SNRU Journal of Science and Technology

Journal home page: snrujst.snru.ac.th

On quasi-ideals in left almost semirings

Pairote Yiarayong

Department of Mathematics, Faculty of Science and Technology, Pibulsongkram Rajabhat University, Phitsanuloke, 65000 Thailand

***Corresponding Author:** pairote0027@hotmail.com

Received: 21 July 2016; **Revised:** 31 December 2016; **Accepted:** 5 January 2017; **Available online** 1 April 2017

Abstract

In this paper, a new approach to left almost semiring theory is proposed by obtaining significant characterizations of regular, bi-ideal and quasi-ideal in left almost semirings via soft intersection left (right) ideals, bi-ideals, quasi-ideals of left almost semirings.

Keywords: left almost semiring; bi-ideal; regular; quasi-ideal; ideal

1. Introduction

A groupoid S is called a left almost semigroup abbreviated as an LA-semigroup, if its elements satisfy the left invertive law [1 – 2], that is: for all Several examples and interesting properties of LA-semigroups can be found in [3 – 6]. It has been shown in [3] that if an LA-semigroup contains a left identity then it is unique. It has been proved also that an LA-semigroup with right identity is a commutative monoid, that is, a semigroup with identity element. It is also known [1] that in an LA-semigroup, the medial law, that is, $(ab)(cd)=(ac)(bd)$, for all $a,b,c,d \in S$ holds. Now we define the concepts that we will used. Let S be an LA-semigroup. By an LA-subsemigroup of [7], we means a non-empty subset A of S such that $A^2 \subseteq A$. A non-empty subset A of an LA-semigroup S is called a left (right) ideal of [8] if $SA \subseteq A(AS \subseteq A)$. By two-sided ideal or simply ideal, we mean a non-empty subset of an LA-semigroup S which is both a left and a right ideal of S.

Yusuf in [9] introduces the concept of a left almost ring (LA-ring). That is, a non-empty set R with two binary operations "$+$" and "$\cdot$" is called a left almost ring, if $(R,+)$ is a LA-group, $(R,\cdot)$ is a LA-semigroup and distributive laws of "$\cdot$" over "$+$" holds. Further in [10] Shah and Rehman generalize the notions of commutative semigroup rings into LA-rings. However Shah and Fazal ur Rehman in [11] generalize the notion of a LA-ring into a near left almost ring. A near left almost ring (nLA-ringl N is a LA-group under "$+$", a LA-semigroup under "$\cdot$" and left distributive property of "$\cdot$" over "$+$" holds.

Shah, Fazal ur Rehman and Raees asserted that a commutative ring $(R,+,\cdot)$ we can always obtain a LA-ring $(R,\oplus,\cdot)$ by defining, for $a,b \in R, a \oplus b = b - a$ and $a \cdot b$ is same as in the ring. There are many mathematicians who added several results to the theory left almost ring (semigroup), see [12 – 14]. Furthermore, in this paper we characterize the regular, bi-ideal and quasi-ideal in left almost semirings.

2. Basic results

In this section we refer to [3] for some elementary aspects and quote few definitions and examples which are essential to step up this study.

Definition 1. [3] A left almost semiring is a triple $(R,+,\cdot)$ of a nonempty set R together with two binary operations "+" and "$\cdot$" (called addition and multiplication respectively) defined on R such that the following hold:

1. $(R,+)$ is a left almost semigroup.
2. $(R,\cdot)$ is a left almost semigroup.
3. $a(b+c) = ab + ac$ and (b+c)a $= ba + ca$, for all $a,b,c \in R$.

If R contains an element 0 such that $0 + x = x$ and $0x = 0$ for all $x \in R$, then 0 is called the left zero element of the left almost semiring R. Throughout this paper, R will always denote a left almost semiring with left zero and unless otherwise stated a left almost semiring means a left almost semiring with left zero.

Definition 2. A left almost semiring R with left identity e such that $e \cdot a = a$ for all $a \in R$, is called a left almost semiring with left identity.

Proposition 3. Let R be a left almost semiring with left identity. Then $RR = R$ and $R = eR = Re$.

Lemma 4. [3] Let R be a left almost semiring. Then $(ab)(cd) = (ac)(bd)$, for all $a,b,c \in R$.

Definition 5. [3] A non empty subset S of a left almost semiring R is said to be a left almost subsemiring if and only if S is itself a left almost semiring under the same binary operations as in R.

Lemma 6. A non empty subset S of a left almost semiring R is a left almost semiring if and only if $a + b \in S$ and $a \cdot b \in S$ for all $a,b \in S$.

Definition 7. [3] A left almost subsemiring I of a left almost semiring R is called a left ideal of R if $RI \subseteq I$, and I is called a right ideal if $IR \subseteq I$ and is called two sided ideal or simply ideal if it is both left and right ideal.

3. Bi-ideals, quasi-ideals in left almost semirings

We start with the following theorem that gives a relation between bi-ideals and quasi-prime ideals in a left almost semiring. Our starting point is the following lemma:

Lemma 8. Let R be a left almost semiring and $\varnothing \neq X \subseteq R$. If $\langle X \rangle = \left\{ \sum_{i=1}^{n} x_i : x_i \in X \right\}$, then $\langle X \rangle$ is a left almost subsemigroup of $(R,+)$

Proof. Let R be a left almost semiring and $x, y \in \langle X \rangle$. Then $x = \sum_{i=1}^{n} x_i$ and $y = \sum_{i=1}^{m} y_i$ where $x_i, y_i \in X$. If $n \geq m$, then

$$\begin{aligned}
x + y &= \sum_{i=1}^{n} x_i + \sum_{i=1}^{m} y_i \\
&= \left(\left(\left(x_1 + x_2 \right) + x_3 \right) + \ldots + x_n \right) + \left(\left(\left(y_1 + y_2 \right) + y_3 \right) + \ldots + y_m \right) \\
&= \left(\left(\left(x_1 + x_2 \right) + x_3 \right) + \ldots + x_n \right) + \left(\left(\left(\left(\left(0+0 \right) + \ldots + 0 \right) + y_1 \right) + y_2 \right) + \ldots + y_m \right) \\
&= \left(\left(\left(\left(x_1 + x_2 \right) + x_3 \right) + \ldots + x_{n-1} \right) + \left(\left(\left(\left(0+0 \right) + \ldots + 0 \right) + y_1 \right) + \ldots + y_{m-1} \right) \right) + \left(x_n + y_m \right) \\
&= \left(\left(\left(\left(x_1 + x_2 \right) + x_3 \right) + \left(\left(0+0 \right) + 0 \right) \right) + \ldots + \left(x_{n-1} + y_{m-1} \right) \right) + \left(x_n + y_n \right) \\
&= \left(\left(\left(\left(x_1 + x_2 \right) + \left(0+0 \right) \right) + \left(x_3 + 0 \right) \right) + \ldots + \left(x_{n-1} + y_{m-1} \right) \right) + \left(x_n + y_m \right) \\
&= \left(\left(\left(\left(x_1 + 0 \right) + \left(x_2 + 0 \right) \right) + \left(x_3 + 0 \right) \right) + \ldots + \left(x_{n-1} + y_{m-1} \right) \right) + \left(x_n + y_m \right) \\
&= \sum_{i=1}^{n} \left(x_i + y \right)_i \in \langle X \rangle \\
&= \sum_{i=1}^{\max\{n,m\}} \left(x_i + y \right)_i \in \langle X \rangle .
\end{aligned}$$

Hence $\langle X \rangle$ is a left almost subsemigroup of $(R,+)$.

Proposition 9. Let R be a left almost semiring and $\varnothing \neq X, Y \subseteq R$. Then $\langle X \rangle \langle Y \rangle \subseteq \langle XY \rangle$.

Proof. Let R be a left almost semiring with left identity and $x \in \langle X \rangle \langle Y \rangle$. Then

$$\begin{aligned}
x &= \sum_{i=1}^{n} x_i \sum_{i=1}^{m} y_i \\
&= \left(\left(\left(x_1 + x_2 \right) + x_3 \right) + \ldots + x_n \right) \sum_{i=1}^{m} y_i \\
&= \left(\left(\left(x_1 \sum_{i=1}^{m} y_i + x_2 \sum_{i=1}^{m} y_i \right) + x_3 \sum_{i=1}^{m} y_i \right) + \ldots + x_n \sum_{i=1}^{m} y_i \right) \\
&= \left(\left(\left(x_1 \sum_{i=1}^{m} y_i + x_2 \sum_{i=1}^{m} y_i \right) + x_3 \sum_{i=1}^{m} y_i \right) + \ldots + x_n \sum_{i=1}^{m} y_i \right) \\
&\in \left(\left(\left(\langle x_1 Y \rangle + \langle x_2 Y \rangle \right) + \langle x_3 Y \rangle \right) + \ldots + \langle x_n Y \rangle \right)
\end{aligned}$$

$$\subseteq \quad \left(\left(\langle XY\rangle + \langle XY\rangle\right) + \langle XY\rangle\right) + \ldots + \langle XY\rangle$$
$$\subseteq \quad \langle XY\rangle,$$

where $x_i \in X$ and $y_i \in Y$. Hence $\langle X\rangle\langle Y\rangle \subseteq \langle XY\rangle$.

Lemma 10. Let R be a left almost semiring with left identity. Then $\langle X \cup RX\rangle$ is a left almost subsemiring of R.

Proof. Let R be a left almost semiring with left identity. Then by lemma 8, we have $\langle X \cup RX\rangle$ is a left almost subsemigroup of $(R,+)$. Consider

$$\begin{aligned}
\langle X \cup RX\rangle\langle X \cup RX\rangle &\subseteq \langle (X \cup RX)(X \cup RX)\rangle \\
&= \langle (X \cup RX)X \cup (X \cup RX)(RX)\rangle \\
&= \langle XX \cup (RX)X \cup X(RX) \cup (RX)(RX)\rangle \\
&\subseteq \langle RX \cup (XX)R \cup R(XX) \cup (RR)(XX)\rangle \\
&\subseteq \langle RX \cup (XR)R \cup (XX)R \cup (XX)R\rangle \\
&\subseteq \langle RX \cup (RR)X \cup (XR)R \cup (XR)R\rangle \\
&\subseteq \langle RX \cup RX \cup (RR)X \cup (RR)X\rangle \\
&\subseteq \langle RX \cup RX \cup RX\rangle \\
&\subseteq \langle RX\rangle \\
&\subseteq \langle X \cup RX\rangle.
\end{aligned}$$

Then $\langle X \cup RX\rangle$ is a left almost subsemiring of R.

Theorem 11. Let R be a left almost semiring with left identity. Then $\langle X \cup RX\rangle$ is a bi-ideal of R.

Proof. Let R be a left almost semiring with left identity. Then by lemma 11, we get $\langle X \cup RX\rangle$ is a left almost subsemiring of R. Therefore

$$\begin{aligned}
\left(\langle X \cup RX\rangle R\right)\langle X \cup RX\rangle &= \left(\langle X \cup RX\rangle\langle R\rangle\right)\langle X \cup RX\rangle \\
&\subseteq \left(\langle (X \cup RX)R\rangle\right)\langle X \cup RX\rangle \\
&= \left(\langle XR \cup (RX)R\rangle\right)\langle X \cup RX\rangle \\
&\subseteq \langle (XR \cup (RX)R)(X \cup RX)\rangle \\
&= \langle (XR)(X \cup RX) \cup ((RX)R)(X \cup RX)\rangle \\
&= \langle (XR)X \cup (XR)(RX) \cup ((RX)R)X \cup ((RX)R)(RX)\rangle \\
&\subseteq \langle (RR)X \cup (XR)(RR) \cup (XR)(RX) \cup (XR)(R(RX))\rangle
\end{aligned}$$

$$
\begin{aligned}
&\subseteq \langle RX \cup (XR)R \cup (XR)(RR) \cup (XR)(R(RR)) \rangle \\
&= \langle RX \cup (RR)X \cup (XR)R \cup (XR)(RR) \rangle \\
&= \langle RX \cup RX \cup (RR)X \cup (XR)R \rangle \\
&= \langle RX \cup RX \cup (RR)X \rangle \\
&= \langle RX \cup RX \rangle \\
&= \langle RX \rangle \\
&= \langle X \cup RX \rangle .
\end{aligned}
$$

Then $\langle X \cup RX \rangle$ is a bi-ideal of R.

Proposition 12. Let R be a left almost semiring with left identity. Then $\langle X \cup XR \cup RX \rangle$ is an ideal of R.

Proof. Let R be a left almost semiring with left identity. Then by lemma 6, we have $\langle X \cup XR \cup RX \rangle$ is a left almost subsemigroup of $(R,+)$. By the definition of a left almost semiring, we get

$$
\begin{aligned}
\langle X \cup XR \cup RX \rangle \langle X \cup XR \cup RX \rangle &\subseteq \langle (X \cup XR \cup RX)(X \cup XR \cup RX) \rangle \\
&\subseteq \langle X(X \cup XR \cup RX) \cup (XR)(X \cup XR \cup RX) \cup \\
&\quad (RX)(X \cup XR \cup RX) \rangle \\
&= \langle XX \cup X(XR) \cup X(RX) \cup (XR)X \cup (XR)(XR) \cup \\
&\quad (XR)(RX) \cup (RX)X \cup (RX)(XR) \cup (RX)(RX) \rangle \\
&\subseteq \langle XR \cup X(RR) \cup X(RR) \cup (RR)X \cup (XR)(RR) \cup \\
&\quad (XR)(RR) \cup (RR)X \cup (RX)(RR) \cup (RX)(RR) \rangle \\
&= \langle XR \cup XR \cup XR \cup RX \cup R(XR) \cup R(XR) \cup RX \\
&\quad \cup (RR)(XR) \cup (RR)(XR) \rangle \\
&= \langle XR \cup RX \cup X(RR) \cup X(RR) \cup RX \cup R(XR) \\
&\quad \cup (RR)(XR) \rangle \\
&= \langle XR \cup RX \cup XR \cup XR \cup RX \cup X(RR) \cup R(XR) \rangle \\
&= \langle XR \cup RX \cup XR \cup X(RR) \rangle \\
&= \langle XR \cup RX \cup XR \rangle \\
&= \langle XR \cup RX \rangle \\
&\subseteq \langle X \cup XR \cup RX \rangle
\end{aligned}
$$

and

$$
\begin{aligned}
\langle X \cup XR \cup RX \rangle R &= \langle X \cup XR \cup RX \rangle \langle R \rangle \\
&\subseteq \langle (X \cup XR \cup RX) R \rangle \\
&= \langle XR \cup (XR) R \cup (RX) R \rangle \\
&= \langle XR \cup (RR) X \cup R(XR) \rangle \\
&= \langle XR \cup RX \cup X(RR) \rangle \\
&= \langle XR \cup RX \cup XR \rangle \\
&= \langle RX \cup XR \rangle \\
&\subseteq \langle X \cup XR \cup RX \rangle.
\end{aligned}
$$

Hence $\langle X \cup XR \cup RX \rangle$ is an ideal of R.

Theorem 13. Let R be a left almost semiring with left identity. Then $\langle X \cup X^2 \cup RX \rangle$ is a left ideal of R.

Proof. Let R be a left almost semiring with left identity. Then by lemma 8, we have $\langle X \cup X^2 \cup RX \rangle$ is a left almost subsemigroup of $(R,+)$. Thus

$$
\begin{aligned}
\langle X \cup X^2 \cup RX \rangle \langle X \cup X^2 \cup RX \rangle &\subseteq \langle (X \cup X^2 \cup RX)(X \cup X^2 \cup RX) \rangle \\
&= \langle (X \cup X^2 \cup RX) X \cup (X \cup X^2 \cup RX) X^2 \cup \\
&\quad (X \cup X^2 \cup RX)(RX) \rangle \\
&= \langle XX \cup X^2 X \cup (RX) X \cup XX^2 \cup X^2 X^2 \cup (RX) X^2 \cup \\
&\quad X(RX) \cup X^2 (RX) \cup (RX)(RX) \rangle \\
&\subseteq \langle RX \cup (XR) R \cup (RR) X \cup RX^2 \cup RX^2 \cup RX^2 \\
&\quad R(RX) \cup R(RX) \cup R(RX) \rangle \\
&\subseteq \langle RX \cup (RR) X \cup RX \cup RX^2 \cup R(RX) \rangle \\
&\subseteq \langle RX \cup RX \cup R(RX) \cup R(RX) \rangle \\
&= \langle RX \cup R(RX) \rangle \\
&= \langle RX \cup (XR) R \rangle \\
&= \langle RX \cup (RR) X \rangle \\
&= \langle RX \cup RX \rangle
\end{aligned}
$$

$$
\begin{aligned}
&= \langle RX \rangle \\
&\subseteq \langle X \cup X^2 \cup RX \rangle
\end{aligned}
$$

and

$$
\begin{aligned}
R\langle X \cup X^2 \cup RX \rangle &\subseteq \langle R \rangle \langle X \cup X^2 \cup RX \rangle \\
&\subseteq \langle R(X \cup X^2 \cup RX) \rangle \\
&= \langle RX \cup RX^2 \cup R(RX) \rangle \\
&\subseteq \langle RX \cup R(RX) \cup R(RX) \rangle \\
&= \langle RX \cup R(RX) \rangle \\
&= \langle RX \cup (XR)R \rangle \\
&= \langle RX \cup (RR)X \rangle \\
&= \langle RX \cup RX \rangle \\
&= \langle RX \rangle \\
&\subseteq \langle X \cup X^2 \cup RX \rangle
\end{aligned}
$$

Then $\langle X \cup X^2 \cup RX \rangle$ is a left ideal of R.

Theorem 14. Let A and B be two bi-ideals of left almost semiring with left identity R. Then $\langle AB \rangle$ is a bi-ideal of R.

Proof. Let A and B be two bi-ideals of left almost semiring with left identity R. Then by lemma 8, we have $\langle AB \rangle$ is a left almost subsemigroup of $(R,+)$. Consider

$$
\begin{aligned}
\langle AB \rangle \langle AB \rangle &\subseteq \langle (AB)(AB) \rangle \\
&= \langle (AA)(BB) \rangle \\
&= \langle AB \rangle
\end{aligned}
$$

and

$$
\begin{aligned}
(\langle AB \rangle R)\langle AB \rangle &\subseteq (\langle AB \rangle \langle R \rangle)\langle AB \rangle \\
&\subseteq \langle (AB)R \rangle \langle AB \rangle \\
&\subseteq \langle ((AB)R)(AB) \rangle \\
&= \langle ((AB)(RR))(AB) \rangle \\
&= \langle ((AR)(BR))(AB) \rangle \\
&= \langle ((AR)A)((BR)B) \rangle
\end{aligned}
$$

$$\subseteq \quad \langle AB \rangle.$$

Then $\langle AB \rangle$ is a bi-ideal of R.

Theorem 15. Let R be a left almost semiring with left identity. If A_i is a bi-ideal of R for all $i \in \beta$, then $\bigcap_{i\in\beta} A_i$ is a bi-ideal of R.

Proof. Assume that A_i is a bi-ideal of R for all $i \in \beta$. Let $x, y \in \bigcap_{i\in\beta} A_i$ and $r \in R$. Then $x, y \in A_i$, for all $i \in \beta$. Since for each $i \in \beta$, A_i is a bi-ideal of R, we get $(xr)y \in (A_i R)A_i$, for all $i \in \beta$. Therefore $(xr)y \in \bigcap_{i\in\beta} A_i$ and hence $\bigcap_{i\in\beta} A_i$ is a bi-ideal of R.

Definition 16. Let R be a left almost semiring. An element $x \in R$ is called regular in R if $x \in (xR)x$. R is called regular left almost semiring if each element of R is regular in R.

Lemma 17. Let R be a left almost semiring. Every right ideal of a regular is an ideal.

Proof. Let R is a regular left almost semiring and let I be its right ideal. Now for each $r \in R$ there exist $x \in R$ such that $r = (rx)r$. If $a \in I$, then

$$\begin{aligned} ra &= ((rx)r)a \\ &= (ar)(rx) \in I(rx) \subseteq I. \end{aligned}$$

Which implies that I is a left ideal. Hence I is an ideal of R.

Lemma 18. Let R is a regular left almost semiring. If A is right ideal and B is left ideal, $AB = A \cap B$.

Proof. Let A is right ideal and B is left ideal of R. It is easy to see that $AB \subseteq A \cap B$. Now let $x \in A \cap B$. Then there exist $a \in R$ such that $x = (xa)x \in AB$. Hence $AB = A \cap B$.

Definition 19. Let R is a regular left almost semiring. A left almost subsemigroup A of $(R,+)$. is called a quasi-ideal of R if $RA \cap AR \subseteq A$.

Remark. Let R is a regular left almost semiring.

1. Each quasi-ideal A of R is a left almost subsemiring. In fact, $AA \subseteq RA \cap AR \subseteq A$.
2. Every right ideal and every left ideal of R is a quasi-ideal of R.

Theorem 20. Let R left almost semiring. The intersection of a left ideal and a right ideal of R is a quasi- ideal of R.

Proof. Let A and B be left and right ideal of R. Consider

$$\begin{aligned} R(A\cap B)\cap(A\cap B)R &\subseteq RA\cap BR \\ &\subseteq A\cap B. \end{aligned}$$

Hence $A\cap B$ is a quasi-ideal of R.

Theorem 21. Let R be a left almost semiring with left identity. If A is a quasi-ideal of R, then $\langle A\cup RA\rangle$ is a left ideal of R.

Proof. Let A be a quasi-ideal of R. Then

$$\begin{aligned} R\langle A\cup RA\rangle &\subseteq \langle R\rangle\langle A\cup RA\rangle \\ &\subseteq \langle R(A\cup RA)\rangle \\ &= \langle RA\cup R(RA)\rangle \\ &= \langle RA\cup(eR)(RA)\rangle \\ &= \langle RA\cup(Re)(RA)\rangle \\ &= \langle RA\cup(RR)(eA)\rangle \\ &= \langle RA\cup RA\rangle \\ &= \langle RA\rangle \\ &\subseteq \langle A\cup RA\rangle \end{aligned}$$

This implies that $\langle A\cup RA\rangle$ is a left ideal of R.

Theorem 22. Let R be a left almost semiring with left identity. If A is a quasi-ideal of R such that $Ae=A$, then $\langle A\cup AR\rangle$ is an ideal of R.

Proof. Let A be a quasi-ideal of R. Then

$$\begin{aligned} \langle A\cup AR\rangle R &\subseteq \langle A\cup AR\rangle\langle R\rangle \\ &\subseteq \langle(A\cup AR)R\rangle \\ &= \langle AR\cup(AR)R\rangle \\ &= \langle AR\cup(AR)(eR)\rangle \\ &= \langle AR\cup(Ae)(RR)\rangle \\ &= \langle AR\cup(Ae)R\rangle \\ &= \langle AR\cup AR\rangle \\ &= \langle AR\rangle \end{aligned}$$

$$\subseteq \quad \langle A \cup AR \rangle.$$

Hence $\langle A \cup AR \rangle$ is an ideal of R.

Theorem 23. Let R be a left almost semiring with left identity .Then every quasi-ideal A of R such that $Ae = A$ is a bi-ideal of R.

Proof. Let S be a left almost semiring with left identity. Consider

$$\begin{aligned}(AR)A &\subseteq (AR)R\\ &= (AR)(eR)\\ &= (Ae)(RR)\\ &= AR\end{aligned}$$

and

$$\begin{aligned}(AR)A &\subseteq (RR)A\\ &= RA.\end{aligned}$$

This implies that $(AR)A \subseteq RA \cap AR \subseteq A$. Hence A is a bi-ideal of R.

4. Acknowledgement

The authors are very grateful to the anonymous referee for stimulating comments and improving presentation of the paper.

5. References

[1] P. Holgate, Groupoids satisfying a simple invertive law, Math. Stud. 61 (1992) 101 – 106.

[2] M.K. Kazim, M. Naseeruddin, On almost semigroups, Aligarh Bull. Math. 2 (1972) 1 – 7.

[3] R. Kellil, On inverses of left almost semirings and strong left almost semirings, J. Math. Sci. Adv. Appl. 26 (2015) 29 – 39.

[4] Q. Mushtaq, Abelian groups defined by LA-semigroups, Studia Sci. Math. Hungar. 18 (1983) 427 – 428.

[5] Q. Mushtaq, Q. Iqbal, Decomposition of a locally associative LA-semigroup, Semigroup Forum. 41 (1990) 154 – 164.

[6] Q. Mushtaq, M. Iqbal, On representation theorem for inverse LA-semigroups, Proc. Pakistan Acad. Sci. 30 (1993) 247 – 253.

[7] Q. Mushtaq, M. Khan, Ideals in left almost semigroup, arXiv:0904.1635v1 [math.GR], 2009.

[8] Q. Mushtaq, M. Khan, A note on an Abel-Grassmann's 3-band, Quasigroups Related Systems. 15 (2007) 295 – 301.

[9] S.M. Yusuf, On left almost ring, Proc. of 7th International Pure Mathematics Conference, 2006.

[10] M. Shah, T. Shah, Some basic properties of LA-ring, Int. Math. Forum. 6(44) (2011) 2195 – 2199.

[11] T. Shah, K. Yousaf, Topological LA-groups and LA-rings, Quasigroups Related Systems. 18 (2010) 95 – 104.

[12] I. Rehman, M. Shah, T. Shah, A. Razzaque, On existence of nonassociative LA-ring, An. St. Univ. Ovidius Constanta. 21(3) (2013) 223 – 228.

[13] M. Shah, I. Ahmad, A. Ali, Discovery of new classes of AG-groupoids, Res J Recent Sci. 1(11) (2012) 47 – 49.

[14] T. Shah, G. Ali, Fazal ur Rehman, Direct sum of ideals in a generalized LA-ring, Int. Math. Forum. 6(22) (2011) 1095 – 1101.