USING FUZZY LINEAR PROGRAMMING TO OPTIMIZE INVENTORY CONTROL POLICY IN A HYBRID MANUFACTURING/REMANUFACTURING SYSTEM

Kittiphan Nuamchit and Navee Chiadamrong*

Received: December 28, 2018; Revised: May 22, 2020; Accepted: May 24, 2020

Abstract

Remanufacturing can bring used or end of life products back to like-new products. The production planning of inventory control policies in a hybrid manufacturing/ remanufacturing system with different prioritizations (manufacturing vs. remanufacturing) is investigated in this study. For this production planning problem, the imprecision of customer demand, related operating costs, number of returned used components and all lead time and production times are uncertain. This uncertainty complicates the planning of the production and inventory control. The fuzzy set theory is employed due to the presence of the imprecise information, and the Fuzzy Linear Programming (FLP) is applied to optimize the model under these uncertainties. The proposed approach maximizes the most likely value of the profit, minimizes the risk of obtaining a lower profit, and maximizes the possibility of obtaining a higher profit for each production planning policy. The results show that the Priority-To-Remanufacturing (PTR) policy shows a higher profit than the policy of Priority-To-Manufacturing (PTM) and FLP can help decision makers to be aware of the risks and effects of uncertainties in their plans. As a result, they can prepare in advance for such scenarios.

Keywords: **Hybrid manufacturing/remanufacturing system, fuzzy linear programming, fuzzy optimization, fuzzy lead time and production time**

Sirindhorn International Institute of Technology, Thammasat University, Thailand. E-mail: navee@siit.tu.ac.th; kittipunhome@hotmail.com

* *Corresponding author*

Introduction

The process of remanufacturing consists of inspection, disassembly, sorting, cleaning, and inspection of used products before reassembling to like- new products or for use as the raw materials of new products. Remanufacturing can be separated into two groups. First, a completed remanufacturing system can be used only with used products that are used for all demands. Second, a hybrid manufacturing/ remanufacturing system can be used when the remanufacturing is required to work along with the existing manufacturing processes. As a result, new components may only be triggered when there is a shortage of raw materials from the returned used components.

The uncertain timing, quantity, and quality of returned products increase the difficulty of production planning and inventory management in remanufacturing and hybrid manufacturing/remanufacturing processes. Decision Makers (DMs) need to make decisions on how many new components or how many used products are required and especially on when such orders should be placed. Moreover, customer demand and related operating costs are subject to uncertainty. Customer demand normally varies with time due to a lack of information, errors in forecasting, and outdated information, while some operating costs are difficult to be fixed in advance. To manage such issues, a specified approach can be used to optimize such problems under the uncertainty. Instead of using traditional linear programming where the uncertainty is ignored, Fuzzy Linear Programming (FLP) is an approach that can be used to incorporate fuzzy data and optimize fuzzy problems. Our main contribution of this study is to introduce the Fuzzy Linear Programming (FLP) model to optimize the profit of the production planning and inventory control in a hybrid manufacturing/remanufacturing system. We can recommend a better policy with optimal operating parameters between the Priority-To-Remanufacturing (PTR) and Priority-To-Manufacturing (PTM) in an uncertain environment, especially under lead time and production time uncertainty. This time uncertainty can complicate the computations and generally has not been incorporated into the model solutions from past research. A sensitivity analysis is then performed with many scenarios to investigate the effects of each policy under the different levels of decision weight from the pessimistic, most likely or optimistic conditions. The results can provide a possible range of profit that occurs from implementing each policy in each scenario, which the indicates possible benefits and hidden risk from a policy.

The remaining paper is presented as follows. Section 2 presents a literature review. Section 3 shows the methodology for solving the stochastic model, based on the Fuzzy Linear Programming model. Section 4 presents a case study of inventory control and optimization in a hybrid manufacturing/ remanufacturing system, in an uncertain environment. Next, Section 5 shows the results and discussion. Section 6 then presents the conclusion.

Literature Review

Remanufacturing is a direct form of reuse that merges returned products, to sell like-new products (again) to customers. The remanufacturing process can be separated into disassembling the returned products, cleaning disassembled parts, replacing or repairing any worn or damaged components, testing the quality of products, updating electro-electronic products, and reassembling the products. Products that pass remanufacturing testing can have the quality of “like-new” products (Abbey *et al.*, 2015). According to Li *et al.* (2009), remanufacturing is normally managed under two business strategy models: dedicated and combined. The dedicated model is mostly applied in North America. It is considered standalone remanufacturing. In contrast, the combined model is mostly applied in European countries where remanufacturing and manufacturing are combined in the same line as a hybrid process.

For production planning and inventory control problems, the focus is placed on different inventory policies such as continuous and periodic reviews. There are many complicated characteristics of remanufacturing such as the uncertain quality of returns, delivery timing, and the need to balance the returns and new components with customer demand. Inderfurth (2004) found that the optimal inventory control policy for a hybrid manufacturing/remanufacturing system under strictly proportional costs and revenues is the order-up-to policy. The system is constructed in a single-period with stochastic returns of used products and customer demand, to use the raw materials of remanufactured products for manufacturing products when a shortage occurs. Wang (2011) investigated the optimal production policy for short life cycle products with stochastic returned products and customer demand. The objective was to minimize the total costs of this system. The optimal total costs are obtained when a combination of manufacturing, remanufacturing, and disposal was applied. A significant reduction in the total cost of the system can be obtained by setting an optimal ratio of remanufacturing products to manufactured products.

With complicated inventory control policies, the optimal settings for the parameters in a production planning and inventory control policy are required to obtain the maximum profitability of the system. Optimization can be performed by either analytical or heuristics models. The analytical models give global optimal solutions but require a longer processing time. The heuristics models may provide only local optimal solutions, which can be good enough solutions, with a shorter processing time. Proper solving method selection is a trade-off between solution quality and computational time. To reduce a long processing time, analytical models require simplified assumptions and present static results. Without the addition of uncertainties, solutions are compromised in real-world problems. Hence, the fuzzy logic is used for managing uncertainties and to produce results that, can reflect real-world problems. As a result, Fuzzy Linear Programming (FLP) can provide practical and optimal results under realistic circumstances (Amid *et al*., 2009).

In practice, input data or related parameters for production planning and control problems are imprecise/fuzzy, such as costs of operations, customer demand, number of returned products, delivery lead time and production lead time. These imprecise/fuzzy data occur because some information is incomplete or uncontrollable. These problems cannot be solved and optimized by a traditional mathematical analytical model such as linear programming because it only operates in deterministic circumstances. To incorporate uncertainty, Zimmerman (1976) introduced the fuzzy set theory into traditional linear programming problems. His study considered linear programming problems with fuzzy goals and constraints. Zadeh (1977) presented the theory of possibility, which is related to the theory of fuzzy sets by applying a possibility distribution as a fuzzy restriction, acting as an elastic constraint. In addition, Buckley (1989) presented Possibilistic Linear Programming (PLP) in a standard form with no equality constraints. Then, Ozgen and Gulsun (2014) applied a two-phase PLP combined with the fuzzy Analytical Hierarchical Process (AHP) to optimize multi objective linear programming. To our knowledge, there are very few research papers that have applied fuzzy lead time to their problems while solving the fuzzy mathematical model. For example, Diaz-Madroflero *et al.* (2015) applied fuzzy multi-objective integer linear programming to a model of the Material Requirements Planning (MRP) problem with the fuzzy lead time. By incorporating the different possibilities of lead times into the crisp MRP, the results showed that by combining the possibility of the existence of the lead times with the MRP model, decision makers would know their risk with the uncertainty of lead times. However, in order to defuzzify the fuzzy timing, considering the different possibilities of each lead time is not compatible with the FLP approach, which is required to solve under three cases (pessimistic, most likely, and optimistic). Our approach decides to use the fuzzy ranking to convert this uncertainty constraint into the crisp constraint. Hence, this research gap is further explored in our study.

As for the main contribution, our study applies the FLP approach for solving the production planning and inventory control policy in a hybrid manufacturing/ remanufacturing system with imprecise forecast demand, related operating costs, machine capacity, number of returned components, ordering lead time of new components, delivery lead time of returned components, and the production lead times of manufacturing and remanufacturing processes. Adding fuzzy ordering and production times to the fuzzy mathematical model is a new research area, which has not been much investigated in the past. The proposed approach can help capture the consequence of time variations with variations from other uncertainties. Our approach finds the most likely value of the imprecise total profits, minimizes the risk of obtaining a lower profit, and maximizes the possibility of simultaneously obtaining higher profits for each policy. This approach can also provide realistic results, which are better than a typical deterministic approach, as it can handle fuzzy data and recommend a range of possible optimal objective values. The result can help decision makers to be aware of possible outcomes in all scenarios from the most likely, optimistic, and pessimistic cases. Therefore, decision makers can prepare themselves in advance for such scenarios.

Methodology

Two optimization-solving methods are selected to evaluate their performances in a hybrid manufacturing/remanufacturing system in an uncertain environment. The system is modeled with both deterministic and stochastic conditions, using Mixed-Integer Linear Programming (MILP) and Fuzzy Linear Programming (FLP), respectively. Figure 1 shows the approach for each solving method.

Deterministic, Using MILP

The system is modeled, and the profit of the model is optimized using MILP. There is no uncertainty in this situation. As a result, the ideal solution from this method can be used as a benchmark for a comparison.

Stochastic, Using FLP

The FLP method can handle a problem with uncertainty. An uncertain environment contains uncertain customer demand in each week, related operating costs, number of arriving returned components in each week, and all related timing. FLP converts these uncertain/fuzzy data into crisp data. There are four main steps to solve FLP:

Model the Uncertainty Data With Triangular (Possibility) Distribution

Figure 2 presents the triangular distribution of an uncertain coefficient

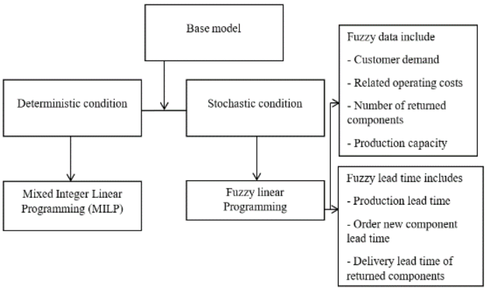

Figure 1. Method of approach

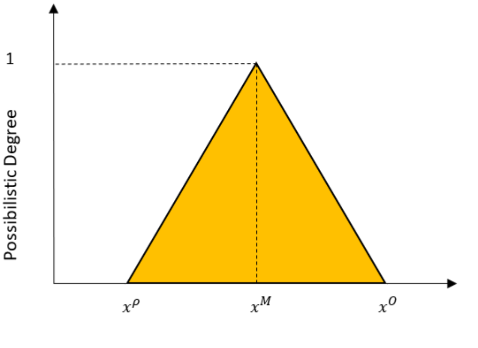

Figure 2. Triangular possibility distribution of $\tilde{x}$

$\tilde{x} = (x^P, x^M, x^O)$. In general, the triangular distribution is based on three important data values as follows.

1. The most pessimistic value (x^P) that has the lowest possibilistic degree for the set of available values (possibilistic degree = 0)
2. The most likely value (x^M) that has the highest possibilistic degree for the set of available values (possibilistic degree = 1)
3. The most optimistic value (x^O) that has the lowest possibilistic degree for the set of available values (possibilistic degree = 0)

Develop Three New Crisp Objective Functions of Multi Objective Linear Programming (MOLP)

The imprecise objective function in this model has a triangular distribution $\tilde{z} = (z^P, z^M, z^O)$. This imprecise objective function is defined by three important points $(z^P, 0)$, $(z^M, 1)$, and $(z^O, 0)$. The imprecise objective can be maximized by pushing three important points to the right. Because the vertical coordinate of the important points is fixed at either 1 or 0, only the three horizontal coordinates are considered. Solving the imprecise objective requires simultaneously maximizing $z^P, z^M, and\ z^O$. Instead of simultaneously maximizing $z^P, z^M, and\ z^O$, the proposed approach maximizes the profit z^M, minimizes the range of the profit of (z^M- z^P) and, maximizes the range of the profit of (z^O- z^M). This proposed approach involves maximizing the most likely value of the imprecise total profit (z^M), minimizing the risk of obtaining a lower profit (z^M- z^P), and maximizing the possibility of obtaining a higher profit (z^O-z^M). Figure 3 presents the strategy for maximizing the imprecise objective functions.

As presented in Figure 3, the possibility distribution $\tilde{A}$ is preferred to the possibility distribution $\tilde{B}$. The results for the three new crisp objective functions are presented as follows:

Max $z_1 = z^M$ (1)
Min $z_2 = (z^M - z^P)$ (2)
Max $z_3 = (z^O - z^M)$ (3)

Equation (1) to Equation (3) are equivalent to simultaneously maximizing the most likely value of the total profit, minimizing the risk of obtaining a lower profit (area A of the possibility distribution in Figure 3), and maximizing the possibility of obtaining a higher profit (area B of the possibility distribution in Figure 3).

Defuzzification Method

Convert uncertain constraints into crisp constraints using the weighted average method

We consider the situation where the number of returned products and customer demand are uncertain and have the triangular distribution with the most likely and least possible values. The problem is to obtain crisp numbers for the uncertainty of the number of returned products, and customer demand by applying the weighted average method to convert them into crisp values, where $w_1, w_2,$ and w_3 denote the weight of the pessimistic, most likely, and optimistic cases respectively. The weights $w_1, w_2,$ and w_3 can be determined by the experience of decision makers and $w_1 + w_2 + w_3 = 1$.

Convert uncertain constraints into crisp constraints using fuzzy ranking

We consider the situation where the lead times of delivery of the returned products, ordering new components, and production processes are uncertain under the triangular distribution. This step obtains crisp constraints from the lead times' variability. Fuzzy ranking is applied to convert them into crisp constraints.

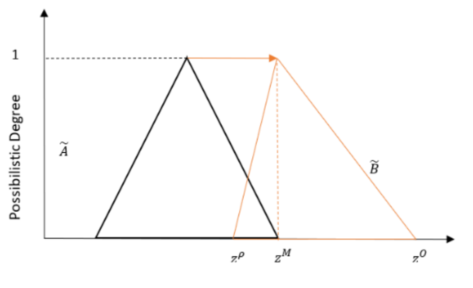

Figure 3. Strategy for maximizing the imprecise objective function

Specify the Linear Membership Functions for the Three New Objective Functions, and Then Convert the Auxiliary MOLP Problem Into an Equivalent Linear Programming Model Using the Fuzzy Decision Method

Find the lower bound and the upper bound of each objective.

To convert the auxiliary MOLP problem into an equivalent single-goal linear programming problem, the fuzzy decision method from Bellman and Zadeh (1970), and the Zimmermann (1978) fuzzy programming method are employed. The Negative Ideal Solution (NIS) and Positive Ideal Solution (PIS) of the three objective functions from Step two (above) are required. The three new crisp objective functions of the multi objective linear programming are:

$$z_1^{PIS} = \max z^M,\ z_1^{NIS} = \min z^M \tag{4}$$
$$z_2^{PIS} = \min (z^M - z^P),\ z_2^{NIS} = \max (z^M - z^P) \tag{5}$$
$$z_3^{PIS} = \max (z^O - z^M),\ z_3^{NIS} = \min (z^O - z^M) \tag{6}$$

Find the corresponding linear membership function of each objective function

The corresponding linear membership function for each objective function is defined by:

$$f1(z1) = \begin{cases} 1 & ,\ z_1 < z_1^{PIS}, \\ \frac{z_1 - z_1^{NIS}}{z_1^{PIS} - z_1^{NIS}}, & z_1^{NIS} \le z_1 \le z_1^{PIS}, \\ 0 & ,\ z_1 > z_1^{NIS}, \end{cases} \tag{7}$$

$$f2(z2) = \begin{cases} 1 & ,\ z_2 < z_2^{PIS}, \\ \frac{z_2^{NIS} - z_2}{z_1^{NIS} - z_2^{PIS}}, & z_2^{PIS} \le z_2 \le z_2^{NIS}, \\ 0 & ,\ z_2 > z_2^{NIS} \end{cases} \tag{8}$$

$$f3(z3) = \begin{cases} 1 & ,\ z_3 < z_3^{PIS}, \\ \frac{z_3 - z_3^{NIS}}{z_3^{PIS} - z_3^{NIS}}, & z_3^{NIS} \le z_3 \le z_3^{PIS}, \\ 0 & ,\ z_3 > z_3^{NIS}. \end{cases} \tag{9}$$

Each linear membership function is obtained by using a case study to specify an imprecise objective value in an interval (0-1). Figure 4 shows a graph of the linear membership functions for Equation (7) and Equation (9). Figure 5 shows a graph of the linear membership functions for Equation (8).

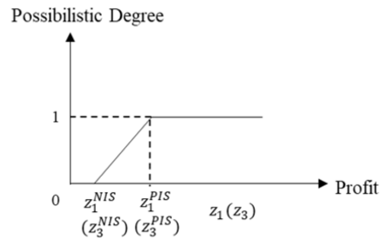

Figure 4. Linear membership function of z_1 and z_3

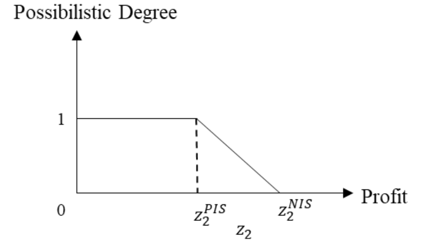

Figure 5. Linear membership function of z_2

Find the maximum overall satisfaction

The fuzzy decision method from Bellman and Zadeh (1970), and Zimmermann (1978) is used to formulate a single goal linear programming model, which is the maximin of the satisfaction values from the three above mentioned objective functions.

Max **λ**

Subject to

$\lambda \le f_i(z_i)$, i = 1,2,3

$0 \le \lambda \le 1$

i = number of linear membership functions of each objective.

Case Study

For the case study of the production planning and inventory control optimization in a hybrid manufacturing/remanufacturing in an uncertain environment, Figure 6 shows a general flow diagram of the system. This system requires two types of components for production: new components and returned components. Returned components indicate the used parts that are returned from customers. Returned components use the periodic review reorder cycle policy to control their inventory, as returned components arrive in a batch at the beginning of each week with a varying lead time from zero to one week (following the triangular distribution). A specified percentage of the returned components need to be disposed of to prevent surplus inventory. Accepted returned components are then stored in the Returned Component Inventory (RCI), waiting for the remanufacturing processes. The remanufacturing time of the returned components varies from zero to one week (following the triangular distribution). The remanufacturing cost per unit can be more expensive than the manufacturing cost per unit depending on the quality of the returned components. As a result, poor-quality components could incur more production expenses.

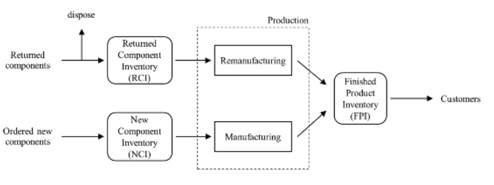

Figure 6. General flowchart of a hybrid manufacturing/remanufacturing system

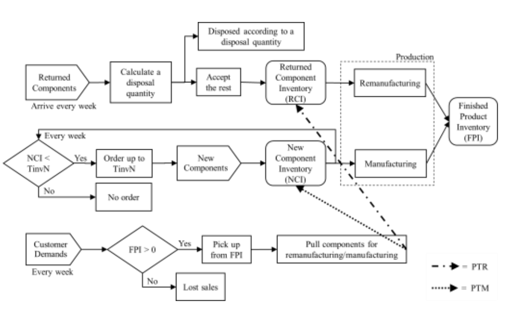

Figure 7. Flow diagram of the PTR and the PTM policies

There is also a New Component Inventory (NCI), which is also reviewed every week. The ordering lead time of the new components varies from one to two weeks (following the triangular distribution). After arrival, these new components are kept in the NCI before they are used in the manufacturing processes. Because of the uncertainty of the production lead time, it can vary from 1 to 2 weeks (following the triangular distribution). Finished products are then stored in a Finished Product Inventory (FPI), waiting for the customer demand.

Customer demand arrives every week. Lost sales occur when there are not enough finished products. Otherwise, customers pick up the finished products from the FPI. For finished products, the quality in remanufacturing and manufacturing is considered to be the same. When the inventory level of the FPI is reduced, the system triggers the production of products. The purpose is to restore the inventory level of the FPI. The initial inventory quantity is equal to the FPI target inventory level, which is one of the decision variables. Two inventory control policies (Priority-To-Remanufacturing vs Priority-To-Manufacturing) are imposed here to decide which process (remanufacturing vs manufacturing) has a higher priority. As shown in Figure 7, the operational details of each policy are presented as follows:

Priority-To-Remanufacturing (PTR)

With priority-to-remanufacturing, returned components are given a higher priority than new components unless they are not available. Manufacturing production is started only when there are not enough returned components in the RCI. The controlled decision variables in this policy include Disposal Rate ($disR$), Target Inventory Level of NCI ($TinvN$), and Target Inventory Level of FPI ($TinvF$). Some returned components are disposed of depending on $disR$. New components are ordered up to $TinvN$ in every review cycle (a week). The initial inventory level of NCI is equal to $TinvN$. When finished products are sold to customers, upstream components are pulled to replenish the taken products by filling the FPI up to $TinvF$ where the initial inventory level of FPI is also equal to $TinvF$.

Priority-To-Manufacturing (PTM)

In contrast to PTR, new components have a higher priority over returned components. Returned components are used for production only when there is a shortage of new components. Inventories are controlled by the decision variables, similar to PTR.

The system is investigated for one year (or 50 weeks). With 5 days a week and 8 h a day, there are 120,000 min a year. Table 1 presents the cost structure. Other relevant information is described below.

Table 1. Cost structure

Parameter	Notation	Cost ($ per unit)
Returned component disposal cost	RD_u	0
New component cost	$\widetilde{NC}_u$	(26,30,35)
Returned component preparation cost	$\widetilde{RP}_u$	(4,5,7)
Manufacturing cost	$\widetilde{MM}_u$	(9,10,13)
Remanufacturing cost	$\widetilde{RM}_u$	((9*(2- y), (10*(2- y)), (13*(2-y))
Returned component holding cost	$\widetilde{RH}_y$	(1.6,2,2.4) per year
New component holding cost	$\widetilde{NH}_y$	(10,12,15) per year
Finished product holding cost (from returned components)	$\widetilde{FHr}_y$	(9.5,11,14) per year
Finished product holding cost (from new components)	$\widetilde{FHm}_y$	(9.5,11,14) per year
Lost sales cost	$\widetilde{LS}_u$	(45,50,58)
Sales price	$\widetilde{Price}_u$	(45,50,58)

Remark: y is the yield of returned components, which is set equal to (0.3,0.65,1). For example, the remanufacturing cost of the most likely case is calculated as (10*(2-0.65)) = $ 13.5 per unit

Other Relevant Information

1. The maximum levels of the target inventory for the new component inventory and finished product inventory depend on the maximum of the production capacity, which fluctuates under the normal distribution in an uncertain environment. The mean and standard deviation of the production capacity is 150 and 20 units per week, respectively.

2. The overall customer service level in this case study is set to at least 85%.

3. The profits of all policies in all scenarios must be higher than or at least equal to $0.

4. The returned component ratio is set to 0.6. This is the ratio of arriving returned components per total customer demand. Both arriving returned components and total customer demand fluctuate under the normal distribution in an uncertain environment. The mean and standard deviation for total customer demand is 100 and 20 units per week, and 60 and 10 units per week for the returned components, respectively.

5. A supplier has the ability to supply new components, but not more than 70 units per week.

6. The holding cost is 40% per year for a unit (i. e., new component cost, returned component preparation cost, finished product cost). It is assumed that the finished product holding costs per unit of both manufacturing ($\widetilde{FHm}_y$) and remanufacturing ($\widetilde{FHr}_y$) products are the same based on the average unit cost value from both types of products.

7. The symbol "~" refers to ambiguous data that are determined to be fuzzy in this study.

LP Formulation (Deterministic case)

The mathematical model presented in this case study is formulated as a linear programming model. Notations and the analytical model formulation are presented below where t refers to the time in weeks, ranging from 1 to 50.

Parameters

Production Planning Parameters (units)

$ReA_{\tilde{t}}$ = Arriving returned components in week $\tilde{t}$

$dispose_t$ = Returned components (disposed of) in week t

$R0_t$ = Returned components accepted to inventory in week t

RCI_t = Ending inventory level in the Returned Component Inventory (RCI) in week *t*

$R1_{\tilde{t}}$ = Returned components sent to remanufacturing in week $\tilde{t}$

$R2_t$ = Finished products from remanufacturing in week *t*

$FPIr_t$ = Ending inventory level in Finished Product Inventory (FPI) from remanufacturing in week *t*

$R3_t$ = Finished products from remanufacturing sent to customer in week *t*

$order_{\tilde{t}}$ = New components ordered in week $\tilde{t}$

NCI_t = Ending inventory level in the New Component Inventory (NCI) in week *t*

$M1_{\tilde{t}}$ = New components sent to manufacturing in week $\tilde{t}$

$M2_t$ = Finished products from manufacturing in week *t*

$FPIm_t$ = Ending inventory level in the Finished Product Inventory (FPI) from manufacturing in week *t*

$M3_t$ = Finished products from manufacturing sent to customer in week *t*

FPI_t = Ending inventory level in the Finished Product Inventory (FPI) in week *t*

D_t = Customer demand in week *t*

LS_t = Lost sales in week *t*

PC_t = Production capacity to produce products in week *t*

Cost Parameters ($)

RD = Total returned component disposal cost

LS = Total lost sales cost

RMM = Total remanufacturing and manufacturing cost

NC = Total new component cost

RP = Total returned component cost

CH = Total component holding cost

FH = Total finished product holding cost

TC = Total costs

$Revenue$ = Total income from selling finished products

$Profit$ = Total income after deducting total costs

Decision Variables

$TinvN$ = Target inventory level in NCI (units)

$TinvF$ = Target inventory level in FPI (units)

$disR_t$ = Disposal rate in week *t* (%)

Objective Function:

Maximize

$$\widetilde{Profit} = \widetilde{Revenue} - \widetilde{TC} \quad (10)$$

subject to

inventory balance constraints:

$$\widetilde{ReA}_{\tilde{t}} = dispose_t + R0_t \quad for\ all\ t \quad (11)$$

$$R2_t = R1_{\tilde{t}} \quad for\ all\ t \quad (12)$$

$$M2_t = \begin{cases} 0 & where\ t = 1 \\ M1_{\tilde{t}-1} & where\ t = 2\ to\ t = 50 \end{cases} \quad (13)$$

$$R3_t + M3_t + LS_t = \widetilde{D_t} \quad for\ all\ t \quad (14)$$

$$FPI_t = FPIr_t + FPIm_t \quad for\ all\ t \quad (15)$$

$$\widetilde{PC}_t \geq R1_{\tilde{t}} + M1_{\tilde{t}} \quad for\ all\ t \quad (16)$$

$$RCI_t = \begin{cases} R0_t - R1_t & where\ t = 1 \\ RCI_{t-1} + R0_t - R1_{\tilde{t}} & where\ t = 2\ to\ t = 50 \end{cases} \quad (17)$$

$$NCI_t = \begin{cases} TinvN - M1_{\tilde{t}} & where\ t = 1 \\ NCI_{t-1} + order_{\tilde{t}-1} - M1_{\tilde{t}} & where\ t = 2\ to\ t = 50 \end{cases} \quad (18)$$

$$FPIr_t = \begin{cases} R2_t - R3_t & where\ t = 1 \\ FPIr_{t-1} + R2_t - R3_t & where\ t = 2\ to\ t = 50 \end{cases} \quad (19)$$

$$FPIm_t = \begin{cases} TinvF - M3_t & where\ t = 1 \\ FPIm_{t-1} + M2_t - M3_t & where\ t = 2\ to\ t = 50 \end{cases} \quad (20)$$

where $\widetilde{ReA}_t$, $\widetilde{D_t}$, and $\tilde{t}$ are uncertain coefficients with the triangular distribution. The objective function (Constraint (10)) maximizes the profit. Constraint (11) and Constraint (12) are inventory balance constraints. For Constraint (11), some of the returned components are disposed of, and the rest is sent to the Returned Component Inventory (RCI). Constraint (12) ensures that the quantity of returned components sent to remanufacturing processes is the same quality as the finished products coming out of the remanufacturing processes within the same week and the remanufacturing time is negligible. Constraint (13) states that the manufacturing lead time is one week. For Constraint (14), customer demand is satisfied by the finished products, otherwise, lost sales occur. For Constraint (15), total units of finished products in remanufacturing and manufacturing are combined in the FPI. Constraint (16) describes the constraint of production capacity for remanufacturing and manufacturing, in which the number of components sent to the manufacturing process must be less than or equal to the production capacity. Constraints (17) to (20) describe the constraints for returned components, new components, finished products for remanufacturing, and finished products for manufacturing, respectively. The ending inventory is equal to the previous ending inventory (week), plus the incoming inventory, minus the outgoing inventory. In the first week, the ending inventory is equal to the incoming inventory, minus the outgoing inventory.

The PTR constraints:

$$RCI_t = \begin{cases} 0 & if\ \widetilde{D_t} \geq R0_t & where\ t = 1 \\ 0 & if\ \widetilde{D_t} \geq R0_t + RCI_{t-1} & where\ t = 2\ to\ t = 50 \\ any\ integer & otherwise & for\ all\ t \end{cases} \quad (21)$$

$$R0_t = \begin{cases} \widetilde{D_t} & if\ \widetilde{D_t} < \widetilde{ReA_t} & for\ all\ t \\ \widetilde{ReA_t} & if\ \widetilde{ReA_t} < \widetilde{D_t} & for\ all\ t \end{cases} \quad (22)$$

$$R1_{\tilde{t}} = \begin{cases} \widetilde{D_t} & if\ \widetilde{D_t} < R0_t + RCI_{t-1} & for\ all\ t \\ any\ integer & otherwise & for\ all\ t \end{cases} \quad (23)$$

where $\widetilde{ReA_t}$, and $\widetilde{D_t}$ are uncertain coefficients with the triangular distribution. Constraints (21) and (23) describe the policies of priority-to-remanufacturing. All returned components in the RCI are sent to remanufacturing when the customer demand is greater than the incoming returned components plus the number of the previous week's ending inventory. Otherwise the number of returned components sent to remanufacturing is equal to the customer demand.

The PTM constraints:

$$NCI_t = \begin{cases} 0 & if\ \widetilde{D_t} \geq TinvN & where\ t = 1 \\ order_{\tilde{t}-1} & if\ \widetilde{D_t} \geq NCI_{t-1} & where\ t = 2\ to\ t = 50 \\ any\ integer & otherwise & for\ all\ t \end{cases} \quad (24)$$

$$M1_t = \begin{cases} \widetilde{D_t} & if\ \widetilde{D_t} < NCI_{t-1} & for\ all\ t \\ any\ integer & otherwise & for\ all\ t \end{cases} \quad (25)$$

where $\widetilde{D_t}$ are uncertain coefficients with the triangular distribution. Constraints (24) and (25) describe the policies of the priority- to-manufacturing, where the number of new components in the NCI that is sent to manufacturing is equal to the customer demand when it is less than the number of new components. Otherwise, all of the returned components are sent to manufacturing. The ending inventory in the NCI is equal to the incoming orders from the previous week when the customer demand is greater than the previous period's ending inventory.

Economic constraints:

$$RD = \sum_t RD\_u * dispose_t \quad for\ all\ t \quad (26)$$

$$\widetilde{LS} = \sum_t LS\_u * \widetilde{LS_t} \quad for\ all\ t \quad (27)$$

$$\widetilde{RMM} = \sum_t \widetilde{RM\_u} * R1_t + \sum_t \widetilde{MM\_u} * M1_t + \widetilde{MM_u} * TinvF \quad for\ all\ t \quad (28)$$

$$\widetilde{NC} = \sum_t \widetilde{NC_u} * (order_t + initial\ NCI + TinvF) \quad for\ all\ t \quad (29)$$

$$\widetilde{RP} = \sum_t \widetilde{RP\_u} * R0_t \quad for\ all\ t \quad (30)$$

$$\widetilde{CH} = \left(\frac{\widetilde{RH\_y}}{50} * (R0_1 + RCI_1)\right)/2 + \sum_{t=2}^{50}\left(\frac{\widetilde{RH\_y}}{50} * (R0_t + RCI_{t-1} + RCI_t)\right)/2 + \left(\frac{\widetilde{NH\_y}}{50} * (initial\ NCI + NCI_1)\right)/2 + \sum_{t=2}^{50}\left(\frac{\widetilde{NH\_y}}{50} * (order_{t-1} + NCI_{t-1} + NCI_t)\right)/2 \quad (31)$$

$$\widetilde{FH} = \left(\frac{\widetilde{FHr\_y}}{50} * (R2_1 + FPIr_1)\right)/2 + \sum_{t=2}^{50}\left(\frac{\widetilde{FHr\_y}}{50} * (R2_t + FPIr_{t-1} + FPIr_t)\right)/2 + \left(\frac{\widetilde{FHm\_y}}{50} * (M2_1 + TinvF + FPIm_1)\right)/2 + \sum_{t=2}^{50}\left(\frac{\widetilde{FHm\_y}}{50} * (M2_t + FPIm_{t-1} + FPIm_t)\right)/2 \quad (32)$$

$$\widetilde{TC} = \widetilde{RD} + \widetilde{LS} + \widetilde{RMM} + \widetilde{NC} + \widetilde{RP} + \widetilde{CH} + \widetilde{FH} \quad (33)$$

$$\widetilde{Revenue} = \sum_t \widetilde{Price\_u} * (R3_t + M3_t) \quad for\ all\ t \quad (34)$$

$$\widetilde{Profit} = \widetilde{Revenue} - \widetilde{Total\ costs} \quad (35)$$

where $\widetilde{LS}_t$, $\widetilde{RM\_u}$, $\widetilde{MM\_u}$, $\widetilde{NC}_u$, $\widetilde{RP\_u}$, $\widetilde{NH\_y}$, $\widetilde{RH\_y}$, $\widetilde{FHr\_y}$, $\widetilde{FHm\_y}$, and $\widetilde{Price\_u}$ are uncertain coefficients with the triangular distribution. Constraints (26) to (32) describe the cost parameters to calculate the total cost (TC). The TC consists of the Returned Component Disposal Cost (RD), Lost Sales Cost (LS), Remanufacturing and Manufacturing Cost (RMM), New Component Cost (NC), Returned Component Preparation Cost (RP), Component Holding Cost (CH), and Finished Product Holding Cost (FH). All holding costs are calculated based on the average inventory level. Constraint (35) states that the profit is calculated by the revenue (Constraint (34)) minus the total costs (Constraint (33)).

Decision Variables:

$$order_{\tilde{t}} = TinvN - NCI_t \quad for\ all\ t \quad (36)$$

$$R1_{\tilde{t}} + M1_{\tilde{t}} = TinvF - FPI_t \quad for\ all\ t \quad (37)$$

$$disR_t = 100 * (dispose_t / ReA_t) \quad for\ all\ t \quad (38)$$

For Equation (36), the order of new components depends on $TinvN$. Equation (37) shows that the total number of new and returned components sent to production is equal to the number of finished products. Equation (38) describes the disposal rate, which is the percentage of disposed of components over the returned components.

Fuzzy Linear Programming (FLP)

We develop the fuzzy linear programming method to solve this problem under uncertainty by developing three new crisp objective functions of the multi-objective linear programming (MOLP) to replace Equations (1)-(3)

$$\begin{aligned} \text{Max } z_1 &= Profit^M \\ &= Revenue^M - TC^M \\ &= Revenue^M \text{-} RD^M + LS^M + RMM^M + NC^M + RP^M + CH^M + FH^M \end{aligned} \quad (39)$$

$$\begin{aligned} \text{Min } z_2 &= Profit^{P-M} \\ &= Revenue^{P-M} - TC^{P-M} \\ &= Revenue^{P-M} - RD^{P-M} + LS^{P-M} + RMM^{P-M} + NC^{P-M} + RP^{P-M} + CH^{P-M} + FH^{P-M} \end{aligned} \quad (40)$$

$$\begin{aligned} \text{Max } z_3 &= Profit^{M-O} \\ &= Revenue^{M-O} - TC^{M-O} \\ &= Revenue^{M-O} - RD^{M-O} + LS^{M-O} + RMM^{M-O} + NC^{M-O} + RP^{M-O} + CH^{M-O} + FH^{M-O} \end{aligned} \quad (41)$$

As the demand of customers, quantity of returned components, and production capacity are fuzzy, a defuzzification method using the weighted average method is applied to solve the imprecise data. Constraints (11), (14), (16), (21), (22), (23), (24), and (25) need to be transformed into crisp constraints as follows:

$$w_1 ReA_{\tilde{t}}^P + w_2 ReA_{\tilde{t}}^M + w_3 ReA_{\tilde{t}}^O = dispose_t + R0_t \quad for\ all\ t \quad (42)$$

$$R3_t + M3_t + LS_t = w_1 d_t^P + w_2 d_t^M + w_3 d_t^O \quad for\ all\ t \quad (43)$$

$$w_1 PC_t^P + w_2 PC_t^M + w_3 PC_t^O \geq R1_t + M1_t \quad for\ all\ t \quad (44)$$

$$RCI_t = \begin{cases} 0 & if\ w_1D_t^P + w_2D_t^M + w_3D_t^O \geq R0_t & where\ t = 1 \\ 0 & if\ w_1D_t^P + w_2D_t^M + w_3D_t^O \geq R0_t + RCI_{t-1} & where\ t = 2\ to\ t = 50 \\ any\ integer & otherwise & for\ all\ t \end{cases} \quad (45)$$

$$R0_t = \begin{cases} w_1D_t^P + w_2D_t^M + w_3D_t^O & if\ w_1D_t^P + w_2D_t^M + w_3D_t^O < w_1ReA_t^P + w_2ReA_t^M + w_3ReA_t^O \quad for\ all\ t \\ w_1ReA_t^P + w_2ReA_t^M + w_3ReA_t^O & if\ w_1ReA_t^P + w_2ReA_t^M + w_3ReA_t^O < w_1D_t^P + w_2D_t^M + w_3D_t^O \quad for\ all\ t \end{cases} \quad (46)$$

$$R1_{\tilde{t}} = \begin{cases} w_1 D_t^P + w_2 D_t^M + w_3 D_t^O \\ \quad \text{if } w_1 D_t^P + w_2 D_t^M + w_3 D_t^O < R0_t + RCI_{t-1} & \text{for all } t \\ \text{any integer} \quad \text{otherwise} & \text{for all } t \end{cases} \tag{47}$$

$$NCI_t = \begin{cases} 0 & \text{if } w_1 D_t^P + w_2 D_t^M + w_3 D_t^O \geq TinvN & \text{where } t = 1 \\ order_{t-1} & \text{if } w_1 D_t^P + w_2 D_t^M + w_3 D_t^O \geq NCI_{t-1} & \text{where } t = 2 \text{ to } t = 50 \\ \text{any integer} & \text{otherwise} & \text{for all } t \end{cases} \tag{48}$$

$$M1_{\tilde{t}} = \begin{cases} w_1 D_t^P + w_2 D_t^M + w_3 D_t^O & \text{if } w_1 D_t^P + w_2 D_t^M + w_3 D_t^O < NCI_{t-1} \quad \text{for all } t \\ \text{any integer} & \text{otherwise} \quad \text{for all } t \end{cases} \tag{49}$$

where $w_1 + w_2 + w_3 = 1$ and the weights of w_1, w_2, and w_3 are determined by the experience and knowledge of decision makers.

As the production lead time, ordering lead time, and returned component delivery time are fuzzy, a defuzzification method using fuzzy ranking is applied to solve these imprecise data. Constraints (12), (13), (18), and (42) need to be transformed into crisp constraints as follows:

For constraint (12):

When the remanufacturing lead time is 1 week

$$R2_1 \leq 0 \quad \text{for all } t \tag{50}$$

$$R2_t \leq R1_{t-1} \quad \text{for all } t \tag{51}$$

When the remanufacturing lead time is negligible

$$R2_t \leq R1_t \quad \text{for all } t \tag{52}$$

Constraints (50) and (51) show the case when the remanufacturing processing time is 1 week. As a result, in the first week, there are no finished product from the remanufacturing processes. Constraint (52) shows the case when the remanufacturing processing time is zero.

For constraint (13):

When the manufacturing processing time is 2 weeks

$$M2_t \leq \begin{cases} 0 & \text{where } t = 1 \\ 0 & \text{where } t = 2 \\ M1_{t-2} & \text{where } t = 3 \text{ to } t = 50 \end{cases} \tag{53}$$

When the manufacturing processing time is 1 week.

$$M2_t \leq \begin{cases} 0 & \text{where } t = 1 \\ M1_{t-1} & \text{where } t = 2 \text{ to } t = 50 \end{cases} \tag{54}$$

Constraint (53) shows the case when the manufacturing processing time is 2 weeks. As a result, there are no finished products from the manufacturing processes in week 1 and week 2 because the manufacturing process takes 2 weeks to produce. Constraint (54) shows the case when the manufacturing lead time is 1 week.

For constraint (18)

When the order lead time is 2 weeks

$$NCI_t \leq \begin{cases} TinvN - M1_1 & \text{where } t = 1 \\ NCI_1 - M1_2 & \text{where } t = 2 \\ NCI_{t-1} + order_{t-2} - M1_t & \text{where } t = 3 \text{ to } t = 50 \end{cases} \tag{55}$$

When the order lead time is 1 week

$$NCI_t \leq \begin{cases} TinvN - M1_t & \text{where } t = 1 \\ NCI_{t-1} + order_{t-1} - M1_t & \text{where } t = 2 \text{ to } t = 50 \end{cases} \tag{56}$$

Constraint (55) shows the case when the order lead time is 2 weeks. There are no new components in week 1 and week 2 since the ordering process takes 2 weeks for delivery. Constraint (56) shows the case when the order lead time is 1 week. New components from the ordering process in week 1 will arrive in week 2.

For constraint (42):

When the returned components lead time is 1 week

$$R0_1 <= 0 \tag{57}$$

$$w_1 ReA_{t-1}^P + w_2 ReA_{t-1}^M + w_3 ReA_{t-1}^O \geq dispose_t + R0_t \quad \text{where } t = 2 \text{ to } t = 50 \tag{58}$$

When the returned components lead time is negligible (assuming 0 weeks).

$$w_1 ReA_t^P + w_2 ReA_t^M + w_3 ReA_t^O \geq dispose_t + R0_t \quad \text{for all } t \tag{59}$$

Constraints (57) and (58) show a "worst case" that the returned component lead time is 1 week. As a result, in the first week, there are no returned components since it takes one week for delivery. Constraint (59) shows that in the case when the returned component lead time is zero, some of the returned components are disposed of depending on the disposal rate in every week, and the rest is sent to the Returned Component Inventory (RCI).

The linear membership functions for the three new objective functions are specified, and we convert the auxiliary MOLP problem into an equivalent linear programming model by the fuzzy decision method.

To find the Negative Ideal Solution (NIS) and the Positive Ideal Solution (PIS) of each objective, the fuzzy decision method as presented in Equations (4) to (6) is introduced to solve MOLP by finding the corresponding linear membership function of each objective function.

$$z_1^{PIS} = \max Profit^M,\ z_1^{NIS} = \min Profit^M \tag{60}$$

$$z_2^{PIS} = \min Profit^{P-M},\ z_2^{NIS} = \max Profit^{P-M} \tag{61}$$

$$z_3^{PIS} = \max Profit^{M-O},\ z_3^{NIS} = \min Profit^{M-O} \tag{62}$$

Equations (7) to (9) are applied to the corresponding linear membership functions for each objective function in this case study as follows:

$$f1(z1) = \begin{cases} 1, & z_1 < z_1^{PIS}, \\ \dfrac{z_1 - z_1^{NIS}}{z_1^{PIS} - z_1^{NIS}}, & z_1^{NIS} \leq z_1 \leq z_1^{PIS}, \\ 0, & z_1 > z_1^{NIS}, \end{cases}$$

$$f2(z2) = \begin{cases} 1, & z_2 < z_2^{PIS}, \\ \dfrac{z_2^{NIS} - z_2}{z_1^{NIS} - z_2^{PIS}}, & z_2^{PIS} \leq z_2 \leq z_2^{NIS}, \\ 0, & z_2 > z_2^{NIS}, \end{cases}$$

$$f3(z3) = \begin{cases} 1, & z_3 < z_3^{PIS}, \\ \dfrac{z_3 - z_3^{NIS}}{z_3^{PIS} - z_3^{NIS}}, & z_3^{NIS} \leq z_3 \leq z_3^{PIS}, \\ 0, & z_3 > z_3^{NIS}. \end{cases}$$

Find the maximum overall satisfaction:

Max $\boldsymbol{\lambda}$

Subject to

$\lambda \leq f_i(z_i)$, where i = 1,2,3

i = number of linear membership function of each objective.

$0 \leq \lambda \leq 1$

$R0_1 <= 0$,

$w_1 ReA_{t-1}^P + w_2 ReA_{t-1}^M + w_3 ReA_{t-1}^O \geq dispose_t + R0_t$ $where\ t = 2\ to\ t = 50,$

$w_1 ReA_t^P + w_2 ReA_t^M + w_3 ReA_t^O \geq dispose_t + R0_t$ $for\ all\ t,$

$R2_1 \leq 0$ $for\ all\ t,$

$R2_t \leq R1_{t-1}$ $for\ all\ t,$

$R2_t \leq R1_t$ $for\ all\ t,$

$$M2_t \leq \begin{cases} 0 & where\ t = 1, \\ 0 & where\ t = 2, \\ M1_{t-2} & where\ t = 3\ to\ t = 50, \end{cases}$$

$$M2_t \leq \begin{cases} 0 & where\ t = 1, \\ M1_{t-1} & where\ t = 2\ to\ t = 50, \end{cases}$$

$R3_t + M3_t + LS_t = w_1 d_t^P + w_2 d_t^M + w_3 d_t^O$ $for\ all\ t$,

$FPI_t = FPIr_t + FPIm_t$ $for\ all\ t,$

$w_1 PC_t^P + w_2 PC_t^M + w_3 PC_t^O \geq R1_t + M1_t$ $for\ all\ t,$

$$RCIt = \begin{cases} R0_t - R1_t & where\ t = 1, \\ RCI_{t-1} + R0_t - R1_t & where\ t = 2\ to\ t = 50, \end{cases}$$

$$NCI_t \leq \begin{cases} TinvN - M1_1 & where\ t = 1, \\ NCI_1 - M1_2 & where\ t = 2, \\ NCI_{t-1} + order_{t-1} - M1_t & where\ t = 3\ to\ t = 50, \end{cases}$$

$$NCI_t \leq \begin{cases} TinvN - M1_t & where\ t = 1, \\ NCI_{t-1} + order_{t-1} - M1_t & where\ t = 2\ to\ t = 50, \end{cases}$$

$$FPIr_t = \begin{cases} R2_t - R3_t & where\ t = 1, \\ FPIr_{t-1} + R2_t - R3_t & where\ t = 2\ to\ t = 50, \end{cases}$$

$$FPIm_t = \begin{cases} TinvF - M3_t & where\ t = 1, \\ FPIm_{t-1} + M2_t - M3_t & where\ t = 2\ to\ t = 50, \end{cases}$$

For PTR

$$RCI_t = \begin{cases} 0 \quad if\ w_1 D_t^P + w_2 D_t^M + w_3 D_t^O \geq R0_t & where\ t = 1, \\ 0 \quad if\ w_1 D_t^P + w_2 D_t^M + w_3 D_t^O \geq R0_t + RCI_{t-1} & where\ t = 2\ to\ t = 50, \\ any\ integer \quad otherwise & for\ all\ t, \end{cases}$$

$$R0_t = \begin{cases} w_1 D_t^P + w_2 D_t^M + w_3 D_t^O \\ \qquad if\ w_1 D_t^P + w_2 D_t^M + w_3 D_t^O < w_1 ReA_t^P + w_2 ReA_t^M + w_3 ReA_t^O & for\ all\ t, \\ w_1 ReA_t^P + w_2 ReA_t^M + w_3 ReA_t^O \\ \qquad if\ w_1 ReA_t^P + w_2 ReA_t^M + w_3 ReA_t^O < w_1 D_t^P + w_2 D_t^M + w_3 D_t^O & for\ all\ t, \end{cases}$$

$$R1_t = \begin{cases} w_1 D_t^P + w_2 D_t^M + w_3 D_t^O \\ \qquad if\ w_1 D_t^P + w_2 D_t^M + w_3 D_t^O < R0_t + RCI_{t-1} & for\ all\ t, \\ any\ integer \quad otherwise & for\ all\ t, \end{cases}$$

For PTM

$$NCI_t = \begin{cases} 0 & if\ w_1D_t^P + w_2D_t^M + w_3D_t^O \geq TinvN \quad where\ t = 1 \\ order_{t-1} & if\ w_1D_t^P + w_2D_t^M + w_3D_t^O \geq NCI_{t-1} \quad where\ t = 2\ to\ t = 50 \\ any\ integer & otherwise \quad for\ all\ t \end{cases}$$

$$M1_t = \begin{cases} w_1D_t^P + w_2D_t^M + w_3D_t^O & if\ w_1D_t^P + w_2D_t^M + w_3D_t^O < NCI_{t-1} \quad for\ all\ t \\ any\ integer & otherwise \quad for\ all\ t \end{cases}$$

$$RD = \sum_t RD\_u * dispose_t \quad for\ all\ t,$$

$$\widetilde{LS} = \sum_t LS\_u * \widetilde{LS}_t \quad for\ all\ t,$$

$$\widetilde{RMM} = \sum_t \widetilde{RM\_u} * R1_t + \sum_t \widetilde{MM\_u} * M1_t + \widetilde{MM}_u * TinvF \quad for\ all\ t,$$

$$\widetilde{NC} = \sum_t \widetilde{NC}_u * (order_t + initial\ NCI + TinvF) \quad for\ all\ t,$$

$$\widetilde{RP} = \sum_t \widetilde{RP\_u} * R0_t \quad for\ all\ t \quad ,$$

$$\widetilde{CH} = \left(\frac{\widetilde{RH\_y}}{50} * (R0_1 + RCI_1)\right)/2 + \sum_{t=2}^{50}\left(\frac{\widetilde{RH\_y}}{50} * (R0_t + RCI_{t-1} + RCI_t)\right)/2 + \left(\frac{\widetilde{NH\_y}}{50} * (initial\ NCI + NCI_1)\right)/2 + \sum_{t=2}^{50}\left(\frac{\widetilde{NH\_y}}{50} * (order_{t-1} + NCI_{t-1} + NCI_t)\right)/2,$$

$$\widetilde{FH} = \left(\frac{\widetilde{FHr\_y}}{50} * (R2_1 + FPIr_1)\right)/2 + \sum_{t=2}^{50}\left(\frac{\widetilde{FHr\_y}}{50} * (R2_t + FPIr_{t-1} + FPIr_t)\right)/2 + \left(\frac{\widetilde{FHm\_y}}{50} * (M2_1 + TinvF + FPIm_1)\right)/2 + \sum_{t=2}^{50}\left(\frac{\widetilde{FHm\_y}}{50} * (M2_t + FPIm_{t-1} + FPIm_t)\right)/2,$$

$$\widetilde{TC} = \widetilde{RD} + \widetilde{LS} + \widetilde{RMM} + \widetilde{NC} + \widetilde{RP} + \widetilde{CH} + \widetilde{FH},$$

$$\widetilde{Revenue} = \sum_t \widetilde{Price\_u} * (R3_t + M3_t) \quad for\ all\ t,$$

$$\widetilde{Profit} = \widetilde{Revenue} - \widetilde{Total\ costs}\ .$$

Results and Discussion

Deterministic Case with the Linear Programming Model

An LP model is constructed and solved by IBM ILOG CPLEX Optimization Studio software. The deterministic results of the PTR and the PTM policies are shown in Table 2.

Table 2 compares the solutions obtained from these two policies. The result shows that the PTR policy is better than the PTM policy in the deterministic case, because the PTR policy can generate a higher profit than the PTM policy. From the result, the PTR policy and the PTM policy do not dispose of any returned components, but the PTM policy has a higher cost from ordering new components (NC). However, it should be noted that the difference of the profits from these two policies is close, with less than a 4% gap.

Stochastic Case with the Fuzzy Linear Programming Model (FLP)

With the lower bound and the upper bound of each objective (Constraints (60) to (62)), Tables 3-4 show the Negative Ideal Solution (NIS) and the Positive Ideal Solution (PIS) of each objective of the PTR and the PTM policies, respectively.

According to Constraints (42) to (49) of the Fuzzy Linear Programming model, the defuzzification of the imprecise customer demand, number of arriving returned components, and production capacity with the weighted average method is subject to the patterns of weight allocation. Different

Table 2. PTR and PTM results

		Policy	
		PTR	**PTM**
Decision Variables	Target inventory of new components (TinvN) (units)	47	40
	Target inventory of finished products (TinvF) (units)	92	167
	Number of disposed returned components (units)	0	0
Cost Parameters	Lost sales cost (LS)	$0	$0
	New Component Cost (NC)	$62,970.00	$66,210.00
	Returned Component Cost (RP)	$15,380.00	$15,005.00
	Remanufacturing and Manufacturing Cost (RMM)	$62,046.00	$62,184.00
	Total Component Holding Cost (CH)	$390.52	$501.18
	Finished Product Holding Cost (FH)	$751.52	$593.34
	Total costs (TC)	$141,538.04	$144,493.52
	Revenue	$256,400.00	$256,400.00
	Profit	$114,861.96	$111,906.48

Table 3. Upper and lower boundary of the PTR policy

PTR	
MOLP	**Profit ($)**
z_1^{PIS} = max $Profit^M$	128,923.50
z_1^{NIS} = min $Profit^M$	45,359.18
z_2^{PIS} = min $Profit^{P-M}$	53,085.93
z_2^{NIS} = max $Profit^{P-M}$	85,315.70
z_3^{PIS} = max $Profit^{M-O}$	81,955.51
z_3^{NIS} = min $Profit^{M-O}$	50,251.90

Table 4. Upper and lower boundary of the PTM policy

PTM	
MOLP	**Profit ($)**
z_1^{PIS} = max $Profit^M$	125,540.90
z_1^{NIS} = min $Profit^M$	0
z_2^{PIS} = min $Profit^{P-M}$	52,459.07
z_2^{NIS} = max $Profit^{P-M}$	85,008.50
z_3^{PIS} = max $Profit^{M-O}$	81,596.26
z_3^{NIS} = min $Profit^{M-O}$	49,675.30

Table 5. Sensitivity analysis of the PTR policy

	Scenario I			**Scenario II**			**Scenario III**			**Scenario IV**		
	P	**M**	**O**	**P**	**M**	**O**	**P**	**M**	**O**	**P**	**M**	**O**
	33%	**33%**	**33%**	**80%**	**10%**	**10%**	**10%**	**80%**	**10%**	**10%**	**10%**	**80%**
Decision Variables												
Target inventory of new components (TinvN) (units)		43			171			41			39	
Target inventory of finished products (TinvF) (units)		196			171			192			204	
Number of disposed returned component (units)		4			8			3			0	
Overall satisfaction		53.69%			37.58%			53.21%			38.16%	
Z_1 ($)		103,420			77,369			97,824			77,270	
Z_2 ($)		68,011			64,014			68,165			73,017	
Z_3 ($)		67,275			62,165			67,122			70,427	
Maximize the most likely value of profit ($)		103,420			77,369			97,824			77,270	
Minimize the risk of obtaining lower profit ($)		35,409			13,355			29,659			4,253	
Maximize of the possibility of obtaining a higher profit ($)		170,695			139,534			164,946			147,697	

patterns of weight allocation can have an impact on the obtained solution. A sensitivity analysis of this weight allocation should be carried out among pessimistic, most likely, and optimistic cases to investigate the impact of such weight allocation on the overall satisfaction of the linear membership functions of each objective function. For a demonstration, a sensitivity analysis is performed for 4 scenarios by varying the weights of pessimistic, most likely, and optimistic values. Tables 5-6 show the results of the sensitivity analysis for both the PTR and the PTM policies.

For example, in Table 5 of the PTR policy with Scenario I, equal weights (33%) are assigned to the pessimistic, most likely, and optimistic values. It is found that the overall satisfaction is 53.69% The outcomes of the maximum most likely value of profit, minimum possibility of obtaining a lower profit, and maximum of the risk of obtaining a higher profit are $103,420, $35,409, and $170,695, respectively.

The result of this sensitivity analysis also shows that Scenario I, which sets the weight of the most likely value to 33% and the weight of optimistic and pessimistic values to 33%, gives the highest overall satisfaction of 53.69%.

Table 6. Sensitivity analysis of the PTM policy

	Scenario I			Scenario II			Scenario III			Scenario IV		
	P	M	O	P	M	O	P	M	O	P	M	O
	33%	33%	33%	80%	10%	10%	10%	80%	10%	10%	10%	80%
Decision Variables												
Target inventory of new components (TinvN) (units)	64			52			58			34		
Target inventory of finished products (TinvF) (units)	197			171			199			208		
Number of disposed returned components (units)	1,315			0			1,088			95		
Overall satisfaction	53.12%			43.42%			52.71%			38.34%		
Z_1 ($)	67,720			63,961			67,852			72,528		
Z_2 ($)	67,720			63,961			67,852			72,528		
Z_3 ($)	66,630			61,744			66,500			69,900		
Maximize the most likely value of profit ($)	67,720			63,961			67,852			72,528		
Minimize the risk of obtaining lower profit ($)	0			0			0			0		
Maximize of the possibility of obtaining a higher profit ($)	134,350			125,705			134,352			142,428		

Scenario I also gives the highest possible value of the most likely profit, the profit from the risk of obtaining a lower profit, and the profit from maximizing the risk of obtaining a higher profit.

In this Scenario I, the system adjusts itself to avoid the high cost of shortage by keeping the high target inventory level of finished products and disposing of only a few units of returned components. However, when the weight of the optimistic value is set to 80%, and 10% for both the pessimistic and the most likely values (Scenario IV), the optimistic value of the end customer demand could lead to too high demand for a limited production capacity, causing a high shortage cost. This would eventually deteriorate the profit of the system.

For example, in Table 6 of the PTM policy with Scenario I, equal weights (33%) are assigned to the pessimistic, most possible, and optimistic values. It is found that the overall satisfaction is 53.12%. The outcomes of the maximum most possible value of profit, minimum possibility of obtaining a lower profit, and maximum of the risk of obtaining a higher profit are $67,720, $0, and $134,350, respectively.

The best result of the PTM policy is from setting the weight of the optimistic value to 80%, and 10% for both the pessimistic and the most likely value (Scenario IV). In contrast to the PTR policy, the profit generated from Scenario IV (the optimistic case) is higher than the profit from Scenario I (equal weight). This is because Scenario IV uses a smaller number of new components and use more returned components than the case of setting an equal weight to all three values (Scenario I). This is because the PTM policy is required to use the new components first but the new components take a longer time to arrive (with the fuzzy delivery time) than the returned components. As a result, to prevent a shortage, the arriving returned components would be selected for production.

In a comparison between the two policies, it is found that the PTR policy can outperform the PTM policy in terms of profit for all objective functions and in all scenarios. In addition, the profits of the PTM policy during the pessimistic case in all scenarios are found to be as low as 0, whereas the lowest profit of the PTR policy is $4,253, and the lowest profit of its best scenario could be up to $35,409. This demonstrates the superiority of the PTR policy, even during the pessimistic case. Focusing on the effects of fuzzy lead time and production time, the actual inventory needs to be calculated at all points based on the worst scenario in each period, using the defuzzification method with fuzzy ranking.

Table 7. Assumed data for calculating values with varying lead times

Parameter (units)	Case 1	Case 2
Ending inventory level in the New Component Inventory (NCI) in week 2 (NCI_2)	10	10
New components ordered in week 1 ($Order_1$)	3	10
New components ordered in week 2 ($Order_2$)	5	5
New components sent to manufacturing in week 3 ($M1_3$)	3	3

For a demonstration, the current values of related values in week 3 are assumed and shown in Table 7.

For Case 1, according to Constraints (55) and (56), when the pessimistic lead time is 2 weeks, the new component inventory in week 3 is calculated as follows:

$$NCI_t \leq NCI_{t-1} + Order_{t-2} - M1_t$$
$$NCI_3 \leq NCI_2 + Order_1 - M1_3$$
Therefore, $NCI_3 \leq 10$ units

However, when the optimistic and the most likely lead time is 1 week, the new component inventory in week 3 is calculated as follows:

$$NCI_t \leq NCI_{t-1} + Order_{t-1} - M1_t$$
$$NCI_3 \leq NCI_2 + Order_2 - M1_3$$
Therefore, $NCI_3 \leq 12$ units

As a result, the actual number of new components would be 10 units for the pessimistic lead time period, as it is the worst case.

For Case 2, according to Constraints (55) and (56) where the pessimistic lead time is 2 weeks, the new component inventory in week 3 is calculated as follows:

$$NCI_t \leq NCI_{t-1} + Order_{t-2} - M1_t$$
$$NCI_3 \leq NCI_2 + Order_1 - M1_3$$
Therefore, $NCI_3 \leq 17$ units

When the optimistic and most likely lead time is 1 week, the new component inventory in week 3 is calculated as follows:

$$NCI_t \leq NCI_{t-1} + Order_{t-1} - M1_t$$
$$NCI_3 \leq NCI_2 + Order_2 - M1_3$$
So, $NCI_3 \leq 12$ units

As a result, the actual number of new components would be 12 units from the optimistic lead time period as it is the worst case. In addition, it was also found that the possibility to obtain the lower profit of the PTM policy can be as low as zero or even a loss as its costs are high due to a longer lead time than the PTR policy, especially in the pessimistic case. This indicates the possible hidden risk from the policy.

Comparison Between the Deterministic and the Best Stochastic Results

Table 8 compares the results between the optimal LP and the best FLP results for

Table 8. Comparison of results

	PTR Policy	
Priority-To-Remanufacturing policy (PTR)	Linear Programming (LP) model	Fuzzy Linear Programming (FLP) model from Scenario I
Objective function λ	Max z	Max λ
(overall degree of satisfaction)	100%	53.69%
z (Profit)	$ 114,861.96	($35,409, $103,420, $170,695)
	PTM Policy	
Priority-To-Manufacturing policy (PTR)	Linear Programming (LP) model	Fuzzy Linear Programming (FLP) model from Scenario IV
Objective function λ	Max z	Max λ
(overall degree of satisfaction)	100%	38.34%
z (Profit)	$ 111,906.48	($0, $72,528, $142,428)

deterministic and realistic stochastic conditions. As seen in Table 8, the most likely profit that is generated from the uncertain circumstance is lower as compared to the profit under no uncertainty because the overall degree of satisfaction is lower. The results from the scenario that yields the lowest degree of satisfaction among the three objectives are recommended, which are maximizing the most likely value of profit, minimizing the risk obtaining a lower profit, and maximizing the possibility of obtaining a higher profit. The PTR policy under Scenario I and the PTM policy under Scenario IV show the best results among the experimental scenarios. However, the PTR policy generates a higher profit than the PTM policy in all cases. With the range of profit from each scenario obtained from FLP, decision makers are prepared for any expected outcome. This is an important feature for decision makers, to build production planning and inventory control systems that are subject to an uncertain environment.

Conclusions

Production planning and inventory control is intermediate planning for finding suitable levels of disposal units, production units, lost sales, inventory, and stock outs. This intermediate planning compensates for uncertainty in the forecast demand and fluctuations of production lead time, order lead time, returned component delivery time, related operating costs, number of returned components, and production capacity. This study finds the optimal inventory control policy of a hybrid manufacturing/ remanufacturing system under two priority policies, Priority-To-Remanufacturing (PTR) and Priority-To-Manufacturing (PTM). Fuzzy linear programming is introduced to optimize and find the optimal result in this uncertain environment. It maximizes the most possible value of the imprecise profit, minimizes the risk of obtaining a lower profit, and maximizes the possibility of obtaining a higher profit by pushing the three prominent points with the triangular distribution towards the right (as profit maximization). Results from this deterministic case show that the PTR policy is slightly better than the PTM policy. The results from the stochastic case also confirm that the PTR policy is better than the PTM policy in every scenario. When the returned component ratio is further reduced to be lower than 60% (0.6), the profit of the PTM is nearly equal to the profit of the PTR policy.

The possibility to obtain the lower profits of the PTM policy can be as low as zero or there may even be a loss as its costs may dramatically increase in the pessimistic scenario, especially when the timing is uncertain. This indicates a possible hidden risk from the policy. As shown in the case study, the proposed method can be used to solve most real- world planning problems involving imprecise parameters through an interactive decision- making process. The proposed method constitutes a systematic framework that facilitates the decision- making process, enabling a decision maker to interactively modify the solution for imprecise data until a satisfactory solution is found. Notably, the optimal goal values using the LP approach are imprecise since the forecasted demand, related operating costs, number of returned components, ordering lead time, production lead time, and returned component delivery time are always imprecise. This situation is closer to real-world problems. Decision makers should have some knowledge in advance to prepare and take necessary action for future uncertainty. This study focuses on multi- periods with a single product from a hybrid manufacturing/remanufacturing problem. It also provides information on different policies in response to any designed variations. Additionally, the approach also considers the actual tradeoffs between the PTM and the PTR policies in each circumstance. This lets us know how each policy would be beneficial or vulnerable depending on the uncertain conditions.

The main limitation of our case study is the assumption of the triangular distribution that represents imprecise data. Decision makers should generate and obtain appropriate distributions based on true judgment and historical resources. Future researchers can also explore different levels of the relative importance of individual goals and different types of distributions, to make their models better fit their practical applications.

Reference

Abbey, J.D., Meloy, M.G., Blackburn, J., and Guide, V.D.R. (2015). Consumer markets for remanufactured and refurbished products. California Management Review, 57(4):26–42.

Amid, A., Ghodsypour, S.H., and Brien, C.O. (2009). A weighted additive fuzzy multiobjective model for the supplier selection problem under price breaks in a supply Chain. Int. J. Product. Econ., 121(2):323–332.

Bellman, R.E. and Zadeh, L.A. (1970). Decision-making in a fuzzy environment. Manage. Sci., 17(4):B-141-B-164.

Buckley, J.J. (1989). Solving possibilistic linear programming problems. Fuzzy Sets Syst., 31(3):329–341.

Diaz-Madroflero, M., Mula, J., and Jiménez, M. (2015). Material requirement planning under fuzzy lead times. IFAC-PapersOnLine, 28(3):242–247.

Inderfurth, K. (2004). Optimal policies in hybrid manufacturing/remanufacturing systems with product substitution. Int. J. Product. Econ., 90(3):325–343.

Li, J., González, M., and Zhu, Y. (2009). A hybrid simulation optimization method for production planning of dedicated remanufacturing. Int. J. Product. Econ., 117(2): 286–301.

Ozgen, D. and Gulsun, B. (2014). Combining possibilistic linear programming and fuzzy AHP for solving the multi-objective capacitated multi-facility location problem. Inform. Sci., 268:185–201.

Wang, J., Zhao, J., and Wang, X. (2011). Optimum policy in hybrid manufacturing/remanufacturing system. Comp. Indust. Eng., 60(3):411–419.

Zadeh, L.A. (1977). Fuzzy sets as a basis for a theory of possibility. Fuzzy Sets Syst., 100:9–34.

Zimmermann, H.J. (1978). Fuzzy programming and linear programming with several objective functions. Fuzzy Sets Syst., 1(1):45–55.