

TWO-DIMENSIONAL FINITE IMPULSE RESPONSE ZERO-PHASE DESIGN BASED ON A NON-UNIFORM FREQUENCY SAMPLING METHOD

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Abstract

A design technique for a 2-dimensional finite impulse response filter based on a non-uniform frequency sampling approach is presented. Employing the zero-phase condition, the numbers of constrained equations are reduced and the filter coefficients of the resulting filter are all real numbers. In this method, a cost function can be constructed by a summation of the magnitude response deviation of all the sampling points. The Lagrange multiplier technique is utilized for the optimal solutions.

Keyword: 2-D FIR filter, zero-phase, non-uniform sampling, optimal filter, Lagrange multiplier

Introduction

Finite impulse response (FIR) filters have been widely used in the field of 1-dimensional and 2-dimensional signal processing. The design of 2-dimensional digital filters has received considerable interest over the last few years. The principal reasons for this are the inherent stability of FIR filters, the ease of design, and the ability to reach a linear (or zero) phase characteristic in the frequency response. The FIR filter is utilized in a variety of applications in many fields such as image processing, geophysical data processing, medical diagnosis, etc. Generally, the design problem for both 1-dimensional and 2-dimensional filters is about determining the

filter coefficients that meet the specification of the filter design. Over the past few decades, several methods for the design of a 2-D FIR filter have been proposed. The standard methods are the window method (Lim, 1990), which is one of the simplest techniques for 2-D FIR filter design, the frequency transformation method (Mersereau, 1980; Pei and Wu, 1982), the optimal filter approaches (Charalambous and Khreishi, 1988; Rajaravivarma and Rajan, 1992; Venkatachalam and Aravena, 1997), and the frequency sampling method (Rozwod *et al.*, 1991; Angelidis, 1995, 1994).

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Among those design approaches, the frequency sampling method has been least studied. This method can be classified into 2 categories. The first is uniform sampling where the desired frequency response values are specified at certain sample points in the frequency domain. 2-D inverse discrete Fourier transform can be efficiently implemented to solve the impulse response coefficients. The apparent disadvantage of the uniform-sampling approach is that there is no flexibility for the sampled frequency placement and the filter order is generally high. The second category is non-uniform sampling. This approach allows frequency sampling to occur anywhere in the (ω_1, ω_2) plane. 2-D FIR filter design based on non-uniform sampling can be implemented by solving the linear equations directly or by other efficient methods that have been proposed in Rozwod *et al.*, 1991; Angelidis, 1994; 1995; Kuang *et al.*, 2012. In this study, a novel method for the design of a 2-D FIR filter based on the non-uniform sampling approach is described. The Lagrange multiplier is implemented to solve the system of equations in this design. This paper is organized as follows: the preliminary definition about zero-phase filters is stated in Section 2; the detail of the proposed method is explained in Section 3 and a design example is demonstrated in Section 4. Finally, the conclusion is discussed in Section 5.

Zero-Phase Filters

A nonlinear phase of filter distorts the proper registration of the different frequency components that make the lines and scratches occur. One of the good characteristics of a zero-phase filter is that it can preserve the shape of the signal component in the passband region, making it very useful, especially in image processing. A zero-phase characteristic is easy to achieve with the FIR filter design. Furthermore, the design implementation of the FIR is simplified if the zero-phase is specified. A digital filter is called a zero-phase filter when its frequency response is a real function (Lim, 1990). This can be expressed

as

$$H(\omega_1, \omega_2) = H^*(\omega_1, \omega_2). \quad (1)$$

From the symmetry properties of the Fourier transform, (1) is equivalent to

$$h(n_1, n_2) = h^*(-n_1, -n_2). \quad (2)$$

Since only real-valued coefficients are considered, (2) is reduced to

$$h(n_1, n_2) = h(-n_1, -n_2). \quad (3)$$

The property (3) indicates that the impulse response of a zero-phase filter is symmetric with respect to the origin.

The frequency response of an $N_1 \times N_2$ filter FIR with the origin symmetry can be written as

$$H(\omega_1, \omega_2) = \sum_{n_1=0}^{N_1} \sum_{n_2=0}^{N_2} a(n_1, n_2) \cos(\omega_1 n_1 + \omega_2 n_2) \quad (4)$$

where

$$a(n_1, n_2) = \begin{cases} h(n_1, n_2) & n_1 = 0, n_2 = 0 \\ 2h(n_1, n_2) & \text{otherwise} \end{cases} \quad (5)$$

From (4) and (5), $H(\omega_1, \omega_2)$ for a zero-phase FIR filter can be expressed as a linear combination of cosine terms of the form $\cos(\omega_1, n_1 + \omega_2, n_2)$. Since the filter in (4) is symmetric with respect to the origin, approximately half of the points in $h(n_1, n_2)$ are independent. This filter is said to have a twofold symmetry.

Proposed Method

A novel method for the design of a 2-D FIR zero-phase filter is described in this section. The method proposed here is based on a non-uniform frequency sampling approach. The sampling points can be selected anywhere in the (ω_1, ω_2) plane where $0 \leq \omega_1 \leq \pi$ and $0 \leq \omega_2 \leq \pi$. The filter coefficients are achieved from equations (4) and (5).

The aim of this method is to minimize the cost function constructed from the non-uniform frequency sampling approach. This

method can achieve an optimal solution for filter coefficients that satisfy the desired frequency response. The number of sampling points, k , is equal to $(N_1 + 1)(N_2 + 1)$. Thus, the frequency response of the designed filter can be written in matrix form, as below:

$$\begin{bmatrix} H(\omega_{11}, \omega_{21}) \\ H(\omega_{12}, \omega_{22}) \\ \vdots \\ H(\omega_{1k}, \omega_{2k}) \end{bmatrix} = \begin{bmatrix} 1 & 2\cos(\omega_{11}0 + \omega_{21}1) & \dots & 2\cos(\omega_{11}N_1 + \omega_{21}N_2) \\ 1 & 2\cos(\omega_{12}0 + \omega_{22}1) & \dots & 2\cos(\omega_{12}N_1 + \omega_{22}N_2) \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 2\cos(\omega_{1k}0 + \omega_{2k}1) & \dots & 2\cos(\omega_{1k}N_1 + \omega_{2k}N_2) \end{bmatrix} \begin{bmatrix} h(0,0) \\ h(0,1) \\ \vdots \\ h(N_1, N_2) \end{bmatrix} \quad (6)$$

$$\Delta = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \end{bmatrix}$$

The cost function, $C(x_1, \dots, x_k)$, for this filter design problem can be constructed by computing the squared magnitude for each sampled point, $x_1, \dots, x_k$, and summing up all the values.

For the sampled points which are in the passband, for which the magnitude is ideally 1, their cost contributions are assigned to be $(1 - x_r^2)^2$. For the sampled points which are in the stopband, for which the magnitude is ideally 0, their cost contributions are assigned to be x_r^2 . The cost function is written as

$$C = \sum_{r=1}^s x_r^2 + \sum_{r=s+1}^k (1 - x_r^2)^2, \quad (7)$$

where s is the number of sampled points which are located in the stopband region and the number of sampled points which are located in the passband region are obviously $k - s$.

The problem then becomes a minimization of the cost function subject to (6). The Lagrange multiplier method is utilized to determine the optimal solution to this problem. The Lagrangian for this problem can be expressed as

$$\begin{aligned} L(x_1, \dots, x_k, h(0,0), \dots, h(n_1, n_2), \lambda_1, \dots, \lambda_k) \\ = C(x_1, \dots, x_k) - \sum_{m=1}^k \lambda_m g(x_m) \end{aligned} \quad (8)$$

where

$$\begin{aligned} g_m = h(0,0) + \sum_{n_1=0}^2 \sum_{n_2=0}^2 2h(n_1, n_2) \\ \cos(\omega_{1k}n_1 + \omega_{2k}n_2) - x_m \end{aligned} \quad (9)$$

In order to determine the stationary points

that satisfy every constraint, equation (8) takes the partial derivatives with respect to all variables, $x_1, \dots, x_k, h(0, 0), \dots, h(n_1, n_2)$ and $\lambda_1, \dots, \lambda_k$. Then, every equation is set to 0. This process is simply expressed as equation (10) or as the matrix form in (11):

$$\nabla L(x_1, \dots, x_k, h(0,0), \dots, h(n_1, n_2), \lambda_1, \dots, \lambda_k) = 0 \quad (10)$$

$$\begin{bmatrix} \frac{\partial L}{\partial x_1} & \dots & \frac{\partial L}{\partial x_k} & \frac{\partial L}{\partial \lambda_1} & \dots & \frac{\partial L}{\partial \lambda_k} & \frac{\partial L}{\partial h(0,0)} & \dots & \frac{\partial L}{\partial h(n_1, n_2)} \end{bmatrix}^T = 0 \quad (11)$$

The system of equations in (11) is divided into 3 sets as follows:

$$\begin{bmatrix} \frac{\partial L}{\partial \lambda_1} \\ \frac{\partial L}{\partial \lambda_2} \\ \vdots \\ \frac{\partial L}{\partial \lambda_k} \end{bmatrix} = 0 \quad (12)$$

$$\begin{bmatrix} \frac{\partial L}{\partial x_1} \\ \frac{\partial L}{\partial x_2} \\ \vdots \\ \frac{\partial L}{\partial x_k} \end{bmatrix} = 0 \quad (13)$$

$$\begin{bmatrix} \frac{\partial L}{\partial h(0,0)} \\ \frac{\partial L}{\partial h(0,1)} \\ \vdots \\ \frac{\partial L}{\partial h(n_1, n_2)} \end{bmatrix} = 0 \quad (14)$$

The Lagrange multipliers in (14) are achieved by finding the eigenvectors corresponding to the polynomial on the left side of the equation. Since the proposed method aims to design a 2-D FIR filter whose filter coefficients are all real values, the eigenvectors which are all real values are considered. Each selected eigenvector is

applied to solve the parts of equation (13) individually. The results of the partial derivatives in (13) can be expressed as

$$\frac{\partial L}{\partial x_r} = \begin{cases} 2x_r - \lambda_r & \text{for stopband region} \\ 2x_r^3 - 4x_r - \lambda_r & \text{for passband region.} \end{cases} \quad (15)$$

The eigenvalues derived from the previous step are substituted back into (15). The value of x_r in the stopband region is simply solved and there is only 1 value of x_r . For the passband area, there are obviously 3 roots of x_r for each iteration. All roots of x_r in the passband region will be used to solve the filter coefficients in (6).

Rewrite equation (6) as

$$A\bar{\mathbf{h}} = \bar{\mathbf{x}} \quad (16)$$

where

$$A = \begin{bmatrix} 1 & 2\cos(\omega_{11}0 + \omega_{21}1) & \dots & 2\cos(\omega_{11}n_1 + \omega_{21}n_2) \\ 1 & 2\cos(\omega_{12}0 + \omega_{22}1) & \dots & 2\cos(\omega_{12}n_1 + \omega_{22}n_2) \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 2\cos(\omega_{1k}0 + \omega_{2k}1) & \dots & 2\cos(\omega_{1k}n_1 + \omega_{2k}n_2) \end{bmatrix} \quad (17)$$

,

$$\bar{\mathbf{h}} = \begin{bmatrix} h(0,0) \\ h(0,1) \\ \vdots \\ h(n_1, n_2) \end{bmatrix} \quad (18)$$

and

$$\bar{\mathbf{x}} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \end{bmatrix} \quad (19)$$

Then, the solutions of the filter coefficient $h(0,0), \dots, h(n_1, n_2)$ are

$$\bar{\mathbf{h}} = A^{-1}\bar{\mathbf{x}}. \quad (20)$$

ince there are 3 roots of x_r for each iteration in the passband, the number of filter coefficient solutions will be 3^q sets where q is the number of the sampled points placed in

the passband area. All sets of filter coefficients will be used to construct a 2-D FIR filter. The best sets of filter coefficients will be analytically selected based on their minimum mean square error (MMSE) performances of the resulting filters.

Simulation Results

In this section, a comparison of frequency-sampling design methods is demonstrated. Both filters are designed to have a 5×5 impulse response. The first example illustrates a 2-D FIR low pass filter with a zero-phase design by our proposed method. This simulation is implemented in MATLAB software to illustrate the performance of the proposed method. Several sets of sample locations are chosen, such that they are well distributed. A set of sampled points that can generate a nonsingular matrix A defined in (17) will be selected. Consider the case where $N_1 = 2$ and $N_2 = 2$. The number of the sample points is equal to $(N_1 + 1)(N_2 + 1) = 9$. The selected sample points for this design example are demonstrated in Figure 1.

This filter is designed to have a unity gain in the passband region $0 \leq \sqrt{\omega_1^2 + \omega_2^2} \leq 0.5\pi$ and a 0 gain in the stopband region. $0.5\pi < \sqrt{\omega_1^2 + \omega_2^2} \leq \pi$. With the sample locations shown in Figure 1, the cost function, $C(\cdot)$, for this design problem can be constructed as

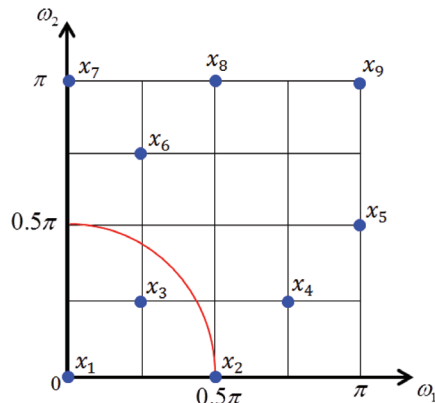


Figure 1. Non-uniform frequency sampling points for low pass filter

$$C(x_1, \dots, x_9) = (1-x_1^2)^2 + (1-x_2^2)^2 + (1-x_3^2)^2 \\ + x_4^2 + x_5^2 + x_6^2 + x_7^2 + x_8^2 + x_9^2$$

$$C(x_1, \dots, x_9) = (1-x_1^2)^2 + (1-x_2^2)^2 + (1-x_3^2)^2 \\ + x_4^2 + x_5^2 + x_6^2 + x_7^2 + x_8^2 + x_9^2 \quad (21)$$

Thus, the Lagrangian of this problem can be written as

$$L = (1-x_1^2)^2 + (1-x_2^2)^2 + (1-x_3^2)^2 + x_4^2 + x_5^2 + x_6^2 + x_7^2 + x_8^2 + x_9^2 \\ + \lambda_1(a+2b+2c+2d+2e+2f+2g+2h+2i-x_1) \\ + \lambda_2(a+2b+2c+0d+0e+0f-2g-2h-2i-x_2) \\ + \lambda_3(a+1.4142b+0c+1.4142d+0e-1.4142f+0g-1.4142h-2i-x_3) \\ + \lambda_4(a+1.4142b+0c-1.4142d-2e-1.4142f+0g+1.4142h+2i-x_4) \\ + \lambda_5(a+0b-2c-2d+0e+2f+2g+0h-2i-x_5) \\ + \lambda_6(a-1.4142b+0c+1.4142d-2e+1.4142f+0g-1.4142h+2i-x_6) \\ + \lambda_7(a-2b+2c+2d-2e+2f+2g-2h+2i-x_7) \\ + \lambda_8(a-2b+2c+0d+0e+0f-2g+2h-2i-x_8) \\ + \lambda_9(a-2b+2c-2d+2e-2f+2g-2h+2i-x_9) \quad (22)$$

where

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} h(0,0) & h(0,1) & h(0,2) \\ h(1,0) & h(1,1) & h(1,2) \\ h(2,0) & h(2,1) & h(2,2) \end{bmatrix} \quad (23)$$

After solving the optimization problem following the procedure described in the previous section, the optimal impulse response of the designed filter is shown in (24).

$$h(n_1, n_2) = \begin{bmatrix} 0 & 0 & -0.0802 & -0.0458 & -0.1028 \\ 0 & 0 & -0.0752 & -0.1501 & -0.0728 \\ -0.0297 & -0.0999 & -0.1233 & -0.0999 & -0.0297 \\ -0.0728 & -0.1501 & -0.0752 & 0 & 0 \\ -0.1028 & -0.0458 & -0.0802 & 0 & 0 \end{bmatrix} \quad (24)$$

where $n_1, n_2 = -2, \dots, 2$

The frequency response of the designed filter is obtained, as illustrated in Figure 2.

The second design example illustrates a 2-D FIR low pass filter design by the non-uniform frequency sampling method proposed in Rozwod *et al.* (1991). This filter is also designed to have a unity gain in the passband region $0 \leq \sqrt{\omega_1^2 + \omega_2^2} \leq 0.5\pi$ and a 0 gain in the stopband region $0.5\pi < \sqrt{\omega_1^2 + \omega_2^2} \leq \pi$. The arrangement of the sample points is demonstrated in Figure 3 and the frequency response of the designed filter is demonstrated in Figure 4.

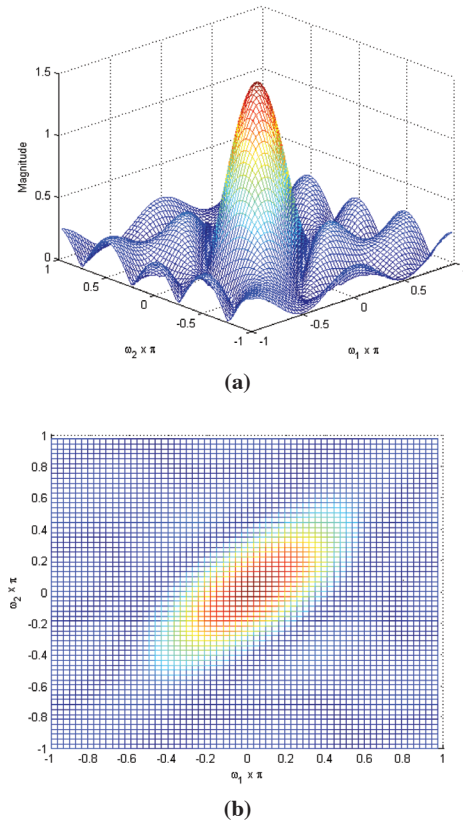


Figure 2. (a) Magnitude response of low pass filter designed by the 5×5 proposed method and (b) Contour plot of the magnitude of the designed low pass filter frequency response

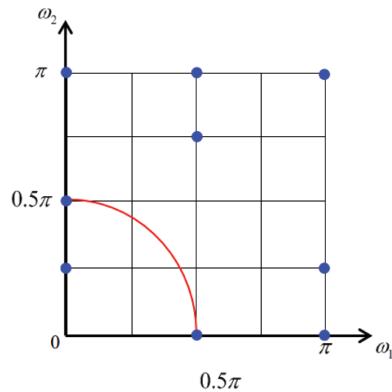


Figure 3. Non-uniform frequency sampling points for a low pass filter design by the method in Rozwod *et al.* (1991)

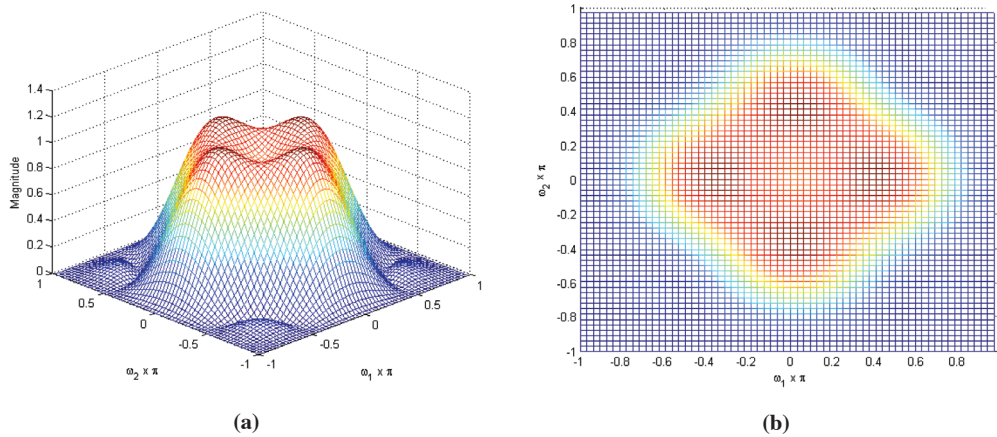


Figure 4. (a) Magnitude of 5×5 low passfilter designed by the method in Rozwod *et al.* (1991) and (b) Contour plot of the magnitude of the low pass filter designed by the method in Rozwod *et al.* (1991)

The performances of these designed filters are evaluated by their MMSE. The reference is the ideal low passfilter, which has the same passband and stopband region. The MMSE of the filter designed by the proposed method, shown in Figure 2, is 35.6947, while the MMSE of the filter designed by the method in Rozwod *et al.* (1991), shown in Figure 4, is 35.8786, which is very similar.

Conclusions

This paper presents a method for a 2-dimensional FIR filter design based on a non-uniform sampling approach. The principal advantages of the proposed method are the flexibility of the sampled point placement and the optimal solution of the resulting filter. This method also yields less computational complexity. Using the symmetry condition for a 2-D zero-phase filter can reduce the number of polynomial equations to be solved. This design method is quite straightforward in terms of the matrix operations. The Lagrange multiplier technique is employed to solve the optimization problem for finding filter coefficients. Since a unique solution is not achieved by the proposed method, the best solution of filter design will be analytically selected. The design example for a 2-D FIR low pass filter is also illustrated to demonstrate

the effectiveness of the proposed method. The comparison between the 2 different approaches is also demonstrated. With the same size of impulse response, the proposed method gives a little better performance than the prior method (Rozwod *et al.*, 1991). In the future, the proposed algorithm may require a method for choosing the sample points and new constraints to be imposed, in order to enhance the computational efficiency.

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