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On ω^*-connected spaces

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Abstract

In this paper, we introduce the notion of ω-separated sets and ω^*-connected spaces. We obtain several properties of the notion analogous to those of connectedness. We show that the continuous image of the $\omega*$-connected space is connected.

Keywords: ω-open set, ω-separated, ω-connected, ω-component.

1. Introduction

Let (X, τ) be a topological space with no separation properties assumed. For a subset H of a topological space (X, τ), $Cl(H)$ and $Int(H)$ denote the closure and the interior of H in (X, τ), respectively.

Definition 1.1. (Kuratowski, 1933) Let H be a subset of a topolgicl space (X, τ). A point p in X is called a condensation point of H if, for each open set U containing p, $U \cap H$ is uncountable.

Definition 1.2. (Hdeib, 1982) A subset H of a topological space (X, τ) is called ω-closed if it contains all its condensation points.

The complement of an ω-closed set is called ω-open.

It is well known that a subset W of a space (X, τ) is ω-open if and only if for each $x \in W$, there exists $U \in \tau$ such that $x \in U$, and $U - W$ is countable (Hdeib, 1989). The family of all ω-open sets that is denoted by τ_ω is a topology on X, which is finer than τ. The interior and closure operators in (X, τ_ω) are denoted by Int_ω and Cl_ω, respectively. Many topological concepts and results related to the ω-closed and ω-open sets appeared in Al Ghour and Zareer (2016), Al-Omari and Noorani (2007a, 2007b), Noiri, Al-Omari and Noorani (2009 a, 2009b), Zorlutuna (2013) and in the references therein. In this paper, we introduce the notion of ω-separated sets and ω^*-connected spaces. We obtain several properties of the no-tion analogous to these of connectedness. We show that the continuous image of the ω^*-connected space is connected. Furthermore, we present a connected space that is not ω^*-connected.

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2. ω-Separated Sets

Definition 2.1. Nonempty subsets A and B of a topological space (X, τ). The pair (A, B) are called ω-separated if $Cl(A) \cap B = A \cap Cl_\omega(B) = \phi$.

Clearly, every separated set is ω-separated, the converse need not be true in general as the following example shown that.

Example 2.2. Let $X = \{a, b, c, d\}$ and $\tau = \{X, \phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}\}$. If $A = \{b\}$ and $B = \{a\}$, then the pair (A, B) form an ω-separated set, but it is not separated. It is clear that the pair (B, A) are not ω-separated set.

Proposition 2.3. Let A be a nonempty open set in a space X and B be a nonempty ω-open set in a space X such that $A \cap B = \phi$, then the pair (A, B) are ω-separated.

Proof. Let $A \cap B = \phi$. Then, $A \subseteq X - B$ and $Cl(A) \subseteq Cl_\omega(X - B) = X - B$, which implies that $Cl_\omega(A) \cap B = \phi$. Again, $B \subseteq X - A$ implies that $C(B) \subseteq Cl(X - A) = X - A$ and so $Cl(B) \cap A = \phi$. Therefore, the pair (A, B) are ω-separated.

Corollary 2.4. Let (X, τ) be topological space. If $A, B \in \tau$ are nonempty open sets such that $A \cap B = \phi$, then A and B are ω-separated.

Proposition 2.5. Let the pair (A, B) be two ω-separated sub-sets in a topological space (X, τ). If $C, D \in X$ are nonempty subsets such that $C \subseteq A$ and $D \subseteq B$, then the pair (C, D) are also ω-separated.

Proof. Suppose that the pair (A, B) are ω-separated and $C(A) \cap B = \phi = A \cap Cl_\omega(B)$. Now, $C \cap Cl_\omega(D) \subseteq A \cap Cl_\omega(B) = \phi$, so $C \cap Cl_\omega(D) = \phi$. Similarly, we can prove that $Cl(C) \cap D = \phi$. Hence, the pair (C, D) are ω-separated.

Theorem 2.6. Let (X, τ) be a topological space. If A and B are ω-separated such that $A \cup B$ is closed set, then one set is closed, and the other is ω-closed.

Proof. Let the pair (A, B) be ω-separated sets and $A \cup B$ is closed. Then, $A \cap Cl_\omega(B) = \phi = Cl(A) \cap B$. For every $A \cup B$ is closed, $A \cup B = C(A) \cup Cl(B)$. Now, $C(A) = Cl(A) \cap [Cl(A) \cup Cl(B)] = Cl(A) \cap [A \cup B] = [Cl(A) \cap A] \cup [Cl(A) \cap B] = A \cup \phi = A$, hence A is closed. Additionally, $B \subseteq A \cup B$ then we have:

$$Cl_\omega(B) \subseteq Cl_\omega[A \cup B] \subseteq Cl[A \cup B] = A \cup B$$

so $Cl_\omega(B) = Cl_\omega(B) \cap [A \cup B] = [Cl_\omega(B) \cap A] \cup [Cl(B) \cap B] = \phi \cup B = B$.

Hence, B is ω-closed.

Theorem 2.7. Let (X, τ) be a topological space. If the pair (A, B) are ω-separated sets of X and $A \cup B \in \tau$, then A and B are ω-open and open, respectively.

Proof. Let the pair (A, B) be ω-separated in X; then, $B = [A \cup B] \cap [X \backslash C(A)]$. Since $A \cup B \in \tau$ and $Cl(A)$ is closed in X, then B is open. Thus, $A = [A \cup B] \cap Cl_\omega(B)]$ since the pair (A, B) are ω-separated in X. Additionally, $A \cup B \in \tau \subseteq \tau_\omega$ and $Cl_\omega(B)$ is ω-closed in X, and then A is ω-open.

Lemma 2.8. (Al-Omari and Noorani (2007a) Let (X, τ) be topological space if Y is an open subspace of a space X and $B \subseteq Y \subseteq X$. Then, $Cl_\omega^Y = Cl_\omega(B) \cap Y$.

Lemma 2.9. Let (X, τ) be topological space and Y is an open subspace of a space X such that $A, B \subseteq Y \subseteq X$. The following statements are equivalent:

1. The pair (A, B) are ω-separated in Y;
2. The pair (A, B) are ω-separated in X.

Proof. This is abvious from Lemma 2.8, $Cl_\omega^Y(A) \cap B = \phi = A \cap Cl^Y()$ if and only if $Cl_\omega(A) \cap B = \phi = A \cap Cl(B)$.

3. $\omega*$-Connected Spaces

In this section, we discuss some properties of $\omega*$-connected space, which is stronger than connected space.

Definition 3.1. A subset A of a topological space (X, τ) is called $\omega*$-connected if A is not the union of any pair of ω-separated sets in (X, τ).

Clearly, every $\omega*$-connected, space is connected, the converse need not be true in general as the following example shown that.

Example 3.2. Let $\mathbb{R}$ be the set of real numbers and $X = \mathbb{Q}$ be the set of all rational numbers. Let $(X, \tau_{l|Q})$ be the relative topology with left ray-topology $(\mathbb{R}, \tau_l)$. Then, $(X, \tau_{l|Q})$ is a connected space, but it is not ω^*-connected.

Example 3.3. Let $\mathbb{R}$ be the set of real numbers and $\mathbb{Q}$ be the set of all rational numbers ($\mathbb{R} - \mathbb{Q} = I$ be the set of all irrational numbers) with the topology $\tau = \{\phi, \mathbb{R}, \mathbb{Q}\}$. Then, $(\mathbb{R}, \tau)$ is con-nected space, but it is not $\omega*$-connected sence the pair $(I, \mathbb{Q})$ are ω-separated sets.

Theorem 3.4. A topological space (X, τ) is $\omega*$-connected if and only if X cannot be written as the disjoint union of a nonempty ω-open set and a nonempty open set.

Proof. Suppose that X is not a union of nonempty disjoint ω-open and open sets A and B that is $X = A \cup B$. Thus, $Cl(A) \cap B = \phi = A \cap Cl_\omega(B)$ since A and B are disjoint. Hence the pair (A, B) are ω-separated sets in X. So, X is not $\omega*$-connected. This is a contradiction.

Conversely, suppose that X is not $\omega*$-connected. There exist a pair (A, B) of ω-separated sets and $X = A \cup B$. By Theorem 2.7, $A, B \in X$ are ω-open and open, respectively. Then, X can be written as the disjoint union of a nonempty ω-open set and a nonempty open set. This is a contradiction.

Theorem 3.5. A subspace Y of a topological space (X, τ) is $\omega*$-connected if and only if there does not exist any pair of ω-separation for Y.

Proof. First, let us assume that the subspace Y is $\omega*$-connected. Thus, we will have to prove that Y does not admit any ω-sepa-ration. Suppose that Y has a pair of ω-separation.

Hence, there exist nonempty subsets A and B of X such that $Y = A \cup B$, $A \cap Cl(B) = \phi = Cl_\omega(A) \cap B$. Now,

$$Cl_\omega^Y(A) = Cl_\omega(A) \cap Y = Cl_\omega(A) \cap [A \cup B] = [Cl_\omega(A) \cap A] \cup [Cl_\omega(A) \cap B] = A$$

This implies that A is an ω-closed subset of (Y, τ_Y). Similarly, $Cl^Y(A) = Cl(B) \cap Y = Cl(B) \cap [A \cup B] = [Cl(B) \cap A] \cup [Cl(B) \cap B] = B$

This implies that B is a closed subset of (Y, τ_Y). Additionally, $Y - B = Y \cap [X - B] = [A \cup B] \cap [X - B] = [A \cap (X - B)] \cup [(X - B) \cap B] = A \cap (X - B) = A$ (since $A \cap B = \phi$). This means that the complement of B with respect to Y is A. Hence, B is ω-open such that $B \neq \phi$ and $B \neq Y$ (since $Y = A \cup B$ and

$A \neq \phi$), and B is both ω-open and closed in Y. This implies that the subspace (Y, τ_Y) is not an $\omega*$-connected space. This is a contradiction, and we arrived at this contradiction by assuming that Y has a pair of ω-separation. Hence, there does not exist any ω-separation for Y.

Conversely, now assume that there is not any pair of ω-separation for Y. Now suppose that the subspace (Y, τ_Y) is not an ω-connected space. Then, there exist nonempty ω-closed set A and closed set B in (Y, τ_Y) such that $Y = A \cup B$ and $A \cap B = \phi$.

Now, $[Cl_\omega(A) \cap B = Cl_\omega(A) \cap [Y \cap B] = [Cl_\omega(A) \cap Y] \cap B = Cl_\omega^Y(A) \cap B = A \cap B = \phi$ (since A is ω-closed in Y, which implies $Cl_\omega^Y(A) = A$). Similarly, $A \cap Cl\ (B) = \phi$. This means that Y has a pair of ω-separation, and this is a contradiction. We arrived at this contradiction by assuming that the subspace (Y, τ_Y) is not an ω^*-connected space. Hence, the assumption is wrong. This means that (Y, τ_Y) is ω^*-connected.

Theorem 3.6. Let (X, τ) not be an ω^*-connected topological space and $\phi \neq A \subseteq X$, A is an open set and ω-closed set in X. Suppose, $\phi \neq Y$ is ω^*-connected subspace of X. Then, either $Y \subseteq A$ or $Y \subseteq X - A$.

Proof. $X = A \cup B$, where $B = X - A$ implies that:
$Y = X \cap Y = [A \cup B] \cap Y = (A \cap Y) \cup (B \cap Y)$.
Also $[A \cap Y] \cap C[B \cap Y] \subseteq A \cap Cl_\omega(B) = A \cap B = \phi$, which implies that $[A \cap Y] \cap Cl[B \cap Y] = \phi$.
Similarly, $Cl[A \cap Y] \cap [B \cap Y] \subseteq Cl_\omega(A) \cap B = A \cap B = \phi$.
Then $Cl[A \cap Y] \cap [B \cap Y] = \phi$. It is given that Y is an ω^*-connected subspace of X. So, it does not imply that $A \cap Y \neq \phi$ and $B \cap Y \neq \phi$. Y cannot admit any pair of ω-separation, since Y is an ω^*-connected subspace of X. Thus, $A \cap Y = \phi$ or $B \cap Y = \phi$ and so $Y \subseteq A$ or $Y \subseteq X - A$.

Theorem 3.7. Let (X, τ) be a topological space and the pair (H, G) are ω-separated sets of X. If A is an ω^*-connected set of X with $A \subseteq H \cup G$, then either $A \subseteq H$ or $A \subseteq G$.

Proof. Let $A \subseteq H \cup G$. Since $A = [A \cap H] \cup [A \cap G]$, then $[A \cap H] \cap Cl[A \cap G] \subseteq H \cap Cl_\omega(G) = \phi$. Again, we have $[A \cap G] \cap [A \cap H] \subseteq G \cap Cl(H) = \phi$. Thus, the pair $(A \cap H, A \cap G)$ are ω-separated sets. If $A \cap H$ and $A \cap G$ are nonempty. So, A is not ω^*-connected. This is a contradiction. Thus, either $A \cap H = \phi$ or $A \cap G = \phi$. Hence $A \subseteq H$ or $A \subseteq G$.

Theorem 3.8. If A is an ω^*-connected set of a topological space (X, τ) and $A \subseteq B \subseteq Cl(A)$, then B is ω^*-connected.

Proof. Suppose that B is not ω^*-connected. There exist a pair (H, G) of ω-separated sets such that $B = H \cup G$. So, H and G are nonempty and $G \cap Cl(H) = \phi = H \cap Cl_\omega(G)$. Then either $A \subseteq H$ or $A \subseteq G$, by Theorem 3.7. If $A \subseteq G$, then $Cl_\omega(A) \subseteq Cl_\omega(G)$ and $H \cap Cl_\omega(A) = \phi$. Thus $H \subseteq B \subseteq Cl_\omega(A)$ and $H = Cl_\omega(A) \cap H = \phi$. Hence H is an empty set. This is a contradiction, since H is nonempty. Now, if $A \subseteq H$. Again, we have G is empty. This is a contradiction. Then, B is ω^*-connected.

Corollary 3.9. If A is an ω^*-connected set of a topological space (X, τ), then $Cl_\omega(A)$ is ω^*-connected.

Theorem 3.10. Let (X, τ) be a topological space. If $\{N_i: i \in I\}$ be a nonempty family of ω^*-connected sets of X with $\cap_{i\in I} N_i \neq \phi$, then, $\cup_{i\in I} N_i$ is ω^*-connected.

Proof. Suppose that $\cup_{i\in I} Ni$ is not ω^*-connected. Then, $\cup_{i\in I} N_i = H \cup G$, where the pair (H, G) are ω-separated sets in X. Since $\cap_{i\in I} N_i \neq \phi$, we have $x \in \cap_{i\in I} N_i$. Since $x \in \cup_{i\in I} N_i$, either $x \in H$ or $x \in G$. If $x \in H$. Since $x \in N_i$ for each $i \in I$, then N_i and H intersect for each $i \in I$. By Theorem 3.7, $N_i \subseteq H$ or $N_i \subseteq G$. Since H and G are disjoint, $N_i \subseteq H$ for all $i \in I$ and hence $\cup_{i\in I} N_i \subseteq H$. Then G is empty. This is a contradiction. Suppose $x \in G$. In a similar way, we have that H is empty. This is a contradiction. Thus, $\cup_{i\in I} Ni$ is ω^*-connected.

Theorem 3.11. Every continuous image of an ω^*-connected space is a connected space.

Proof. Let $f : X \to Y$ be a continuous function and X be an ω^*-connected space. Suppose that $f(X)$ is not a connected subset of Y. Thus, there exists nonempty separated sets A and B with $f(X) = A \cup B$. Since f is continuous and $A \cap Cl(B) = \phi = Cl(A) \cap B$, $Cl(f^{-1}(A)) \cap f^{-1}(B) \subseteq f^{-1}(Cl(A)) \cap f^{-1}(B) = f^{-1}[Cl(A) \cap B] = \phi$, $f^{-1}(A) \cap Cl_\omega(f^{-1}(B)) \subseteq f^{-1}(A) \cap Cl(f^{-1}(B)) \subseteq f^{-1}(A) \cap f^{-1}Cl(B)) = f^{-1}[A \cap Cl(B)] = \phi$. Since A and B are nonempty, $f^{-1}(A)$ and $f^{-1}(B)$ are nonempty. Hence, $f^{-1}(A)$ and $f^{-1}(B)$ are a pair of ω-separated and $X = f^{-1}(A) \cup f^{-1}(B)$. This is a contradiction since X is ω^*-connected. Therefore, (X) is connected.

Question: Is a continuous image of an ω^*-connected space ω^*-connected?

Theorem 3.12. If every distinct points of a subset H of a space X are elements of some ω^*-connected subset of H, then H is an ω^*-connected subset of X.

Proof. Suppose H is not ω^*-connected. Then, there exist nonempty sets $A, B \in X$ and $Cl(A) \cap B = \phi = A \cap Cl_\omega(B)$ and $H = A \cup B$. Since A and B are nonempty, there exists $a \in A$ and $b \in B$. By hypothesis, a and b must be elements of an ω^*-connected subset C of H. Since $C \subseteq A \cup B$, by Theorem 3.7, either $C \subseteq A$ or $C \subseteq B$. Consequently, either both a and b are in A or in B. Let $a, b \in A$. Hence, $A \cap B \neq \phi$, which is a contradiction to the fact that A and B are disjoint. Therefore, H must be ω^*-connected.

Theorem 3.13. If A is an ω^*-connected subset of an ω^*-connected topological space (X, τ) such that $X - A$ is the union of a pair (B, C) of ω-separated sets, then $A \cup C$ are ω^*-connected.

Proof. Suppose $A \cup B$ is not ω^*-connected. Then, there exist a pair (G, H) of nonempty ω-separated sets such that $A \cup B = G \cup H$. Thus, $A \subseteq A \cup B = G \cup H$, since A is ω^*-connected and, by Theorem 3.7, either $A \subseteq G$ or $A \subseteq H$. If $A \subseteq G$. Since $A \cup B = G \cup H$, $A \subseteq G$, then $A \cup B \subseteq G \cup H$ and hence $G \cup H \subseteq G \cup B$. Hence, $H \subseteq B$. Since a pair (B, C) are ω-separated, also a pair (H, C) are ω-separated. Thus, H is ω-separated from a pair (G, C). Now, $C(H) \cap [G \cup C] = [Cl(H) \cap G] \cup [Cl(H) \cap C] = \phi$, and $H \cap Cl_\omega[G \cup C] = H \cap [Cl_\omega(G) \cup Cl_\omega(C)] = [H \cap Cl_\omega(G)] \cup [H \cap Cl_\omega(C)] = \phi$.

Therefore, H is ω-separated from $G \cup C$. Since $X - A = B \cup C$, $X = A \cup [B \cup C] = [A \cup B] \cup C = [G \cup H] \cup C$, and since $A \cup B = G \cup H$, $X = [G \cup C] \cup H$. Thus, X is the union of a pair of nonempty ω-separated sets ($G \cup C$, H), which is a contradiction. There is a similar contradiction if $A \subseteq H$. Then, $A \cup B$ is ω^*-connected. Also, we can prove that $A \cup C$ is ω^*-connected.

Theorem 3.14. If A and B are $\omega*$-connected sets of a topological space (X, τ) and none of them is ω-separated, we have $A \cup B$ is ω^*-connected.

Proof. Let A and B be ω^*-connected in a space X. If $A \cup B$ is not $\omega*$-connected. Then, there exist a pair of nonempty disjoint ω-separated sets (G, H) and $A \cup B = G \cup H$. By Theorem 3.7 and since A and B are ω^*-connected, either $A \subseteq G$ and $B \subseteq H$ or $B \subseteq G$ and $A \subseteq H$.

Now, if $A \subseteq G$ and $B \subseteq H$, then $A \cap H = B \cap G = \phi$.

Therefore, $[A \cup B] \cap G = [A \cap G] \cup [B \cap G] = [A \cap G] \cup \phi = A \cap G = A$.

Additionally, $[A \cup B] \cap H = [A \cap H] \cup [B \cap H] = \phi \cup [B \cap H] = B \cap H = B$.

Similarly, if $B \subseteq G$ and $A \subseteq H$, then $[A \cup B] \cap G = B$ and $[A \cup B] \cap H = A$. Now, $[(A \cup B) \cap H] \cap Cl_\omega[(A \cup B) \cap G] \subseteq (A \cup B) \cap H \cap Cl_\omega(A \cup B) \cap Cl_\omega(G) = (A \cup B) \cap H \cap Cl_\omega(G) = \phi$ and $Cl[(A \cup B) \cap H] \cap [(A \cup B) \cap G] \subseteq Cl(A \cup B) \cap Cl(H) \cap (A \cup B) \cap G = (A \cup B) \cap Cl(H) \cap G = \phi$.

Therefore, the pair $((A \cup B) \cap H, (A \cup B) \cap G)$ are ω-separated sets. Thus, by Proposition 2.5, we have that the pair (A, B) are ω-separated, which is a contradiction. Hence, $A \cup B$ is ω^*-connected.

Example 3.15. Let $X = \{a, b, c, d\}$, $\tau = \{ X, \phi, \{b\}, \{b, c\}, \{a, b, d\} \}$. If $A = \{c\}$ and $B = \{d\}$, the A and B are ω^*-connected. But $A \cup B = \{c, d\}$ is not ω^*-connected since the pairs (A, B) and (B, A) are ω-separated.

Example 3.16. Let $X = \{a, b, c, d\}$, $\tau = \{ X, \phi, \{b\}, \{b, c\}, \{a, b, d\} \}$. If $A = \{b\}$ then A is ω^*-connected. But $Cl(A) = X$ is not ω^*-connected since the pair (H, G) are ω-separated where $H = \{a, c, d\}$ and $G = \{b\}$.

Definition 3.17. Let (X, τ) be topological space and $x \in X$. The union of all ω^*-connected subsets of X containing x is called the ω-component of X containing x.

Lemma 3.18. Each ω-component of a topological space (X, τ) is a maximal ω^*-connected set of X.

Lemma 3.19. The set of all distinct ω-components of a topological space (X, τ) forms a partition of X.

Proof. Let A and B be distinct ω-components of X. If A and B intersect. Then, by Theorem 3.10, $A \cup B$ is ω^*-connected in X. Since $A \subseteq A \cup B$, then A is not maximal. Hence, A and B are disjoint.

Lemma 3.20. Each ω-component of a topological space (X, τ) is an ω-closed in X.

Proof. Let A be an ω-component of X. By Corollary 3.9, $Cl(A)$ is ω^*-connected and $A = Cl_\omega(A)$. Thus, A is an ω-closed in X.

Example 3.21. Let $X = \{a, b, c, d\}$, $\tau = \{ X, \phi, \{b\}, \{b, c\}, \{a, b, d\} \}$. If $A = \{b\}$ then the ω-component of A is A which is not closed.

Theorem 3.22. Each ω^*-connected subset of a space X which is open and ω-closed is ω-component of X.

Proof. Let A be an ω^*-connected of X which is open and ω-closed. For $x \in A$. If C is the ω-component containing x, then $A \subseteq C$ (since A is an $\omega*$-connected subset of X containing x). Let $A \subset C$. Then, $C \neq \phi$ and $C \cap (X - A) \neq \phi$. Thus, $X - A$ is closed and ω-open and $[A \cap C] \cap [(X - A) \cap C] = \phi$ (since A is open and ω-closed). Additionally, $[A \cap C] \cup [(X - A) \cap C] = [A \cup (X - A)] \cap C = C$. Also, A and $X - A$ are nonempty disjoint open and ω-open sets, respectively, and $A \cap Cl(X - A) = \phi = Cl_\omega(A) \cap (X - A)$. Then we have that $(A \cap C) \cap Cl[(X - A) \cap C] = \phi = Cl_\omega(A \cap C) \cap [(X - A) \cap C]$. Hence the pair $[A \cap C]$ and $[(X - A) \cap C]$ are ω-separated sets. This is a contradiction, then $A = C$. Hence, A is an ω-component of X.

4. Conclusions

In the present work, we have continued to study the properties of connected spaces. We introduced ω-separated sets and ω^*-connected. Moreover, we have also established several results and presented their fundamental properties with the help of some examples.

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