^{\geq 0} \;\; \forall j$.
6) Compute $\Psi_j^{\geq 0} = T_{\Psi_j} \Lambda_{\Psi_j}^{\geq 0} T_{\Psi_j}^T \;\; \forall j$.
7) Compute $Z \equiv \Phi + \sum_{j=1}^{r} [(h_{uj} - h_{lj}) \Psi_j^{\geq 0}]$.

When the bounded perturbation vector w(t) presents as in real practices, we cannot employ Theorem 1 to assert exponential convergence of trajectories to the origin. Rather, we can assert that trajectories converge into a neighborhood about the origin. The extent of this neighborhood is determined by the size of w(t). This is known as input-to-state stability [6], and is addressed in the following corollary:

Corollary 1 If Theorem 1 is satisfied with $w(t) \neq 0$, then trajectories of Eq. (3.2) converge to a neighborhood about the origin. The extent of this neighborhood is defined by

$$\Omega = \{x \mid V(x) \leq (1/2) \max(\lambda(P)) \gamma^2\}$$

where V(x)=(1/2) $x^T P x$, $P = P^T > 0$ is obtained from Theorem 1, $\gamma = 2\phi \max(\lambda(P)) / |\max(\lambda(Z))|$, and ϕ is a bound on w(t).

Proof Following the proof of Theorem 1, we have along the trajectories of Eq. (3.2) that

$$\dot{V}(x,t) \leq (1/2) x^T Z x + (\partial V / \partial x) w(t)$$

where V(x)=(1/2) $x^T P x$ with $P = P^T > 0$. Let w(t) be strictly bounded by $\phi \in \Re^+$ and notice that $\partial V / \partial x = x^T P$. Thus,

$$\dot{V}(x,t) \leq (1/2) x^T Z x + \max(\lambda(P)) \phi \|x\|$$

where we have $\max(\lambda(P)) > 0$ because $P = P^T > 0$. Because $\max(\lambda(Z)) < 0$ when Theorem 1 is satisfied, it follows that

$$\dot{V}(x,t) \leq -(1/2) |\max(\lambda(Z))| \|x\|^2 + \max(\lambda(P)) \phi \|x\|$$

It is clear that $\dot{V}(x,t) < 0$ when $\|x\|$ is sufficiently large. By direct substitution, one can verify that $\dot{V}(x,t) < 0$ when

$$\|x\| > \frac{2\phi \max(\lambda(P))}{\max(\lambda(Z))} \equiv \gamma \qquad (3.3)$$

Because V(x) ≤ (1/2) $\max(\lambda(P)) \|x\|^2$, it follows that $\dot{V}(x,t) < 0$ where V(x) > $(1/2) \max(\lambda(P)) \gamma^2$. By Lyapunov stability theorem [6], trajectories originating within Ω remain therein, while those out of Ω converge into Ω . This completes the proof.

With Theorem 1 and Corollary 1, input-to-state stability of the perturbed system in Eq. (3.2) is guaranteed when the adaptive integral gain k_{Ia} varies between h_{lj} and h_{uj} $\forall j$. If required, it is certainly possible to fix k_{Ia} at any value between the two bounds. Now that our adaptive law for the integral gain and the tools for assessing stability of the control systems are readily available, we demonstrate in the next section their applications and results.

4. Design Examples

In this section, we apply our adaptive integral gain to a DC motor position control system, and a two-mass position control system. It is assumed that a stabilizing PID controller is available for each of the systems, and we want to reduce overshoot by augmenting our adaptive integral gain to the controller.

Example 1 Consider the Maxon model 353297 DC motor position control system with gearbox model 223085 in [16]. The nominal unperturbed error dynamics of this SISO system can be approximated by Eq. (2.1), in which

$$A = \left[\begin{array}{c:c:c} 0 & 1 & 0 \\ \hdashline 0 & 0 & 1 \\ \hdashline 0 & 0 & -20489.5 \end{array}\right], \text{ and}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ -2514.82 \end{bmatrix}.$$

Here, the state vector $x \in \Re^3$, with state variables $x_1 = \int_0^t e(t)dt$, $x_2 = e(t)$, and $x_3 = \dot{e}(t)$. The error is defined as $e = \theta_R - \theta_M$, where θ_R is the reference signal, and θ_M is the angular displacement of the motor shaft. Note that the armature inductance L of the motor is truncated when deriving the above error dynamics because it is very small when compared to other parameters of the motor. This also reduces the number of feedback signals by one. Using the Linear Quadratic Regulator (LQR) theory, we obtain the PID control law $u = -Kx$, in which $K = [-1 \,\vdots\, -4.17 \,\vdots\, -0.06]$. Note that this control law minimizes the cost function

$$J = \int_0^\infty (x^T Q x + u^T R u)dt \qquad (4.1)$$

where $Q = I_{3\times3}$, and $R = [1]$.

To investigate transient response characteristics resulting from the above PID controller, we perform numerical simulations using the following full-order state equation:

$$\dot{q} = A_T q + B_T u + NT_d \qquad (4.2)$$

where the state vector $q \in \Re^4$, with $q_1 = \int_0^t \theta_M dt$, $q_2 = \theta_M$, $q_3 = \dot{\theta}_M$, and $q_4 = i$. Note that we now include the armature current i and the armature inductance into our simulation model. The relevant matrices are:

$$A_T = \left[\begin{array}{c:c:c:c} 0 & 1 & 0 & 0 \\ \hdashline 0 & 0 & 1 & 0 \\ \hdashline 0 & 0 & -0.7 & 1828.4 \\ \hdashline 0 & 0 & -6324.5 & -2189.4 \end{array}\right],$$

$$B_T = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1552.8 \end{bmatrix}, \text{ and } N = \begin{bmatrix} 0 \\ 0 \\ 473.2 \\ 0 \end{bmatrix}.$$

When the system is subjected to a square-wave reference signal $\theta_R = 10\text{sign}(\sin(0.15t))$ and a square wave disturbance torque $T_d = 0.5\text{sign}(\sin(0.5t))$, we obtain simulation results for output θ_M shown in Fig. 3. We see that the control system spends approximately 2 s to drive the output to reach the reference, and exhibits approximately 2.6 rad or 13% of maximum overshoot. Transient response characteristics appear satisfactorily, and should be acceptable for many applications.

We now explore application of the adaptive integral gain. Notice that the constant integral gain $k_{Ic} = -1$ is K(1,1). Because k_{Ic} is negative, we want the adaptive integral gain k_{Ia} to be positive to counter the slowly-varying nature of integral control that causes overshoot. For this, it is convenient to select zero as the lower bound on k_{Ia}, and then find an allowable positive upper bound. Using Theorem 1, a satisfactorily large allowable upper bound on k_{Ia} is found to be $h_u = 0.55$. This corresponds to the following matrices:

$$Z = \begin{bmatrix} -14.8 & 20.9 & 2.1 \\ \dots & -33.6 & 3.0 \\ \dots & \dots & -45.4 \end{bmatrix}, \text{ and}$$

$$Q = \begin{bmatrix} 23 & 0 & 0 \\ 0 & 24.2 & 0 \\ 0 & 0 & 24 \end{bmatrix}.$$

Theorem 1 is satisfied with $\max(\lambda(Z)) = -1.1$, noting that Z is symmetric. Using the adaptive law for k_{Ia} in Eq. (2.4) with $\beta = 2$, numerical simulations give the resulting output θ_{Ma} as shown in Fig. 3. We see that the rise time of θ_{Ma} is virtually the same as that of θ_M. However, the overshoot is now approximately 1.3 rad, showing approximately 50% overshoot reduction. Note that convergence of θ_{Ma} to the reference is slightly slower than that of θ_M.

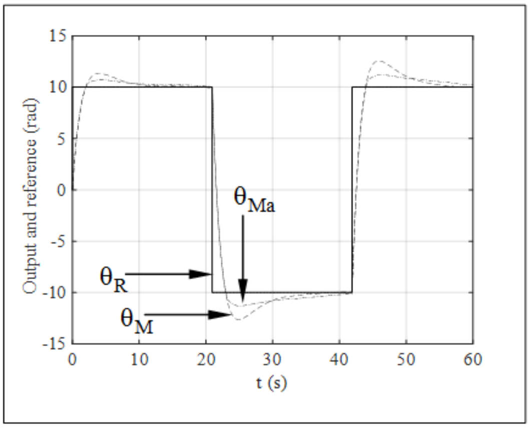

Fig. 3. Outputs and reference resulting from PID controls (θ_M : without adaptive integral gain, θ_{Ma} : with adaptive integral gain)

Figure 4 shows the corresponding trajectory of k_{Ia}, which is bounded by $h_l = 0$ and $h_u = 0.55$. Note that increasing value of β beyond 2 hardly affects transient response characteristics because the corresponding trajectory of k_{Ia} is already faster than that of θ_{Ma}.

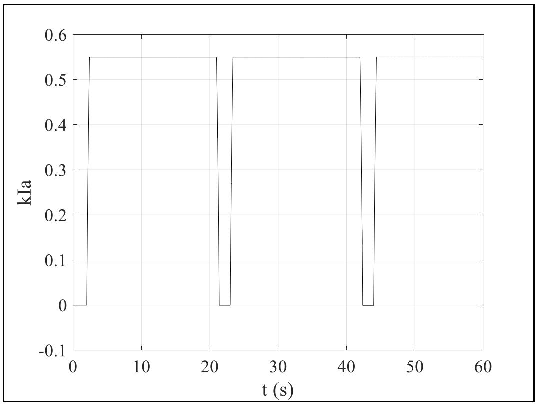

Fig. 4. Trajectory of adaptive integral gain k_{Ia}, augmented to K(1, 1)

Example 2 Consider the two-mass system depicted in Fig. 5. The nominal unperturbed error dynamics of this MIMO system can be approximated by Eq. (2.1), in which

$$A = \begin{bmatrix} 0_{4\times2} & \multicolumn{4}{c}{I_{4\times4}} \\ 0_{2\times2} & -2c_1 & c_1 & -2c_2 & c_2 \\ & c_3 & -c_3 & c_4 & -c_4 \end{bmatrix},$$

$$B = \begin{bmatrix} 0_{4\times2} \\ -I_{2\times2} \end{bmatrix},$$

c_1=k/m_1, c_2=b/m_1, c_3=k/m_2, c_4=b/m_2, spring constant k=1000N/m, mass $m_1 = 10$kg, mass $m_2 = 20$kg, damper coefficient $b = 100$N.s/m, and control vector $u = [f_1 \;\; f_2]^T$.

The control force f_i is applied to m_i, i = 1, 2. There are 6 state variables, namely $x_1 = \int_0^t e_1 dt$, $x_2 = \int_0^t e_2 dt$, $x_3 = e_1$, $x_4 = e_2$, $x_5 = \dot{e}_1$, and $x_6 = \dot{e}_2$. We also define error variable $e_i = r_i - \theta_i$, in which r_i and θ_i are the i-th reference position and the i-th output associated with m_i respectively. Using the LQR theory yields the following constant feedback gain matrix for the PID control law $u = -Kx$:

$$K = \begin{bmatrix} -63.2 & 2.5 & -28.6 & -33.2 & -47 & -6.1 \\ -2.5 & -63.2 & 2.06 & -92.6 & -6.1 & -60.6 \end{bmatrix}$$

The above control law minimizes the cost function J in Eq. (4.1), with $Q = 4000 I_{6\times6}$, and $R = I_{2\times2}$.

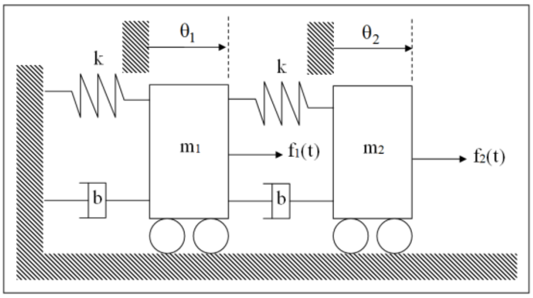

Fig. 5. Schematic of two-mass system.

In the same fashion as the previous example, we employ numerical simulations to estimate responses of the above PID control law. The corresponding model used for this is Eq. (4.2), for which we note that $A_T = A$, and $B_T = -B$. The two-mass system is not used for driving external loads, so we set $T_d = 0$. The state vector $q \in \Re^6$, in which $q_1 = \int_0^t \theta_1 dt$, $q_2 = \int_0^t \theta_2 dt$, $q_3 = \theta_1$, $q_4 = \theta_2$, $q_5 = \dot{\theta}_1$, and $x_6 = \dot{\theta}_2$. The same square-wave reference position r_1=r_2=sign(sin(0.15t)) is used for both masses to facilitate comparisons of the simulation results shown in Fig. 6. It appears that the rise time of θ_1 is significantly larger than that of θ_2. Also, maximum overshoot of θ_1 is negligible while that of θ_2 is approximately 0.28 m, or 14%. Again, the LQR theory is able to yield satisfactory results.

From the above simulation results, we only want to decrease overshoot of θ_2. To do this, notice that u_2 is associated with two integral gains, $K(2,1) = -2.5$ and $K(2,2) = -63.2$. We augment k_{Ia} only to the latter, because it is much larger than the former, and is directly associated with m_2. In the same fashion as in Example 1, we elect that the lower bound for k_{Ia} is zero, and employ Theorem 1 to find a sufficiently large allowable positive upper bound h_u. It turns out that Theorem 1 is satisfied with $\max(\lambda(Z)) = -4.1$ when $h_u = 37.9$. The relevant matrices are:

$$Z = \begin{bmatrix} -30 & -0.28 & -0.04 & -0.13 & 0 & -0.08 \\ \ldots & -15.0 & 4.37 & 15.42 & 0.18 & 9.72 \\ \ldots & \ldots & -42.4 & 2.1 & 0.02 & 1.33 \\ \ldots & \ldots & \ldots & -42.0 & 0.09 & 4.69 \\ \ldots & \ldots & \ldots & \ldots & -31.5 & 0.05 \\ \ldots & \ldots & \ldots & \ldots & \ldots & -41.05 \end{bmatrix}$$

and $Q \in \Re^6$ is a diagonal matrix, whose diagonal elements are 15, 23.5, 21.5, 24.75, 15.75, and 22.

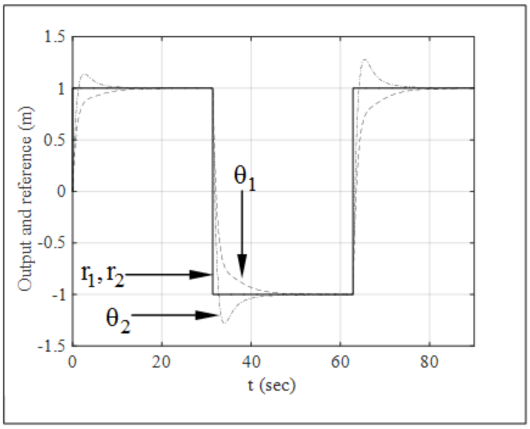

Fig. 6. Outputs and reference resulting from PID controls.

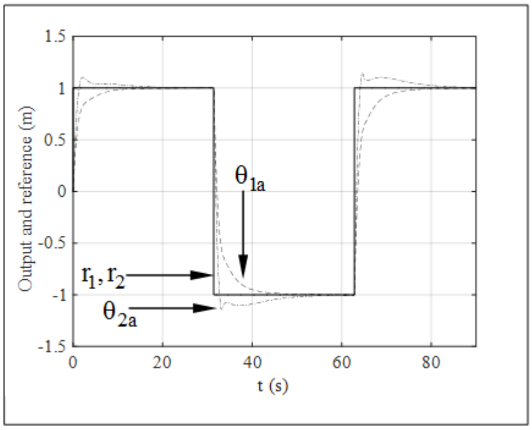

Fig. 7. Outputs and reference resulting from PID controls with adaptive integral gain.

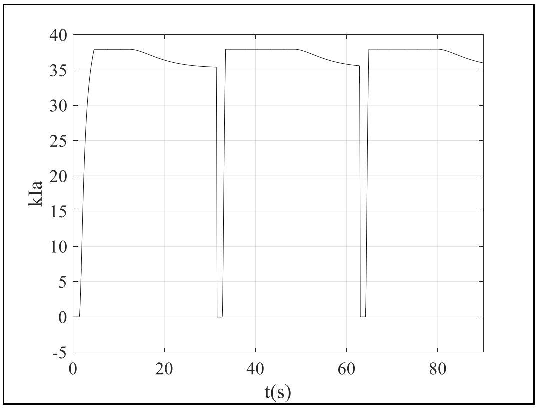

Fig. 8. Trajectory of adaptive integral gain k_{Ia} augmented to K(2, 2).

According to the adaptive law for k_{Ia} in Eq. (2.4), we set $\beta = 10$ and obtain simulation results for output θ_{1a} and θ_{2a} as shown in Fig. 7. When comparing simulation results in Fig.6 and Fig.7, we see that θ_1 and θ_{1a} are almost identical. The rise time of θ_2 is approximately the same as that of θ_{2a}. However, maximum overshoot of θ_{2a} is approximately 0.13 m, representing a 54% reduction when compared to that of θ_2. The corresponding trajectory of k_{Ia} is shown in Fig. 8. Note that θ_2 converges to r_2 faster than θ_{2a} does, and that increasing value of β beyond 10 affects transient response characteristics very slightly because k_{Ia} is already faster than θ_{2a}.

5. Conclusion

In PID controllers, the integral control is generally incorporated to reduce or eliminate steady-state error due to unknown disturbances and parameter variations in the system of interest. While the integral control is useful for this purpose, it can reduce relative stability of the resulting control system. Reducing relative stability can be useful for reducing rise time of the output, but it can also cause excessive overshoot when the reference signal changes abruptly. The derivative control can be employed to suppress this undesirable characteristic at the usual expense of increasing rise time. The derivative signal is also very sensitive to noises and measurement errors. Accordingly, using a large derivative gain is normally prohibited because it can degrade performance of the control system significantly.

To suppress excessive overshoot without totally relying on the derivative control of an existing PID controller, we propose that an adaptive integral gain be used. Our original adaptive integral gain is associated with a smooth adaptive law, making the resulting control signal smooth. Input-to-state of the resulting adaptive PID control system is guaranteed when the gain variation is within the correspondingly allowable upper and lower bounds. Simplicity and effectiveness of the adaptive integral gain is shown in two design examples. Numerical simulations indicate that the adaptive gain can reduce maximum overshoot that occurs when reference signal changes abruptly by approximately 50%.

In light of the results obtained in this paper, other benefits of adaptive gains for enhancing robust performance of existing PID control systems are to be investigated in the near future. In particular, disturbance rejection in PID motion control systems is now our topic of interest.

References

[1] Ziegler JG, Nichols NB. Optimal settings for automatic controllers. Trans ASME 1942; 64: 8, 759-768.

[2] Killingsworth N, Krstic M. PID tuning using extremum seeking. IEEE Control Systems Magazine 2005; November.

[3] Ang KH, Chong G, Li Y. PID control system analysis, design, and technology, IEEE Trans On Control Systems Technology 2005; 13: 4: July 559-576.

[4] Li Y, Ang KH, Chong G. PID control system analysis, and design. IEEE Control Systems Magazine 2006; 26: 1: February: 32-41.

[5] Balestrino A et. al. From basic to advanced PI controllers: a complexity vs. performance comparison. Advances in PID Control. InTech; 2011.

[6] Khalil HK. Nonlinear Systems: Third Edition. Prentice-Hall 2002; NJ.

[7] Visioli A, Legnani G. On the Trajectory Tracking Control of Industrial SCARA Robot Manipulators. IEEE Trans on Industrial Electronics 2002; 49: 1: February: 224-232.

[8] Mizumoto I, Iwai Z. Adaptive PID control system design based on ASPR property of systems. Advances in PID Control. InTech; 2011.

[9] Guerra R, Gonzalez S, Reyes R. Adaptive gain PID control for mechanical systems. Advances in PID Control. InTech; 2011.

[10] Su W. A model reference-based adaptive PID controller for robot motion control of not explicitly known systems. International Journal of Intelligent control and systems 2007; 12: 3: September: 237-244.

[11] Peng YQ et. al. Model reference fuzzy adaptive PID control and its applications in typical industrial processes. Proceeding of the IEEE International Conference on Automation and Logistics 2008; Qingdao: China.

[12] Zue X et. al. Adaptive PID and model reference adaptive control switch controller for nonlinear Hydraulic actuator. Hindawi Mathematical Problems in Engineering 2017.

[13] Habibi H, Nohooji HR, Howard I. Adaptive PID control of wind turbines for power regulation with unknown control direction and actuator faults. IEEE Access 2018; 6.

[14] Ngamsom P, Hoberock LL. Using Robust Stability Analysis Theorems for Robust Controller Design. ASME Journal Of Dynamic Systems, Measurement, and Control 2003; 125: 4, 669-671.

[15] Ngamsom P. A graphical robust PID controller design for time-varying uncertain systems with applications to trajectory tracking of a load-varying rigid SCARA. Thammasat International Journal of Science and Technology 2016; 21: 2: 34-44.

[16] Ngamsom P. Design of trajectory tracking system for DC motors with uncertain time-varying inertial loads. Science & Technology Asia 2018; 23: 2: 54-65.